Defects in Crystalline Solids (Part -I)
Prof. Shashank Shekhar
Department of Materials Science and Engineering
Indian Institute of Technology, Kanpur

Lecture - 21
Stress and Strain Fields

So, last time we derived the strain relation for a screw dislocation.

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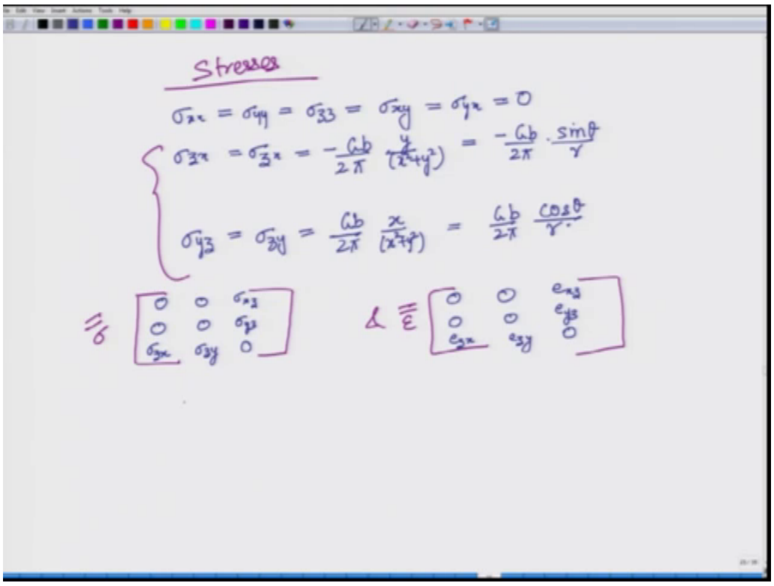

And then based on that we also got to the relation for stresses around a screw dislocation, and we found that there will be some zero quantities, and there will be some non-zero quantities. So, if we write it in the tensor format, how would these look like. Now, I left it empty, hoping that you would fill in.

So, let us see if you have gotten it right. So these four terms are 0 for stresses, and as you can see sigma x z; and if sigma x z is there, then sigma that z x must also be there. Then we have sigma y z, and therefore sigma z y should also be there. And, what about the sigma z z that is also 0. So, we have thus this gives the tensor format for the stresses around screw dislocation.

Now, how would it look like for the strain, matrix or the strain tensor; these are 0, this is e xz, this is e yz, this is e zx, e zy, and so it is a similar to the stress matrix or the stress tensor. So, these are the two stress and strain tensors for which depict the field around a screw dislocation.

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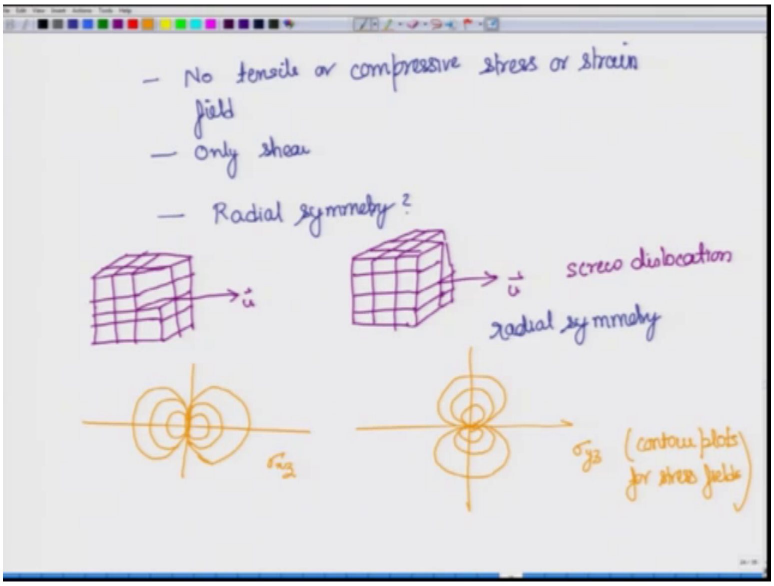

So, what do we see? There are certain points that we can note down from here that there are note what are the zero components that we looked over there, those are actually tensile and compressive. So, there are no tensile or compressive the diagonal matrix that which is which represent that tensile or compressive the normal stresses. So, there are no tensile or compressive stress or even strain field. What we have is only shear that two also only in certain direction, we do not have any tensile or compressive.

Now, another thing is what do you note about the radial symmetry, should we have a radial symmetry or we should not have a radial symmetry; what does the matrix tell us. In the x, y, z the coordinate system that we have used what we see is that we do not see radial symmetry, but is it expected, do you think that there should not be it should be any radial symmetry or not. Let us look at our old drawing again.

Now, if you had remember what we said that the screw dislocation the cut can be anywhere, which would mean that the radial symmetry should be there; meaning that the cut is not really defining any particular direction. And therefore stresses and strains should show a radial symmetry or we can also depict it using the diagram like this. I have showed you this diagram earlier also, but what I am trying to show here right now is that this particular diagram drawing that we have been using, does show shows that it should have a radial symmetry, meaning there should not be any particular line direction or any particular cut direction.

So, for example, here it may look on the first appearance that this is some particular direction, which can be taken as a reference, and there therefore there cannot be any radial symmetry. But, let us look at it, we can also draw it something like this. So, what we see is that both of these drawings represent the same screw dislocation. So, first try to convince yourself that this is thus one and the same screw dislocation that is being represented in the two diagrams.

And now, when you realize that this is the one, and the same is screw dislocation. What you will realize is that there is no reference direction. Meaning that this particular around this direction which is the line direction, there is no particular perpendicular to this direction, there is no particular line or direction which can be taken as a reference, meaning all directions are same, which what does that mean that there should be or by default there should have been some radial symmetry.

But, if you look if we go by the stress and strain field, and we draw if we want to draw, all they would look like. Let us see how would they look like. So, we have the two components sigma x z and sigma y z. If we try to draw them, they will look like something like this. So, one side, you will have the positive. So, let us say this is sigma x z. For sigma y z, it will not be very different; it will be just that the orientation would be different, it will now be along y direction. So, what I am drawing here are contours contour plots for stresses. So, each line here represents a particular value of stress, so each particular value of sigma y z. And in one direction, it is positive; in the other direction, it is negative of almost similar value.

So, what you see here is that from this drawing it looks like, there is no radial symmetry. But, our earlier drawing, when we draw it like this, and also we have discussed earlier that the cut when we use the voltra diagram or the voltra model, the cut can be anywhere. And therefore there should be a radial symmetry. So, you must think about how or why are we not getting radial symmetry, but you know we will get a get back to that a little later. For now, let us come back to our; what are the other points that we can understand from what we have from the stress relations that we have obtained. So, what are the other things that we see?

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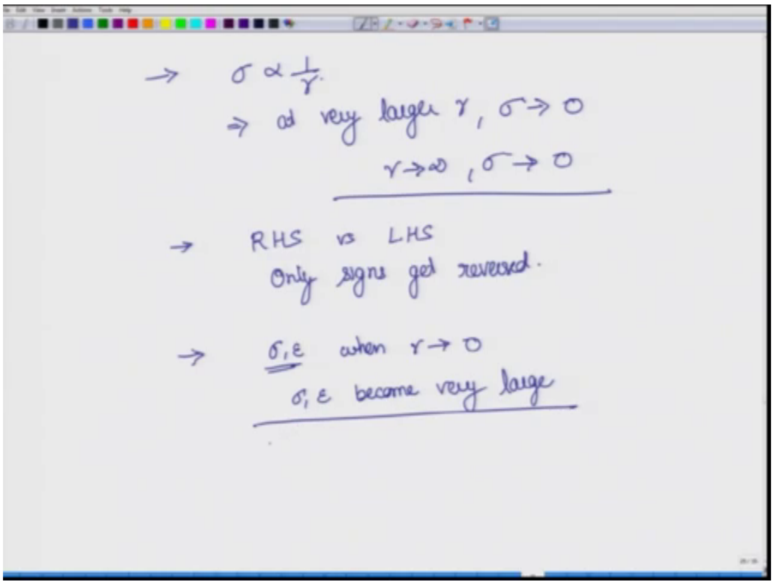

We see that the stress is proportional to 1 over r. Now, what does that imply, it implies that at very large r stress tends to 0, meaning as r tends to as r tends to infinity, stress tends to 0. Now, this is something that we described that we said earlier is actually the definition of a dislocation as compared to a disclination.

So, in disclination, the stresses and strain fields do not go to 0 at large distances, while in dislocation; at very large distances, they do go to 0. So, this our relation that we have obtained, we can say it is cross verified, when we see that it is proportional to 1 over r, which means that at very large values of r, it will it will shrink to 0, it will it will come down to 0.

We have a described it, but whatever stress and strain fields that we have described, we have described it for one type of screw dislocation, let us say it was for left handed screw dislocation. What will happen in the case of right handed screw dislocation compared to left handed screw dislocation; only the signs will reverse. As you can cross check yourself, you can go back and describe and derive these relations what you would see is that what changes when you grow from left hand screw to right hand screw is that only signs get reversed.

Another point that we should observe from what we have derived about the stress and strain field, what will happen to these stress and strain, when r tends to 0. So, you have seen that stress and strains are proportional to 1 over r. Now, that would mean that when a stress and that when r tends to 0, what will happen to these, these will become stress and strain become very large ok.

So, we will come back to this, but what I will let me point out at this point that what this implies is that at such at some point, our assumptions that linear elasticity principles hold will become invalid. Therefore, when we reduce the r, there is certain value of r below which these relations will not be valid that is what this implies. When you keep reducing r, stress and strain become very large, they become so large that our initial assumptions that we used when deriving these will not hold. And therefore, these cannot be used for all the way up to 0, there is there is has cut off radius. And we will see what that cut off radius is later on.

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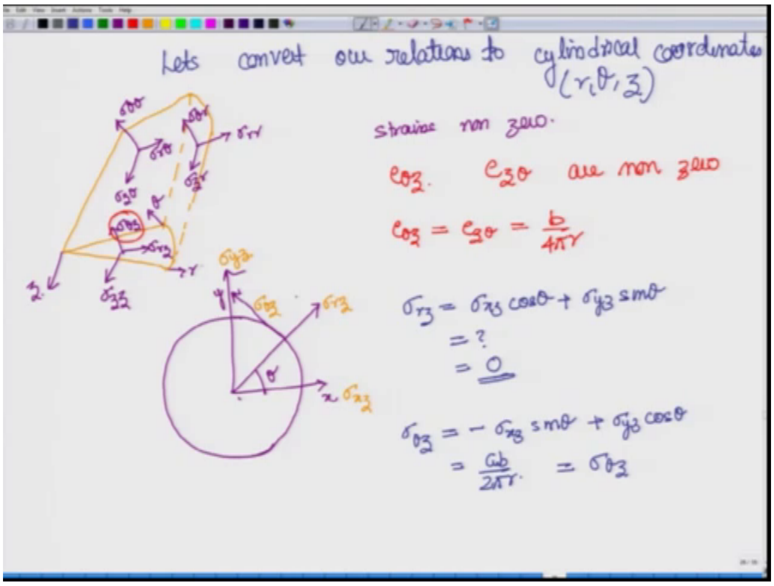

But, for now we are interested in understanding, why we do not see radial symmetry; so, for that what we will do is let us convert, whatever we have obtained in x y, z coordinate to r, theta, z or cylindrical coordinate system. So, let us convert our relations to cylindrical coordinates ok. So, before we move to r, theta, z; let me show you how would this similar to the x, y, z there will be different normal stresses, and there will be shear components.

So, let us describe or define it. These are standard definitions we are not going to the details of it, we are just showing how it is commonly used. So, this is a section from your screw dislocation or from that ultra-model only that it is we are taking a cylinder not a hollow cylinder. So, we have this small slice out of this, and so this is our theta direction, this is our r direction, and this is our z direction. So, this is z, and this is r. And if you go in this direction, then it will be r direction sorry this will be theta direction.

Now, let us look at the different components if on if you look on this phase, you will again have three components just like in x, y, z coordinates. So, this is theta phase. So, this is sigma theta theta, this will become sigma r theta, this will become sigma z theta. Now, this is the z phase. So, here this will become sigma z z, this will become sigma r z, this will become sigma theta z. Now, this is the r phase. So, over here the three components are with respect to the r phase. So, this is normal r r, shear sigma theta r, this is sigma z r.

So, now first let us try to understand; what are the components that will be 0. And remember that I have shown it with respect to sigma, but similar thing will be true for (Refer Time: 14:25) we had taken epsilon or which is the strain. So, we could have similarly pointed out what are the different kinds of strain epsilon theta theta, epsilon r theta, epsilon z theta over here, it would be epsilon theta z or z and z z and so on.

So, first let us look at what are the components, which will become zero in the strain. In fact, it will be easier to say which are non-zero. So, let us look at non-zero. Now, if you look over here, which are the particular directions, where you saw that there was some amount of strain was there any strain being, where this was being pulled out in the r direction. We know there was nothing like that was there anything normal in the theta theta direction, nothing like that, was there anything in the z direction nothing like that. There was if you remember that it was a helical type of structure, so there was a small amount of strain in the z direction and along the theta phase.

So, what we had as non-zero component was this one sigma theta z. And if sigma theta z is non-zero, then to balance if this is the shear stress sorry so I am talking about stress here strains here. So, this is epsilon epsilon theta z is non-zero, which means epsilon z theta is also non-zero. And it is not very difficult to convince that all other terms in terms of strain would be zero. The only two non-zero components are the ones that I have already mentioned over here. And you would also be in a position to show that this based on the older drawing that we had used that this will be equal to b by 4 pi r.

Now, let us get to the stress the stress relation, which will be non-zero. Now, again let us look at, so we said that there are strain along the z direction, there was some amount of strain along the z direction. So, let us draw the drawing with where we can relate x, y, z with the r, theta, z. So, this is our x direction. And the z direction will be along on that perpendicular to this circle that we have drawn. So, what we have drawn here is a circle or section of which is with the z in the z direction is shown over here. So, the sigma x z which direction would the sigma x z line, sigma x z would lie along this sigma y z. Sigma r if we look at sigma r z it should lie, it will not be dependent on x and y, it will anywhere along the radial direction, if you go out that will be sigma r z.

Similarly, sigma theta z if you go along anywhere along the circle, where you take a tangent, and that will represent sigma theta z. So, this is a tangent, let me draw it more accurately. So, what we are trying to do here is now we are we have already seen the relation for strain. Now, we are trying to do the similar kind of analysis for stress. So, we are trying to relate different stresses with respect to what we already know, which is x z and y z. So, sigma x z and y z are the only non-zero component.

If you look at the x, y, z coordinate system, and we have already said that there was no z direction strain. So, we have looking only the x y plane. So, now looking at the x y plane, we have sigma r z and sigma theta z. And now if we try to find a relation, you can show that or we will see that sigma r z is nothing but the sum of the components from x z on to this and y z on to this. And we have value theta here that there. The angle here is theta. Therefore, if you know the theta, then you can find a component of x z in direction r z. Similarly, you can find component of y z along r z, and it will become sigma x z cos theta plus sigma y z sine theta.

Now, what is this equal to you go back, and you will see that it had sine theta term, they said y theta term everything else was same. So, now what it has become is a quantity into sine theta cos theta minus quantity into sine theta cos theta, because it had also a negative sign. Therefore, this will boil down to 0. And as we expected sigma r z is also 0, we should have already predicted looking at the strain values e theta z and e z theta are only non-zero component.

So, now let us go to the sigma theta z sigma theta z is along this direction. So, what we get here is the negative component of sigma x z in that direction minus sigma x z sine theta plus sigma y z cos theta. This is simply taking components again for x z and y z along z theta. And what you get is G b by 2 pi r and this is also equal to sigma theta z, because this is a shear stress. So, there has to be a balancing stress, and this is the sigma theta z component. So, sigma theta z is equal to G b by 2 pi r.

And now as soon as you see this relation, what you would realize that we have found our radial symmetry. So, this is we were not able to see the radial symmetry, because we were looking in the x y coordinate system, when the strains or the when the stresses had a radial symmetry, which can be described only when we are when we are using cylindrical coordinate system.

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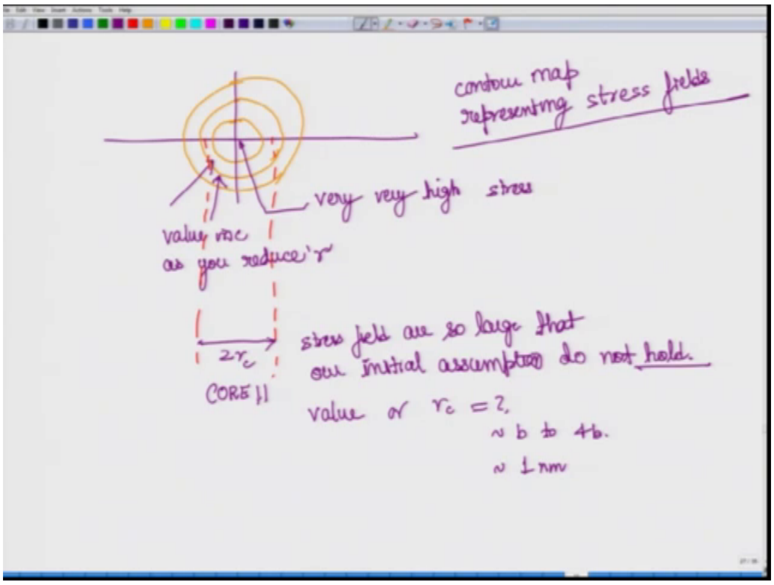

Now, if you look at the stress field, which will be similar to the strain field; how would it look like now. So, what are these lines as I said earlier, these are contours representing a particular stress level, if we are talking about a stress field. And a strain level, if you are talking about strain fields. So, these are in increasing levels of stresses or strains, and these are contour map, so let me write here.

So, for now I will restrict it to stress field. So, contour map representing stress field. And it is it should be obvious that we are increasing the stress as we are keep going inside the radius. So, this is higher value, this is a lower value, this is a lower value. So, you can imagine that it is like a cone, which is dropping like this. So, there at the core you have a very large value, and the value is dropping as you go away from the centre point, which is x equal to 0, y equal to 0.

And this at this point, so like I said we are restricting our self to stress fields, we the strain fields will also be similar to this. And the values are increasing as you keep going inside. So, at certain point, you would realize that the strains or stresses have become so large. So, let us say somewhere around this. Somewhere at this 2 r, let us call it critical value.

Stress fields are so large that our initial assumption do not hold anymore ok. So, there must be a critical radius will under which if you try to calculate the stresses or the strain. Then our initial assumptions are not valid, which means that we should not use our regular model of calculating stress and strain field over here. And this particular the region is called core of a dislocation. And this part the energy is stored in a different way, it is the core energy.

The energy outside it is the elastic energy, and the energy inside it is called the core energy. It is stored in a different way or in a using a you will have to use a different model to understand the energy stored in this particular region. So, what is this value of r critical? If you remember, we earlier said that it is somewhere of the order of b to 4 b or another as another approximation is one nanometre.

So, depending on what is the condition you are taking, you can take r critical or the r radius for the core as b to 4 b or you can take it approximately one nanometre. So, within this core you will have a different model or you will have to calculate the energy separately. And outside it the energy would be calculated the way we have been calculating so far. So far we have what we have looked at or the stress and strain fields around a screw dislocation.

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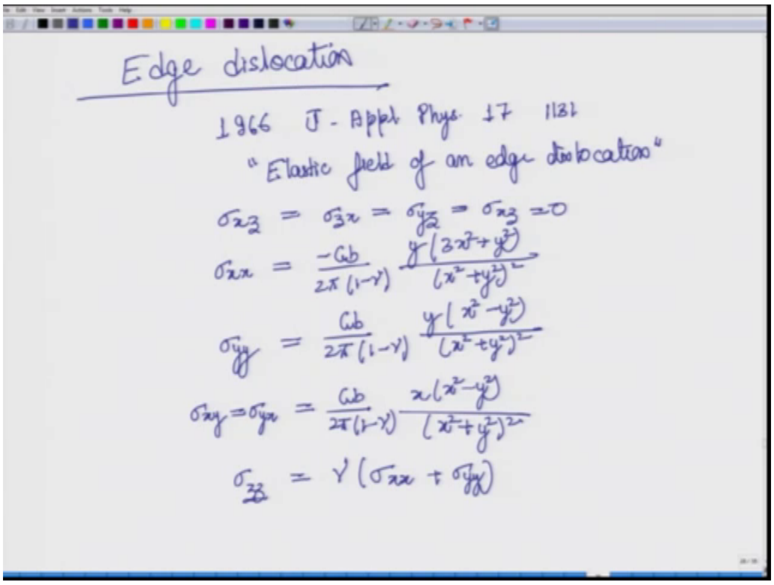

Now, there is also a derivation for edge dislocation. It is not as straightforward as the one that we described about the screw dislocation. However, it is not very complicated either. So, if you are interested, you can go back and read this very classic paper in Journal of Applied Physics and the topic is Elastic field of an edge dislocation.

So, here they have in this particular work, they have discussed the elastic field for edge dislocation. You would be able to calculate the strains, they have basically they have derived the strains strain fields around a edge dislocation, stress fields around a edge dislocation. And we will not go through it, but I will simply give you the values of those strain and stress fields that you have obtained and it comes out like this.

So, these are the components, they have found turns out to be 0. So, a lot of these shear stress components in the edge dislocation are 0. And what is not zero are this one of them is the x x, the normal stress along the x direction. And this has been derived to be now in this particular relation, you would see that a factor Nu creeps into the relation. Nu is the Poisson ratio, which defines how much the lateral deformation or lateral dilation or compression would be if you are compressing in one direction, so because the volume has to remain conserved, if you deform it in one direction, there will also be simultaneous deformation in the other direction.

And that ratio is given by this parameter nu which is Poisson ratio and because we are talking about normal stresses. This ratio creeps into this edge dislocation relation. So, this is sigma x x, now this is sigma y y, this is the normal stress along the y direction. Again you would see whenever we are writing the normal direction will have the new parameter.

In fact, even in the shear stress for edge dislocation, you would see there is new parameter, because in here we have a very large normal direction because of which there is also contraction in the other directions. So, we have x x y y and we have x y or y x. And since, there are x x and y y you can also go back. And see that there has to be a z z component, which will again be related to this parameter nu Poisson ratio.

So, these are the relations for edge dislocation. So, we will leave it here, and come back continue with this in the next lecture.