Fundamentals of Materials Processing (Part- II)

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Lecture – 04
Strain Tensors and Mohr Circle for Strains

So last time we looked at Mohr circle in our general case which is still drawn over here and then we also looked at Mohr circle which had only compressive stresses.

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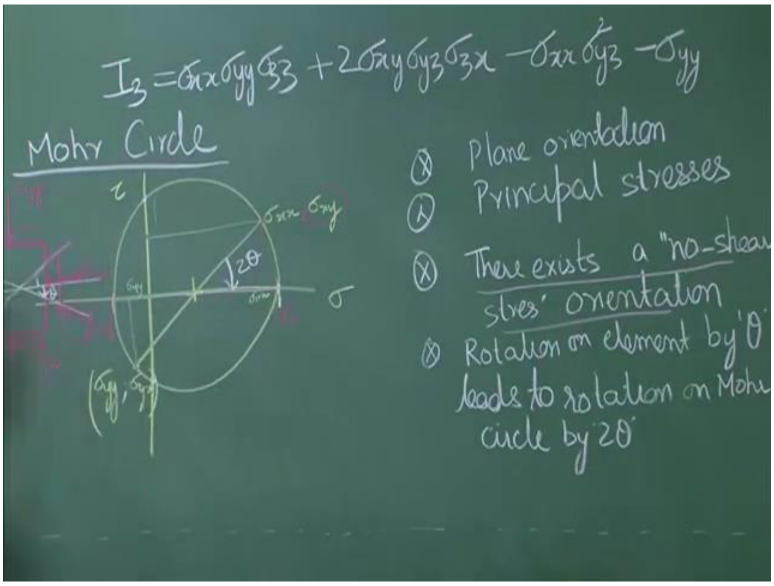

Then we looked at Mohr circle which had only tensile stresses and we also said that there exists a no-shear stress orientation, which we said is this one when you have the diameter or the circle cutting the x axis. Now the other question we want to answer over here, does there exists a plane where no normal stresses exists. So, in general of course, they do not exists such plane but let us look at a particular condition, what will be that particular condition.

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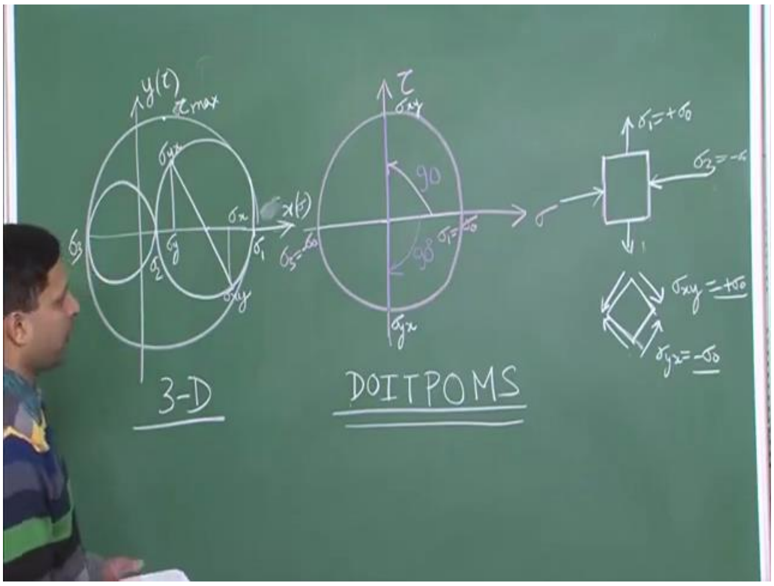

So let us draw it like this, so we have a Mohr circle; you remember what is the x axis, x axis is normal stresses, y axis is shear stresses tau. Now if we take a particular element which is in a condition such that the normal principle stress is sigma naught which is equal to sigma 1 and again we are looking at two dimension and sigma 3 is equal to minus sigma naught meaning one principle stress is equal in magnitude to the third principle stress, but their signs are different, meaning one is the tensile the other is compressive.

Now in that case what happens, this y axis is now bifurcating this circle. So first let me draw that small orientation, when I say this what I have is like this. So, this is sigma 3 compressive which is equal to minus sigma naught and we have sigma 1 equal to plus sigma naught, this is in tensile condition. Now if I rotate; this is this line represent orientation of that element, if I rotate that line or the orientation such that the orientation is now represented by this line; see this the diameter still crosses through the x axis thus 0 0. So, now our new orientation is this and here I had to rotate, I can say counter clock wise 90 degree or clock wise 90 degree meaning I am rotating this element by half of that 45 degree. So since this is square element, I can rotate it this way or that way; it does not matter, which is also reflected in the Mohr circle, so now my element is represented like this.

Now in this particular condition what has happened is I have a clock wise shear stress. So, if I call this as sigma xy this is positive and clock wise is called as represented by positive, so this is sigma xy and the other one is anti clock wise, so this is my sigma yx. So, this is sigma yx and guess what; what is the magnitude, can we say from this; yes we can say, this is equal to minus sigma naught, this is negative and this is equal to plus sigma naught; why because this is the diameter.

So 2 sigma naught is the diameter and over here also the diameter will of course; for the circle remains constant, so this length is represented by 2 sigma naught and therefore, this is half of that which is plus sigma naught and this is minus sigma naught. So our shear stresses; we know the magnitude of these shear stresses and we also know that there does not exist any principle stress or any normal stress in this particular element. So, this is a particular condition or a particular case you must be remember; not every element in a material or in a component which is being deformed or under some stress will have this kind of case.

But you may be able to find some elements where you have positive stress equal to negative stress at that particular point and no shear stress existing then you can rotated by 45 degree and the same amount of stress actually there is the shear stress and there is no normal stress acting on it at that particular point. Now since we are talking about this Mohr circle, if you are interested because this is a very interesting way to understand stresses under very simple way to understand stress, you must visit this website; you can just search on your Google or some search engine DOITPOMS, this is the website by some of the academic institutes in UK.

So, it is Dissemination of IT for Promotion of Material Science, so you will get not only information about this and many other related material science, material engineering related concept but they also provided it in a very; you can say animated way. So you will get lots of animation, you can rotate, you can change theta angle, you can change sigma values, you can change sigma x y value and see how the circle changes and so you get; to get a feel about the numbers and the construction of this Mohr circle.

So, I will strongly advise you to visit DOITPOMS just search on it and you will get, it is DOITPOMS dot ac dot uk, if I remember right and you will be able to get not only information about this like I said, you can also get many other in a information related to manufacturing and characterization and other processes.

Now, we have looked at a two dimensional case so far remember, so we had no stresses acting in the third direction. How would the Mohr circle look like? When you have the third circle or the third direction also having some stresses that is not very difficult to judge or difficult to say from here actually, it is a just simplified or extended version of this. If you have a three dimensional stress condition, it is not represent actually it is not represented by a sphere, but still by circle but now instead of one circle, you will have three circles.

So the x axis still represents normal stresses, y axis still represents your shear stresses and over here this will represent your sigma 1, sigma 2, sigma 3 and if you look at this, you will be able to get for a particular orientation; what is the value of sigma x, sigma y, sigma xy, sigma yx and so on and this will represent the maximum shear stress that exists. So, now you have three principle stresses being defined; sigma 1, sigma 2 and sigma 3 when you draw it like this. So, in this particular case we have three dimensions stresses in even outside the plane, so in that case this is how you will draw and again you can get the values of principle stresses using this. We will not get into this again like I said you can find out, once you have the proper understanding of this extension two, three dimensions would be natural and it will be very easy. So, you can look about it in the books or even on like websites like academic websites like this.

So we will move on from here to our next part which is strains, so we have looked at Mohr circle for stresses so far, we will now try to understand a little bit about strains. Remember stresses were a tensor quantity, what about the strains actually strains are also tensor quantity and most over thing that we have described so far can again be very easily extended to our understanding for strain.

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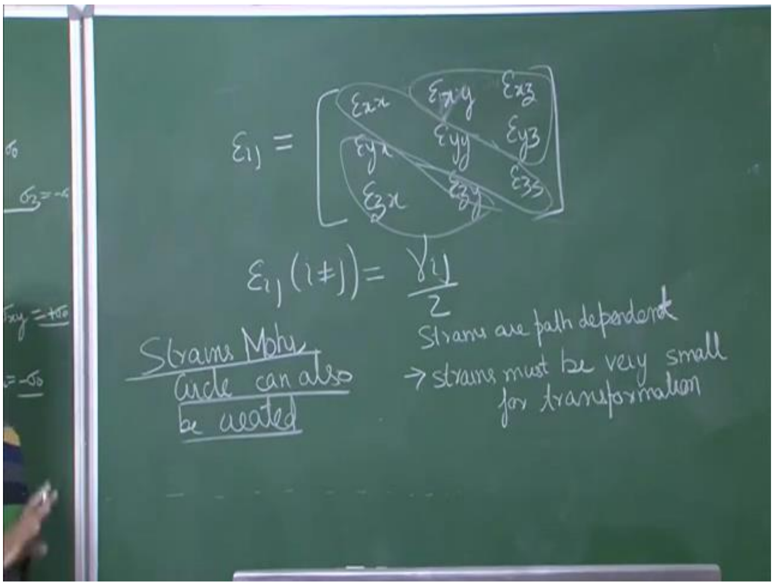

All you need to understand first of all is that strain would be a tensor given like this. So, again we have some components, some normal strains and some others are shear strains, so these are our shear strains. Again there is this similarity with stresses that we discussed earlier, another thing that when we are talking about the streams that you need to be aware of is that when we are talking about the shear strains when i is not equal to j, what you measure is usually gamma and that gamma i; j by 2 phase, what is taken in this tensor matrix, tensor table.

So, epsilon i j where i is not equal to j is actually gamma i j by 2 that is one thing that you need to be aware of and another thing is that you know, you must be aware that strains are path dependent which implies that if you want to do some kind of transformation on these tensors it is actually you have to be very careful. We will look at some of those constrains later on, but for the time being it is (Refer Time: 11:39) to say that the strains must be very small for any such transformation, mathematical transformation. If you are transforming from one axis to another or something or your doing some kind of rotation then in those cases you must check that the strain are sufficiently small, otherwise the transformation where the values that may get may not really be a true representation of strains in that particular condition.

So, other things that we talked about like Mohr circle, so for strains also you can create Mohr circle. So, the Strains Mohr circle and the Stress Mohr circle as long as the strain values that we are talking about are very small. So, just remember these values must be very small and if we have those values are very small then we can create circles just like that we created for sigma and just treat it as if it works similar kind of treatment. You can rotate it; you can get the values all as long as the values of the strains are small enough.

So, strain Mohr circle can also be created if you just as good and there will be as good as the Mohr circle that we created for stresses. So, you can have principle strains, you can have strain normal strains and shear strains, you can get particular orientation in which you have only normal stresses which are called the principle stresses; sorry principle strains which will be only have normal strains and you can in particular cases you can get only shear strains. For example the condition we saw over here, where we have some positive strains in the normal direction and the negative strains in the other direction. So, all those kind of manipulation and understanding can again we extended to strain Mohr circle.

A few more things before we move on to something more or not more important but something where we will use these as the tool.

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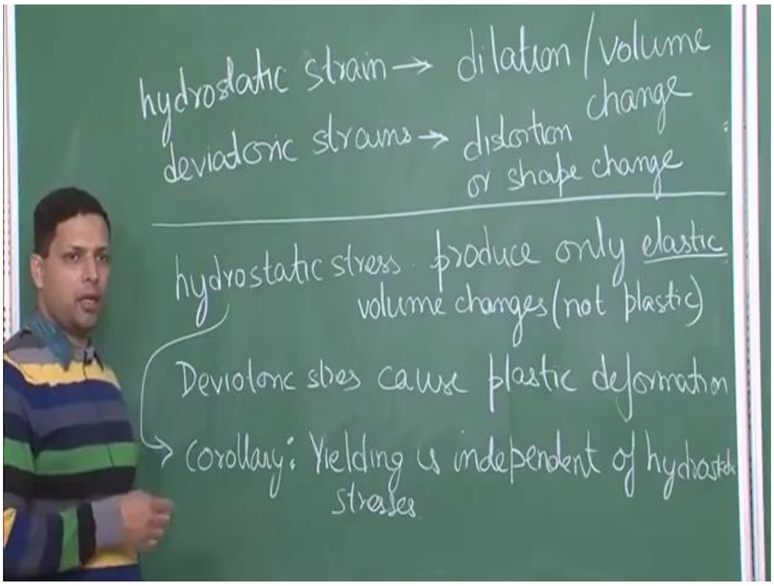

Hydrostatic strain, what is the hydrostatic strain; hydrostatic components are the diagonal components and (Refer Time: 14:36) component are the non diagonal elements. So, hydrostatic strains are the one which cause dilation or volume change and the deviatoric strains because we are going to deal with deformation, so we need to understand which component actually causes volume change and that is why we are listing it out here. So, there are two main components of strain; hydrostatic strains and deviatoric strains.

So, hydrostatic strains are the ones which we call as dilation or volume change. On the other hand deviatoric strains or kind of shear strains, will cause distortion or shape change that is for about strains. Now let us look at hydrostatic stress, again the diagonal components are the hydrostatic stress. Hydrostatic stresses produce only elastic volume changes; not the plastic. So, we have a explicitly said it only elastic volume changes; not plastic, but when we are doing deformation which is the kind of deformation or the volume change that takes place, not the volume change the deformation is plastic in nature.

So, the plastic deformation that takes places must be deviatoric in nature, so deviatoric stress cause plastic deformation. If you remember your tensile stress strain curve then we have the elastic region and the plastic region and these two are separated by what is called the yield stress. So, in other words we can say whether the yielding in a material, so the corollary of these two is yielding. These are very important statements that you must remember if you are dealing with metal forming.

Yielding is independent of hydrostatic stress, so this like I said these are very important statements which will be converted into mathematical forms or equations which will help us get to our goal and what is our goal will see, I mean we know that, but we will put it in more in a clear expressive towards very soon. So, the corollary of this is first is that yielding is independent of hydrostatic stresses. So, you may have very large hydrostatic stresses but it does not mean that the material will yield, it will deform plastically. Only when you have deviatoric stress; components in element, in the material that it will actually start to deform. Now having said that let us try to understand what is yielding, we have so far always been talking about deformation and deformation is plastic in nature and that plastic deformation takes place once the materials start to yield.

So now let us look at yielding we will start again with our simple understanding that we have already built over the years from our high school study, high school knowledge and our undergraduate knowledge which is of another uniaxial tensile stress.

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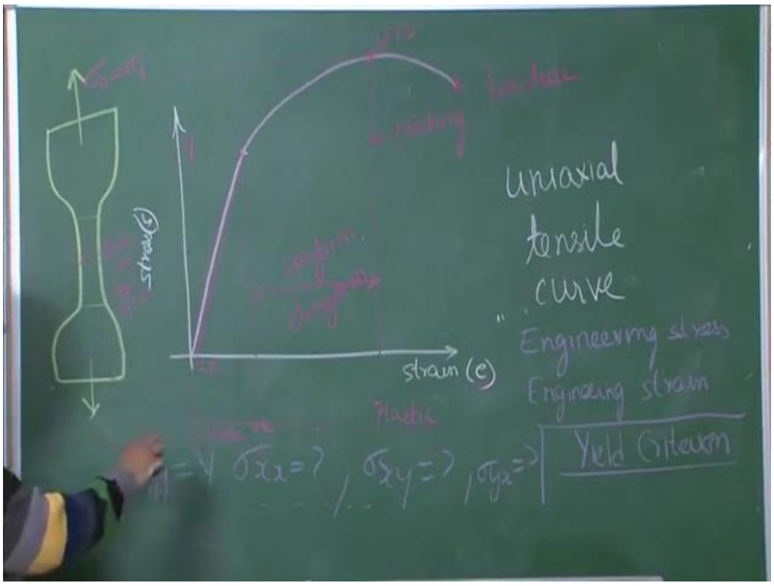

So, we have, so this is the uniaxial curve, on the y axis you have the strain and the x axis here the strain under in the y axis you have stress and how does this plot look like. So, this is our simple tensile stress strain curve. What are the different components of it, we know one of this; this is what we call as our proof stress. At 0.2 percent at the 0.2 strain, if you make a parallel line along with the Yang's modules of the or the slope of the stress strain curve in the elastic region, then it will start to intersect the curve at some point and it is this point which is defined as yield stress or y.

So, it is a stress value at which the material starts to deformed and again let me for the sake of completion, let me draw a simple. So, this is simple (Refer Time: 20:47) which you may be stretching in tensile condition. So, you have put it in a UTM which is the Universal Tensile Machine and you are stretching on the using two cross sides on to these side and this is the region which gage length, which undergoes deformation and it is in this region that we are looking at all the stress and the strain part.

Over here up to this point, we say this is elastic and beyond this the material is plastic. Now when we are interested in deformation, we are always interested in the plastic zone of the material; not in the elastic zone. So, this is the region that we are interested in beyond this 5. And that is why we are interested to know where does this yielding take place; now here in a uniaxial tensile test, if this is your sigma naught, this is also equal to your sigma.

Other sigma 2 and sigma 3 are 0; sigma xy and sigma yx also does not exist, therefore it is very easy; we are just concerned about one particular stress value which is sigma 1 or sigma yy it can be. So, we know the all other stresses are 0 and therefore we can just look at this value; this value reaches y, this material is yielding, but do you remember when we are talking about a Mohr complex shapes, the stress conditions are different, there is not only one stress anymore; it is not only sigma yy or sigma xx, there may be as many as seven different components and therefore, there we need to find out what particular condition, what will be the values of those at which it will start to yield or it will start to deform.

But before that let us look at some more of these parameters, this is the point where which is called UTS or Ultimate Tensile Strength. So, this is again as stress value and the strain related to this is also unique; not unique, but it also very special or it has some significance. It only up to this region, you get uniform elongation, so if you are stretching it; this is the gage length on to which you have applied the stress. So, this whole region is getting elongated uniformly, at each and every point the strain is constant or similar. However, beyond this point what happens something happens and it is called necking.

So, in some region you may develop something like this, so you see now this is a neck or a thinner region and in this thinner region most of the stress is now concentrated and so most of the deformation will be concentrated in this region and therefore, it is the elongation that you would see beyond this is no more uniformed, it is concentrated in only this part and therefore, this is non uniform elongation, beyond this are called necking and at the end point you get what is called fractured.

So, this is the stress strain plot that we have being seen from long time, from as early as high school and this is what it represents and the at this points since now we have already talked about engineering stress versus true stress, engineering strain versus true strain; what do you think does this one represent, is it engineering stress strain or true stress strain. By looking at these e and s; you may have guess that it is actually engineering stress strain; engineering stress, engineering strain. But there is still another way to find out whether what we are dealing with or the plot that we have drawn is actually for engineering stress, engineering strain or is it true stress and true strain and that is to look until the point of fracture.

Now if this curve at all comes back, why is this coming back? Because over here you remember in engineering strain we start with just the original area as the base area, but in the true strain we look at the instantaneous area. So in the instantaneous area, if we are looking at the stress; the stress should always increase because the stress is f by a, so the instantaneous area is getting reduced. So, the force by area term which is the stress at that particular region will keep on increasing, but on the other hand when we are talking about engineering stress our air remains constant.

So although the force may be limited, the load may be limited to this region in a very small region, but we are still looking at of very large area, so because of that the plot seems to be curving down, when in reality the stress; the required load is still increasing and that is how you can look at the plot if it is coming back down, you can say it has to be engineering stress strain and the true stress strain plot, it will not come down; it will keep on increasing because as long as you are applying load, the stress must be increasing and in fact, there is something called as work hardening will look at because of the work hardening the stress has to keep on increasing and therefore, the plot should keep on going up and it should not come down.

Anyhow, so coming back, so this is our stress strain plot or to be precise engineering stress, engineering strain plot for a uniaxial tensile condition and as we saw that here we were concerned with only one parameter as soon as sigma yy or sigma xx whatever it is reaches y, value y. So start with 0 and up to some point it will reach y, as soon as it reaches y what happens; it is starts to yield or the material can deformed is actually plastically deforming; however, as dimension it couple of times before, now that when you have more than one stress components, more than one stresses acting on each and every element then you have to find out not only our sigma yy, we have to also say what should be sigma xx, what should be sigma xy, what should be sigma yx all these.

So, should it be independently defined, so let me rewrite it over here; sigma what should be sigma xy, what should be sigma yx, what should be the value of these components if we want to find out, if the material will deform or not and that is why we need a way to define the yielding of the material and this is why it is where we need what is called as yield criterion. One thing that in elastic region; the form of the equation in the elastic deformation is very straight forward and in fact, it is very exact, it is given by Hooke's law. So, you know that y is equal to e times epsilon; where e is your Yang's modules and y is your stress, so or you can say sigma is equal epsilon time e.

So over the elastic region, the equation is exact. In the plastic region even for a uniaxial condition, the deformation; the form of the equation is not exact, we do not know we have empirical knowledge about that and some of the equations that are used are something like power law. So, it is given by sigma equal to k epsilon to the power n, this if this is defending how much stress you need for a given deformation where k is some constant. So, these are some of the forms of the equation and again this is only related to, so for what we are talking about is uniaxial.

In again multiaxial condition, again you will have to think about how to defined this plastic deformation, where should be defined only for 1 sigma which can be sigma yy or sigma xy or sigma xy sigma xx and so on; again up to 7 or even in a simple two dimensional case up to 4 values. So, all those things you have to be able to define and look at when we are talking about yielding and the plastic deformation.

So, will come back to this in the next lecture and we will start with discussing what is the called as the yield criterion for a material which is subjected to multiaxial stress, how do we define, at what particular condition it will start to deform and that is what is called the yield criterion there are mainly two Tresca criterion and Von Mises criterion. So, we will look at these two in somewhat detail in next class.