Fundamentals of Materials Processing (Part-I).
Professor Shashank Shekhar.
Department of Materials Science and Engineering.
Indian Institute of Technology Kanpur.
Lecture-09.
Heat Flow (Insulating Mold Condition).

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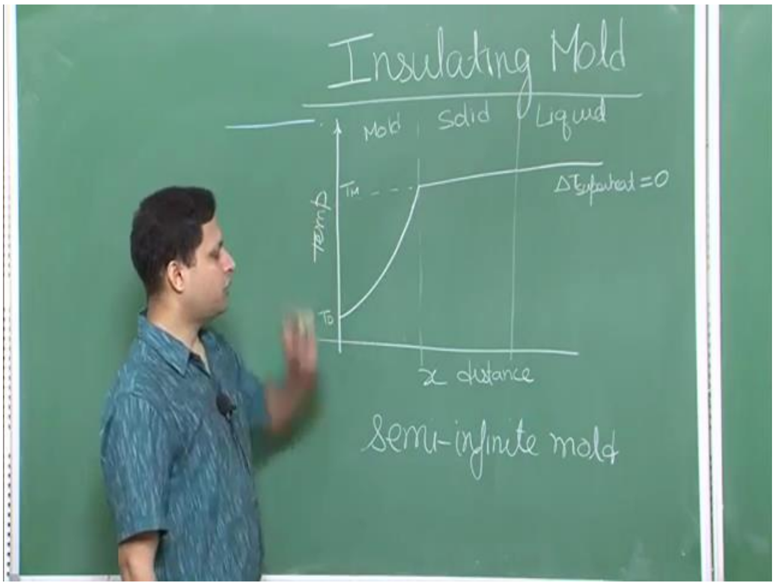

Okay, so let us continue with our discussion on insulating mold condition for solidification. So we are talking about insulating mold condition. Let me 1st draw the, I am drawing the schematic of the temperature profile that you will see in an insulating mold condition. So there are 3 layers mold, solid, and liquid. So when we say insulating mold condition, what we mean is that mold insulation properties are the dominating condition for the flow. So here Y is our temperature axis and this is the distance.

So when we are talking about temperature, when temperature drop will be only taking place in a mold and not in the solid or in the liquid, so this is TM and this is T0. So remember that word, we are saying the temperature drop takes place in mold, we are not saying that we have fixed the outside temperature to equal to T0, that is a difference between that when we look at that in a moment. So there are some more things that we need to understand over here. What you will see is that the temperature does not drop in the solid.

Why is that because we are assuming that the solid, remember from the previous class that solid is very very Conducting. Much more Conducting than the insulating mold. Of course it is insulating, so it has very low conductivity. And since it has very high conductivity, therefore any amount of heat this point in the solid which is at the interface will get transferred directly to the interface of solid and mold. Another thing that we have done here is Delta T super heat is equal to 0. See, in actual practice whenever you are pouring liquid, you will not post it at exactly the melting point, it will be a little above the melting point.

But for simplification of calculation and to keep analytical solution simple, we assume that the liquid is exactly exactly Delta T or you can say just a little above Delta TM, it is not Delta T, it is little TM which is the melting point of a, of the material. So it is just a little about the melting point, so it is liquid state, solid is just a little below the melting point, it is in solid-state. And the temperature dropped from the TM to T0 takes place in the mold. I am saying, saying both the things right now that the temperature drop takes place here and is not equivalent to saying that T0, outside of the mold is T0.

And at the same time I have also given it as T0. Because we have invoked another condition what is called as semi-infinite mold. So for the insulating mold, you remember we talked about 3 different conditions. One is that, we are inherently is assuming some of the, we are taking some inherent assumptions. One of them is that solid is very Conducting because of which there is no temperature drop here, there is no super heat in the liquid and therefore the temperature here they are also melting point.

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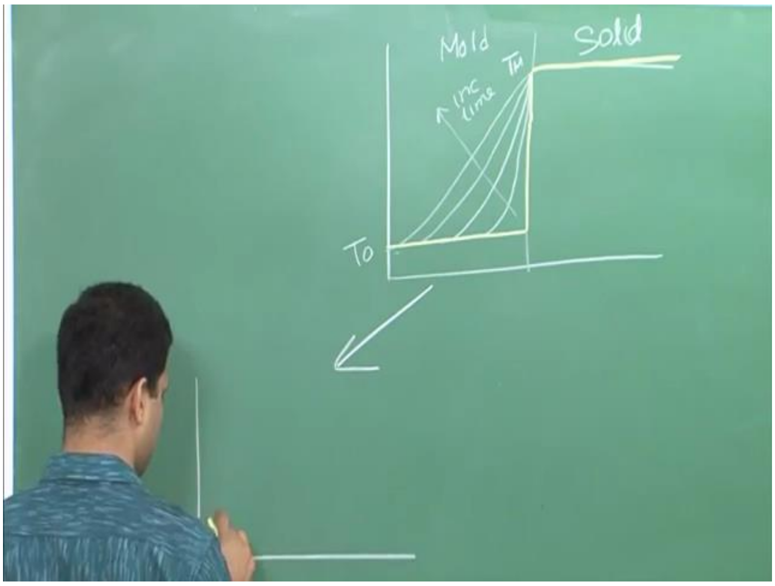

And insulating mold, so the temperature drop is taking place only in mold and along with that there is that semi-infinite mode. So what is that semi-infinite mold? Let me draw little bit to explain it that to you. Let us say this is an interface and it can be any interface, may not be the solid, liquid we are talking about but just for the sake of comparison with what we are discussing over here, let us say this is the mold and this is a solid. So just at the beginning, what will happen?

Just at the beginning, just at the start, mold is at T0 which is ambient temperature. And solid is at TM temperature. So the temperature profile is a step function like this. Now as soon as the heat starts to flow, what will happen? The temperature will, profile will change, it will become like this. So the mold is getting heated up and the temperature near the solid mold interface is rising. And therefore you can see these, these are the points which have rose, the temperature here has rose from T0 to this point, this point and so on.

And this is with increasing time, remember. So we are talking about… Now at this point, there are 2 things that can happen. So this was the temperature profile and from this point onwards, it may happen that temperature, so this is the outside, this is the start of the mold, remember this is distance. I am sorry that I did not label it over here but you must remember that this is distance from also this is the outside of the mold, this is the solid mold interface.

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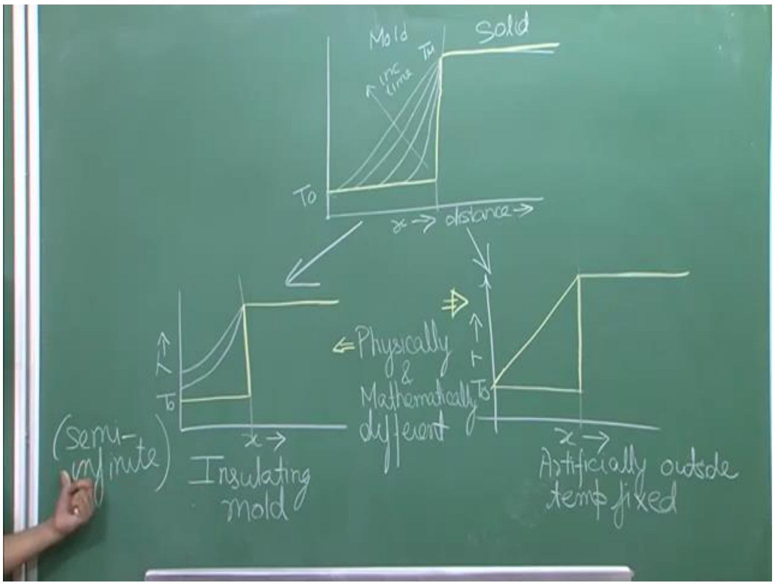

Now at the outside of the mold so far we have T0 but beyond a certain time, again we are increasing time this way. And this is T, this is X. Beyond a certain time, the profile may become like this, meaning temperature of the outside as the mold has also increased beyond T0. Other case, what can it be? So this is again temperature X, other case could have been that beyond a certain point this will become like this. In this case, what will happen is that this is in a steady, you can say it is in a steady state.

And why is it different? It is different because here the outside temperature is actually fixed, it is equal to T0, not actually, rising like in this particular case. So these are the 2 different places and physically and mathematically both of them are different. This is actually the condition which we call insulating mold. And this is a particular kind of condition, configuration when you have artificially fixed the boundary condition that is the mold air interface equal to T0.

So this is you can say, I am not giving it any name here, artificially outside temperature is fixed. And these 2 is actually possible in some kind of casting. What you can do is, you can have a cooling mechanism on the outside surface of the mold, for example you can have a very cold water running around this. So it will have something like 4 degree or 5 degree at the outside of the mold. And there you are artificially fixing the temperature. So this is also possible but this is not the condition we are talking about.

What we are talking about is this condition which is insulating mold where the temperature after certain time, it can actually rise beyond T0. But then why are we interested in this, or what this basically happening over here? If it is not fixed at T0, how are we able to earlier say that it is T0? It is because of this condition, semi-infinite mold. So this is insulating mold with semi-infinite condition. Okay, let me write it a little over here. So this is semi-infinite insulating mold.

What is different here is that if the time that we are talking about, the timeframe that we are talking about is small enough, you can see that the outside of the mold will remain equal to T0. But if the time goes beyond a certain value or it becomes very very large, then eventually it will start to rise. So when we are talking about the incident in mold, we can say that we are looking only at a condition where the solidification will be so fast or it will be soon enough that the temperature of the outside of the mold does not increase beyond a certain point.

And therefore we can hold a condition of semi-infinite insulating mode. And this particular condition means we are not using any artificial mechanism of cooling the mold from outside. In here, you remember it cannot be, if you are talking about a simple sand more casting which we are doing in, in our usual laboratory processes, we are not actually using this because we are not keeping the temperature fixed from outside by any mechanism, it is just the ambient temperature.

And at a, after certain point, the mold temperature starts to rise, we are not trying to bring it down by any mechanism. So that is why these 2 are different in both of them can be used in different conditions. But for now let us stick with this which is our insulating mold with semi-infinite mold condition. So that is our, that is the semi-infinite mold condition that we are talking about. Now let us get to what we want over here. What we want here is in the end we are, we are taking all the simplification so that in the end we can get an equation of T which is temperature is a function of X and T.

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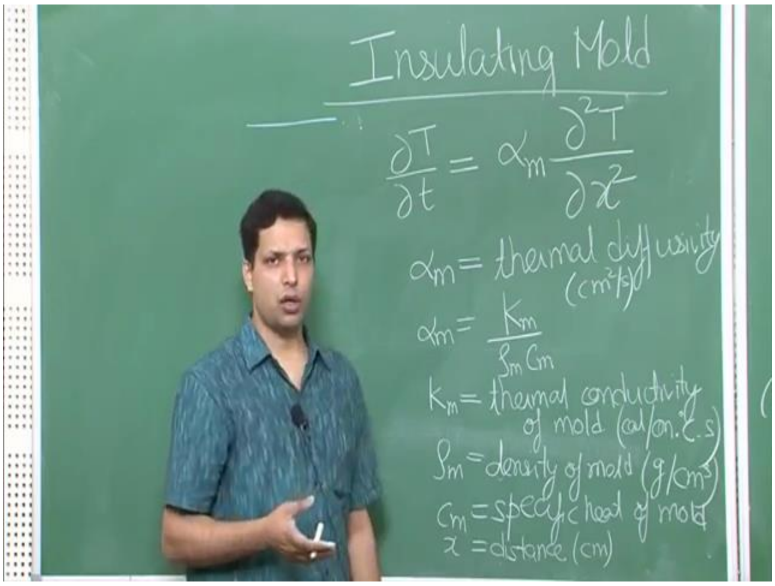

X is the distance and T is the time. So we want temperature is a function of distance and time and since the temperature change is taking place only in the mold, so here we are interested in the temperature inside the mold as a function of X and T. For that, the equation that is invoked is that heat balance equation. And in the, the heat balance equation that originates from Fourier Law gives us the heat equation which can be written as, so the uppercase T is the temperature, the lower case t is the time, alpha M is also called the thermal diffusivity and we will see what it composes of.

Again T the temperature and X is the distance. So this is, if you remember us equation, you must have in this kind of equation elsewhere, in actually diffusion. So this is very similar to the diffusion equation. And that is why this term over there it is called diffusivity term and therefore in analogy to that, it is called thermal diffusivity in this particular case. So this is originating, this equation is coming from heat balance equation. From there we get this differential form of the equation and when we solve this using the initial conditions and boundary conditions, we will be able to get temperature as a function of X and T.

And then we can do some more, once we have the temperature is a function of distance and time, we will be able to solve other things. For example what is the solidification weight, what is the total time taken for a mold and so on. Okay, so now let us look at some other terms over here. So I said alpha M is thermal diffusivity, it is in centimetre square per second. Since this is composed of other terms and those terms are KM again you remember is the thermal conductivity and the subscript M denotes the mold.

So we are talking about the thermal diffusivity of mold and when we say KM, it is the thermal conductivity of the mold, rho M means the density of the mold and CM means heat capacity of the mold. So for the sake of completeness, I will mention all the parameters that I am talking about here. Thermal conductivity of the mold and along with the unit, so these calories per centimetre per degree C per second. Rho M is density where subscript M denotes mold, so this will be in gram per centimetre cube.

And CM we have already said is specific heat of mold and other parameters we know T is the time, X is the distance in centimetre. So this is the you can say the governing equation that we will use that leads to for getting the temperature as a function of X and T. And remember this is one-dimensional form of that equation. We can also get them three-dimensional form of the equation or two-dimensional form of the equation depending on the geometry. But for the sake of simplicity we will just assume that it is a single dimensional object, for example a rod or something like that.

And that does, that does not take away any information from us, we can still do all the manipulations to get the form of the equation for this. And you will see even though this is a single dimensional solution that we will obtain from this, we will be able to generalise it to event 2-D and 3-D. Next what we need is to define what other initial and final conditions.

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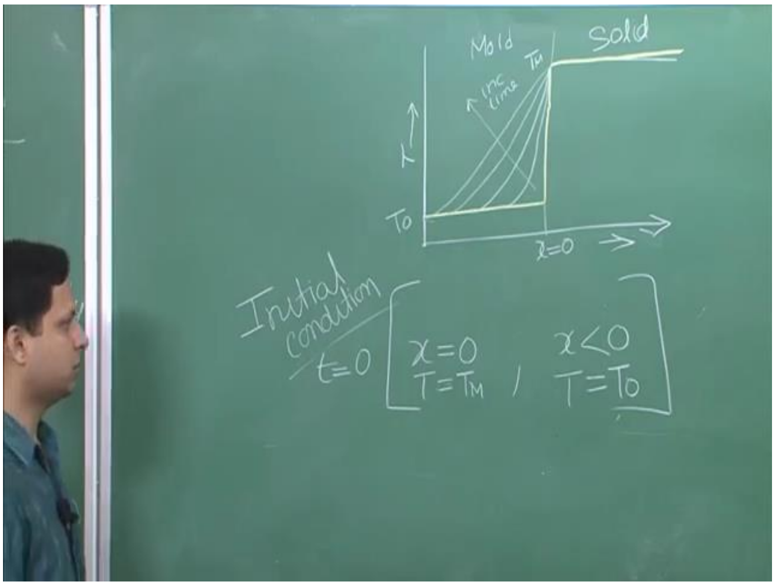

So we know the equation, the governing equation but what are the conditions? There have, there has to be certain, because we are using a simplification, there is most likely certain conditions that we can use over here. Let us look at some of those conditions. Initial condition. We need, you know what was the temperature profile at the start, we know that part, right. So can we look at those conditions, what if the temperature profile, it is this yellow curve that you look over here, right. This is the temperature profile that was there at the very beginning.

And what does it say? If we take this as X equal to 0 and this is the temperature, remember, so this is X equal to 0, then what it is saying is that at X equal to 0, T is equal to TM. So we are assuming that X equal to 0 point lies in the solid, so for T equal to 0, that is the initial condition for T equal to 0. So we are talking about for T equal to 0 at X equal to 0, T is equal to TM. And as you will see that since we, since we are having temperature drop only in the mold, so we are interested only when X equal to 0 to X less than 0 part of the plot.

We are not interested in the X greater than 0 part. So at X equal to 0 we have T equal to TM. What about the rest? For X less than 0, we can say that T is equal to T0. So this becomes our initial condition. For any X less than 0, T will remain equal to T0, it is the starting condition, that is the initial condition, that is one thing, that is set up. And remember these initial and boundary conditions are also useful when you are trying to formulate this problem using some finite element method. For example, using Matlab. Although we will get analytical solution here but you can also get the numerical solution by using some finite element method using 1st principle.

So you will not have to solve the analytical solution, you can just directly go back to your fundamental equation which is the governing equation with which we call as equivalent of the diffusivity equation. So you can use this, use the finite element methods and get the solution for not only this, I went for more complicated condition or complicated temperature profile. So that, in that sense initial condition and boundary condition is important to be known.

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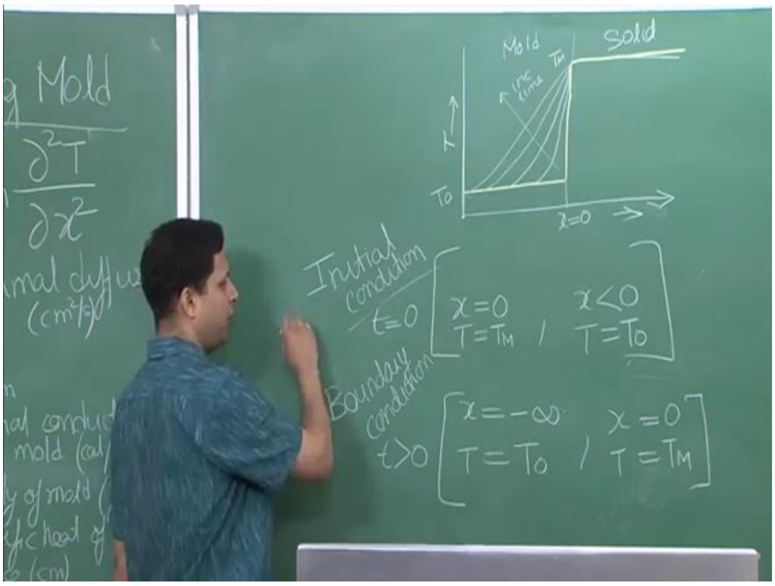

Now boundary condition is something that will be true for all times. So for all T greater than 0, what are the things that we can say? One thing that we can say is that, it is a semi-infinite. And we said that at very very large distance, the temperature will always be T0. We are not saying at this particular distance, this is not a real infinite distance. At a real infinite distance which is X equal to - infinity, T will be equal to T0. That is another boundary condition that we are invoking over here. And another thing that we are invoking over here is again that X equals to 0 T remains equal to TM.

So we have our initial conditions and the boundary conditions. If you use this, then you would see that there are very standard solutions for this kind of equation under those conditions. And what is that solution? So let us look at.

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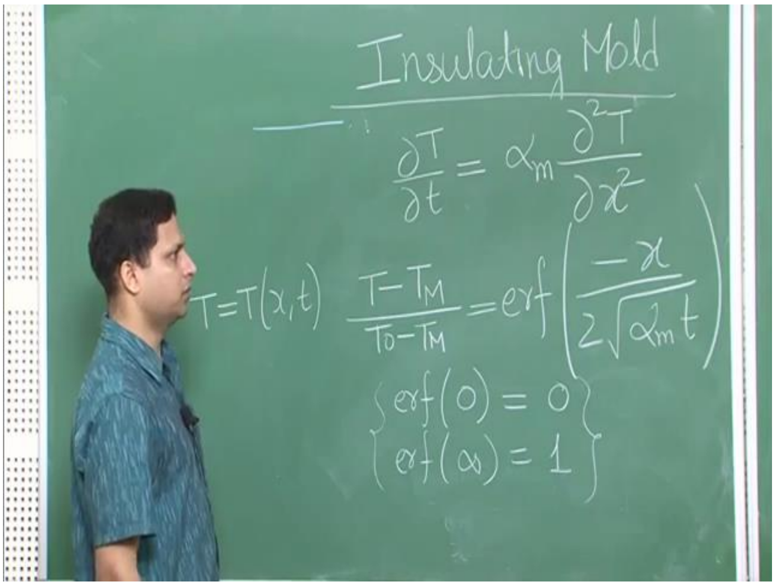

So the solution you may see would be of this form. And this is just a general solution that you can obtain for a differential equation of this form. ERF is the error function. So this is the final equation and you see this what we are getting is T is a function of X and T, which is, what our initial goal was. So we are able to get temperature is a function of X and which is distance and time. And with that equation where ERF is the error function. Since we are talking about error function, and some of you may not be aware of the error function. So let me give you some values that you may find interesting. One is that ERF 0 is equal to 0 and that ERF infinity is equal to 1.

If we have, actually if you use these 2 values of ERF, you would be able to cross check that all your initial and boundary conditions are actually coming out to be true. How? Let us see. ERF 0 is equal to 0. So this is our solution, now how will this term be 0? It can be 0 if you put X equal to 0. And what do we have at X equal to 0. At X equal to 0 we have T equal to TM for all-time for T equal to 0 and for T greater than 0. So at X equal to 0, T should come out equal to TM.

And we can get this term, whole term as 0 by putting X equal to 0, no matter what the value of T time you put over here. So this will be 0, which means ERF, the overall value of this function will be equal to 0, therefore T - TM will be equal to 0, therefore T will come out equal to TM, right. So, we actually do get T equal to TM when you put X equal to 0. Another thing, if you put T equal to 0 which is a small t, the time t, if you put this value equal to 0, this whole thing becomes infinitely.

And if you have ERF infinity, then you know that ERF infinity is equal to1. So this value becomes 1. And before I get to the final solution, let us see what do we have for T equal to 0. For T equal to 0, for whatever X you use, you should get T equal to T0, okay. So here if you put infinity, this will become, this whole thing becomes 1 and that means T - TM is equal to T0 - TM, which means T equals to T0. Therefore this is again coming out what we expected at X equal to 0, T is equal to, sorry at T equal to 0, for all explain than 0, T is coming out to be T0.

So no matter what value of X you put here, actually in the negative only, if you put whatever negative value of X you put over here, you will get T equal to T0. So this is also coming out to be exactly as you would have expected. So this equation that ensures that our equation is all right. Now let us move onto the next stage. We have the form of the equation for this inserting mold condition, why not make use of it to get some more information from this.

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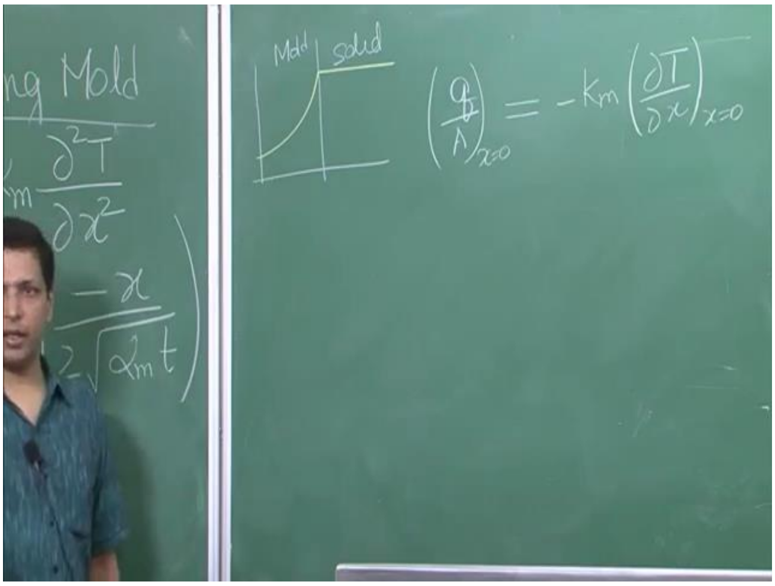

So what are those information, we will see. One of the things that you can obtain is how much solidification would have taken place in a given time. So how do we get to that? So for that we will have to use our equation, this is you can say similar to 2nd law is fixed, although there is nothing like that. But what is commonly referred to as fixed 2nd law, this is in the same form. If you call that the fixed 1st law, this becomes the fixed 2nd law and this is your Q over A is equal to - KM Dell T over Dell X.

What it is saying is what is the heat coming or going into an interface that can be obtained as a product of connectivity time thermal gradient at that point. That means that if we have, if we use our sand, sand mold and solid condition and assume that there is some temperature profile, what we are trying to do next is trying to equate what is the heat going into this mold. And we will also get to that value of heat by different method and then equate these 2. And then equate these 2 we will be able to get the length of solidification as a function of time.

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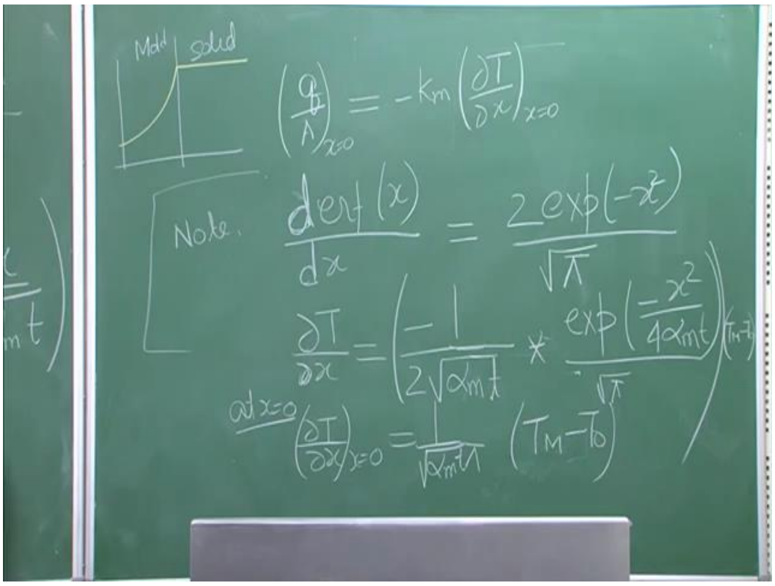

Let us move on. So this is our equation Q over A is equal to - KM Dell T over Dell X. So what we need is to differentiate our equation which is this equation in with respect to X. How do we differentiate Dell T over, this function with respect to X. So for that we also need how do we differentiate ERF function. So again quick bit of information. So this is differential factor dell, differential of ERF X is given by. Now if you know this, then we will be able to differentiate our this equation.

Now if we look at this equation, if you want to differentiate it this way, you will have to invoke what is called as chain rule. So this is ERF is a function of something which is a function of X. So it is a function of function of X. Temperature here is a function of function of X. So H is equal to F of G of X, then what we are saying is that 1^{st} differentiate H with respect to G and then differentiate G with respect to X and then we will be able to obtain the equation, obtain the final differential form of T with respect to X.

Let us move on and we have Dell T, if we use this, we can, you can see that this is how it will come out, Dell T over Dell X. 1^{st} we are differentiating with respect to , we are differentiating X with respect to G which is this form. And then differentiating G with respect to X, actually vice versa, this is differentiating G with respect to X and this is differentiating H with respect to G. And there is a TM - T0 term over here. So see, we have used this equation and this fact that differential of ERF X is given by this and also we have used the chain rule of differentiation, so we are able to get Dell T over Dell X. And there is, there is this TM - T0 which is actually from here.

And there would also be TM when you differentiate with that X, that becomes 0, so we are only left with these terms. So this is our Dell T over Dell X, which is just one part of it. If we want to get Q over D at X equal to 0, my, difficult to contributing times gradient at X equal to 0. So next thing we have to do is we have to put X equal to 0 in over here. So if you put X equal to 0 over here, you can see that as you put at X equal to 0 you get, I am putting the equation and you can easily see that it comes out from the other equation.

So this is the Dell T is over Dell X at X equal to 0 and then we will have to multiply it. So we will come back to this in the next lecture. So we will leave it over here and we will use the same equation in the next class. Okay, thank you.