Fundamentals of Materials Processing (Part–1)
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Lecture Number 08
Heat Flow (Continued)

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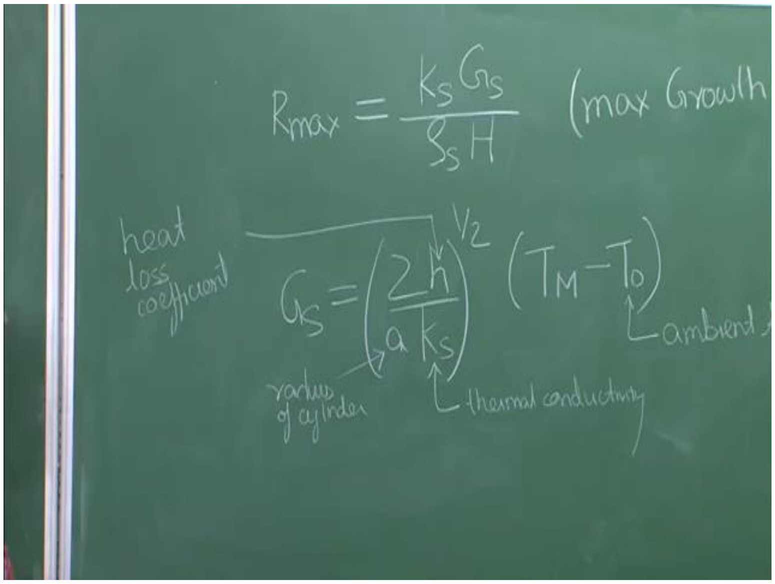

$$R_{max} = \frac{K_S G_S}{\rho_S H}$$ (max Growth

$$G_S = \left(\frac{2h}{a K_S}\right)^{1/2} (T_M - T_0)$$

Okay so let us get back to our equation that we were discussing in the previous lecture. So this is the equation for GS, which depends on so many factors; H, which is the heat loss co-efficient; A, the radius of the crystal, if it is, considering it is cylindrical; KS, which is the thermal conductivity of solid, and TM minus T0, which is the difference between the melting temperature and the room temperature, or the ambient temperature. So these are the factors. Now let us get back and try to recollect what we have said so far, and we will solve a problem onto this in just a moment; so now let us look at the slides a little bit.

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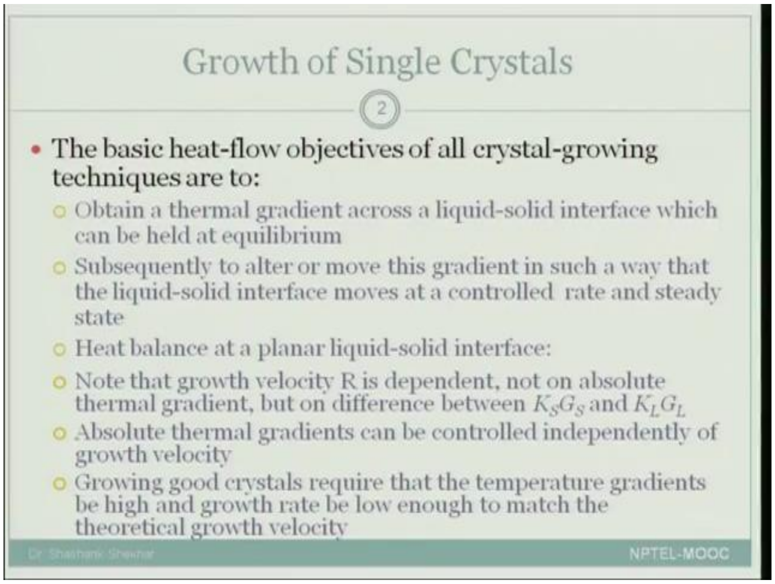

Growth of Single Crystals

- **The basic heat-flow objectives of all crystal-growing techniques are to:**
  - Obtain a thermal gradient across a liquid-solid interface which can be held at equilibrium
  - Subsequently to alter or move this gradient in such a way that the liquid-solid interface moves at a controlled rate and steady state
  - Heat balance at a planar liquid-solid interface:
  - Note that growth velocity R is dependent, not on absolute thermal gradient, but on difference between $K_S G_S$ and $K_L G_L$
  - Absolute thermal gradients can be controlled independently of growth velocity
  - Growing good crystals require that the temperature gradients be high and growth rate be low enough to match the theoretical growth velocity

We note that the growth velocity R is dependent not on absolute thermal gradient, but on difference between KSGS and KLGL. Absolute thermal gradients can be controlled independently of the growth velocities; so if you, even if you have the same R, you can have two different combination of GS and GL that can give you the same growth velocity. So absolute thermal gradient can be controlled independently of the growth velocity. Growing good crystals require that the temperature gradients be high which I already mentioned, and the growth rate be low enough to match the theoretical growth velocity.

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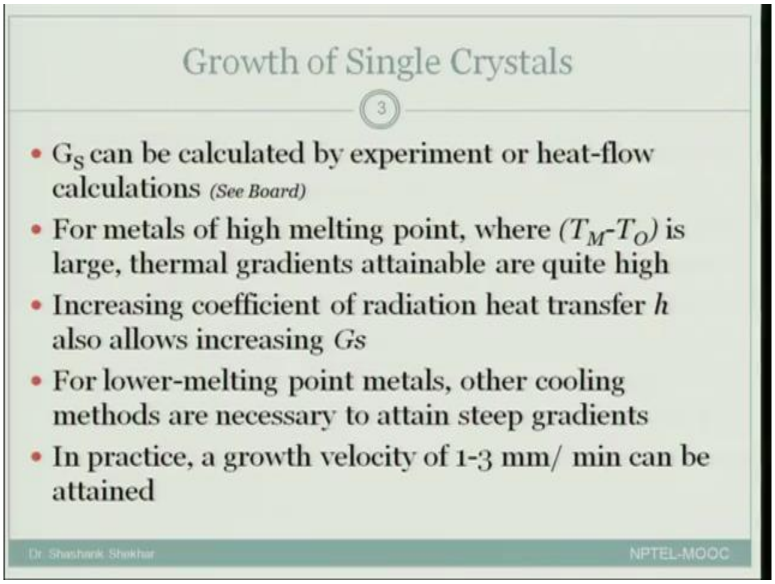

Growth of Single Crystals

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- G_S can be calculated by experiment or heat-flow calculations *(See Board)*
- For metals of high melting point, where (T_M-T_O) is large, thermal gradients attainable are quite high
- Increasing coefficient of radiation heat transfer h also allows increasing Gs
- For lower-melting point metals, other cooling methods are necessary to attain steep gradients
- In practice, a growth velocity of 1-3 mm/ min can be attained

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We also found that GS, which is this equation, can be calculated by experiment or heat flow equation; in this particular case it was obtained using heat flow equation, the details of which we have not discussed. But this is the final form of the GS value, which is the thermal gradient in the solid. For metals of high melting point, where TM minus T0 is large, thermal gradients of high values can be obtained. So that is again a something that is , that you can understand when you look again at the equation; GS is proportional to TM minus T0. Compare the TM minus T0 for aluminum versus let us say steel, so you know that steel is not a good example because steel is a alloy, not a single component material. So let us say we are talking about iron. So this iron will have 1500 plus melting point, on the other hands, the melting point of aluminum is 660 Celsius.

So TM minus T0 for iron will be much larger and therefore you can get much larger thermal gradient of solid in iron than in aluminum. For lower melting point metals, some other cooling methods can be employed. Now let us say TM is fixed, but T0 is something that you can change. How can you change T0? You can reduce by some cooling mechanism, you can make it cryo, less than less than zero or sub-zero temperature. That way again you can increase the value of TM minus T0, and so still you can get high GS value. In practice, the growth velocity of 1 to 3 millileter per minute can be attained. Now having said this, let us try to solve a simple equation, a simple problem based on this.

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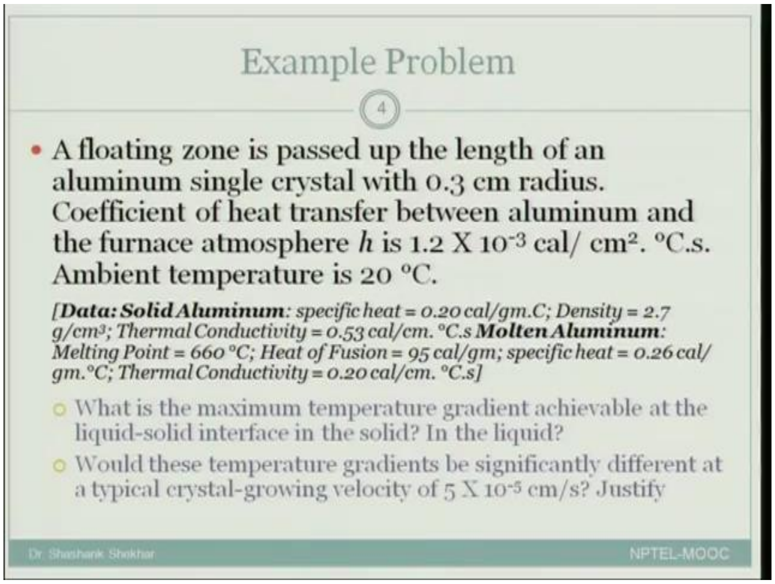

Example Problem

- A floating zone is passed up the length of an aluminum single crystal with 0.3 cm radius. Coefficient of heat transfer between aluminum and the furnace atmosphere *h* is 1.2 X 10^{-3} cal/ cm^2. °C.s. Ambient temperature is 20 °C.

  *[**Data: Solid Aluminum**: specific heat = 0.20 cal/gm.C; Density = 2.7 g/cm^3; Thermal Conductivity = 0.53 cal/cm. °C.s **Molten Aluminum**: Melting Point = 660 °C; Heat of Fusion = 95 cal/gm; specific heat = 0.26 cal/ gm.°C; Thermal Conductivity = 0.20 cal/cm. °C.s]*

  - What is the maximum temperature gradient achievable at the liquid-solid interface in the solid? In the liquid?
  - Would these temperature gradients be significantly different at a typical crystal-growing velocity of 5 X 10^{-5} cm/s? Justify

We talked about the floating zone technique, which is for growing single crystals where there is no mould; so you can see that it is a very sophisticated technique, which does not require mould, it is not in contact with any outside surface and therefore, heterogeneous nucleation will not take place and so there will not be nucleation taking place all around the walls. Now in our floating zone technique, we are trying to solidify aluminum, and the radius of the crystal is point 3 centimeter. Co-efficient of heat transfer is given and you are also given values of the specific heat of the solid aluminum molten aluminum, heat of fusion and all the values that you need.

Now the question is, how will you find out what is the maximum temperature gradient achievable at solid-liquid interface in the solid? First let us look at the maximum temperature gradient achievable at the liquid-solid interface in the solid. Remember we are always talking about the thermal gradient at the interface, (all) whether it is in the solid or liquid but at the interface. So, what let us look at what is the maximum thermal gradient that you are going to obtain in solid.

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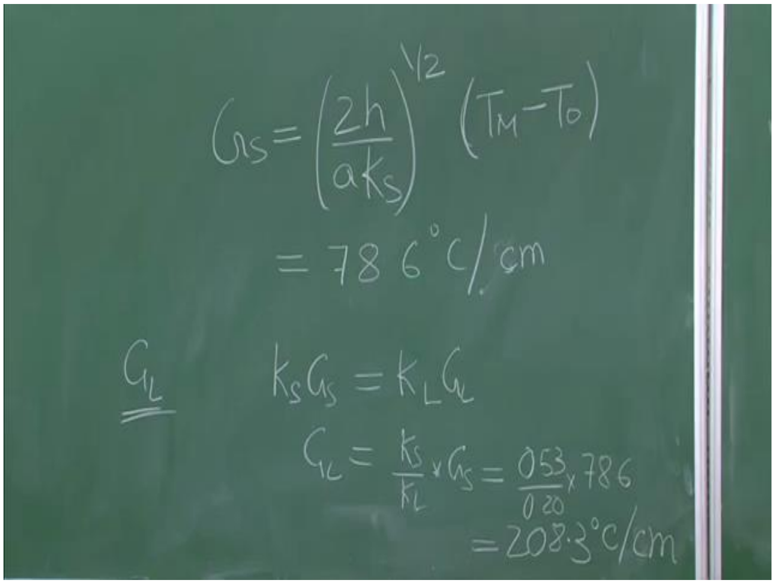

$$G_S = \left(\frac{2h}{a k_S}\right)^{1/2} (T_M - T_0)$$

$$= 78\ 6\ ^\circ C/cm$$

$$\underline{\underline{G_L}} \qquad k_S G_S = k_L G_L$$

$$G_L = \frac{k_S}{k_L} \times G_S = \frac{0\ 53}{0\ 20} \times 78\ 6$$

$$= 208{\cdot}3\ ^\circ C/cm$$

So what we need is for the solid; first let us look at solid, so this is the equation, we have already derived over here. You have to look at this equation. Now the question is asking for maximum thermal gradient in the solid. We have already seen that these are parameters that are already given, you cannot change that. Now, if the material is given, TM is fixed. And assuming that and in fact T0 is also given, so this is also fixed. So the maximum or the only value that you can calculate for GS is by taking all the values that are already given; you cannot change it, given that TM is fixed, T0 is fixed, h and a, KS are all fixed. So you will get only one value, and if you put in all the values over there, you would see that it comes out to 78 point 6 degree Celsius per second. So this is for GS. Now what about GL?

We want to find out what is the maximum value of GL? Now what will happen if you keep increasing GL? If you keep increasing GL, you remember it is a negative term, so the rate growth velocity keeps decreasing. So there will reach a point where growth will (())(05:57) become 0. It can never become negative, you could you cannot come down or you cannot start melting; so the highest GL would be the condition where KSGS is equal to KLGL, meaning the rho S rho S HR term has gone to 0, basically R has gone to 0, and therefore rho S HR term has gone to 0; therefore KSGS is equal to KLGL. And from here, you would (see) say that GL is equal to KS by KL into GS, which is just the basically multiplying GS by a factor and therefore, this factor happens to be point 53 by point 2 into 78 point 6, and you get 208 point 3 degree celsius per second.

It is a growth it is a thermal gradient, so sorry this this is my (())(07:02) again let me recorrect it. It is a thermal gradient, it is not per second, it is per centimeter. The units, if you check properly, it will come out to degree celsius per centimeter. It is a thermal gradient, we are not talking about the heat rate or temperature rate. So it is degree celsius per centimeter, 78 point 6 degree c per centimeter for GS, and GL, maximum value you can get is 208 point 3 degree c per centimeter. So that is our first part of the problem. Now let us look at the second part of the problem. It says, would this temperature gradients, which we have calculated just now, which are 78 point 6 and 208 point 3; will this temperature gradients be significantly different at a typical crystal growth velocity of 5 into 10 to power minus 5 centimeter per second?

So, in the second part, you are being given the growth velocity R, and then you have to calculate what is going to be GS and GL; or basically you have to comment on whether GS and GL would be very different. The easiest way is why not calculate GS and GL again, given that we already have the R value, we already have rho S HR, so, why not calculate all those.

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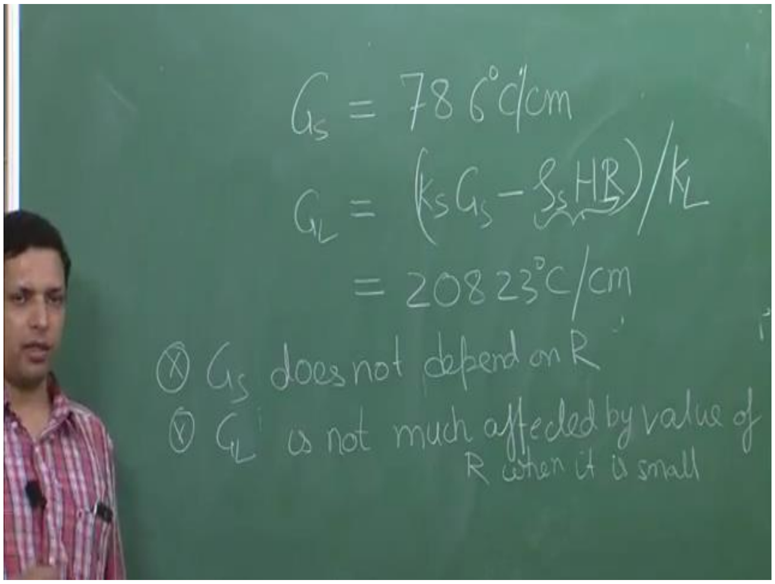

$G_s = 78\ 6\,^\circ C/cm$

$G_L = (K_s G_s - \rho_s H R)/K_L$

$= 208\ 23\,^\circ C/cm$

⊗ G_s does not depend on R

⊗ G_L is not much affected by value of R when it is small

So, let us calculate GS for this given growth velocity, but we see that, GS does not depend on R, which means, that GS would remain same, that we had calculated earlier, 78 point 6 degree c per centimeter. What about GL? If GS is given, R is given, then, GS would actually depend on R from the early, most the first equation that we talked about in terms of single crystal growth. So this will be; so this would be the equation that will give you the value of GL. And when you put these values over here, what you would find is that GL comes out to 208 point 23 degree c per centimeter.

Now you look at these values, and compare it with the values that we obtained earlier. So GS remained same, of course because it does not depend on our GL which is a imposed value, which does depend on R, is actually not very different; at a typical growth velocity of 10 to the power minus 5 centimeter per second. So, our answer would be that, no, the GS and GL, the thermal gradients, do not depend much or do not vary much when we are talking when from the maximum value, when we are talking about a typical growth velocity, and why is that? The reason is that, GS does not depend on R.

What about GL? GL does depend on R, but then, GL is not much affected by value of, when R is very small. So if the R value is very small, this whole term becomes almost negligible, and what you have in GL is effectively KSGS by KL, which is equivalent to our maximum gradient, thermal gradient that we were obtaining. So this value does not make much (different) difference to GL; and therefore the answer is, that whenever you have a slow growth rate, which is the typical growth rate velocity, it will not influence the GS and GL at all.

Now at this question, at this point let me add another question to this. You have all the conditions given; like someone asks you how can you increase GS and GL even further? What would you do? Again we will come back to this. You know that KS cannot be changed, okay so first, let us go back to first the GS, because GS is the one independent. So here, we cannot change KS, we cannot change TM, but we can make a difference to some of these quantities h, heat loss co-efficient, meaning a if you will start, or if you allow more heat loss to the furnace environment, then this quantity becomes larger; so GS can become higher. a is a radius; if you decrease the radius, then again GS can increase, but then that would also imply that you are getting a very very thin single crystal, which beyond a point, it may not be meaningful.

So you may not like to make much change to this. We already said TM is not something you can change, but T0 is something that you can change. You can put some additional cooling mechanism; so your ambient temperature is no more T0, but let us say some minus 50 degree celsius or minus 100 degree celsius. So if you put a additional cooling mechanism, T0 can be reduced, and this can, this factor can become large, and GS will become larger. And when GS becomes large, then GL will also become large, so the these are the ways that you can increase both thermal gradients of solid and liquid near the interface. So that is all we will talk about the heat flow in single crystal. Next we will talk about the heat flow during the solidification of castings and ingots.

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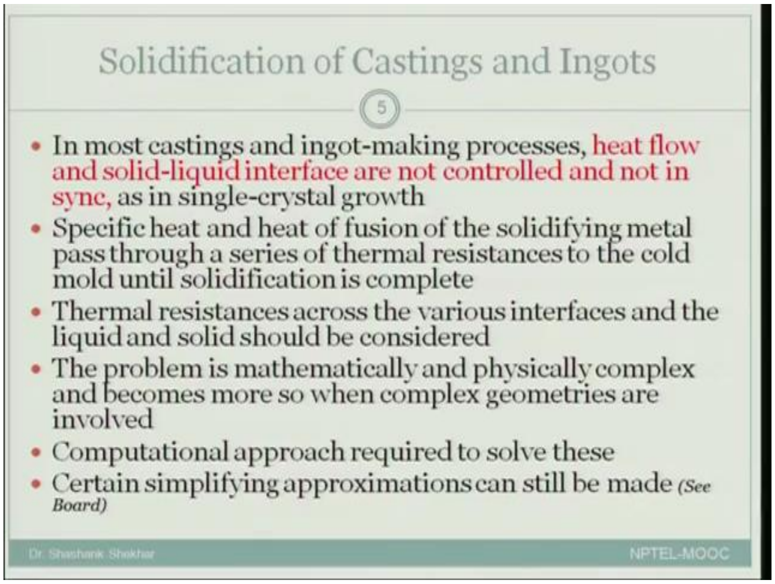

Solidification of Castings and Ingots

- In most castings and ingot-making processes, heat flow and solid-liquid interface are not controlled and not in sync, as in single-crystal growth
- Specific heat and heat of fusion of the solidifying metal pass through a series of thermal resistances to the cold mold until solidification is complete
- Thermal resistances across the various interfaces and the liquid and solid should be considered
- The problem is mathematically and physically complex and becomes more so when complex geometries are involved
- Computational approach required to solve these
- Certain simplifying approximations can still be made *(See Board)*

In most casting and ingot making process, what you will see is that heat flow and solid-liquid interface are not controlled. You remember in the single crystal growth, we were talking about the controlled rate of interface movement, and that is what allowed us to have a single crystal, but that is not the case when we are talking about the ingot making or the castings of for example, if you are making a sand caste mould, and you make some casting out of that, over there, you are not you are not controlling at what rate the interface should move, not even at what rate the thermal gradient should move. All those are automatically controlled adjusted, and hence what we get is polycrystal materials and therefore it also means that you do not have to have a very small growth rate velocity, growth rate; you can have very large growth rate, in fact you can get a very large casting in matter of few minutes, but that was not the case for single crystal.

In the castings and ingots, the amount of heat which we have, which is the specific heat as well as the heat of fusion, of the solidifying metal and cooling solid, it must pass through a series of thermal resistances, and to the outside of the mould, and only then, your heat flow process has completed. This itself is a very complex or it involves a lot of you can say different heat flow equations, or thermal resistances; you have to take into account several conditions to formulate such problems. Thermal resistances across the various interfaces and the liquid and solid should also be considered.

All in all, the problem (this) that we are describing here becomes physically and mathematically very complex, and it will become even more complex if we are talking about complex geometries; if we are talking about, let us say just a simple rod, then this can still be a tractable problem, but if we are talking about let us say, a engine head, over there you have somewhere convex shapes, somewhere concave shapes, and then the geometry also adds to the complexity of heat flow. And in that case, you may need some computational approach, meaning, you may have to have a finite element method kind of a solution to find out the actual rate of heat flow.

But, we are nowhere happy with that, we always want to simplify the assumptions, and find some analytical solutions, and certain, , so that is what we will do over here, we will have some certain simplifying assumptions to be able to get to some simple solutions under some simple conditions. So what are those simple conditions, we will look at. But before that, let us talk about what are the different, in a very very general condition, what will be the different layers that through which the heat has to flow, which makes it so complex?

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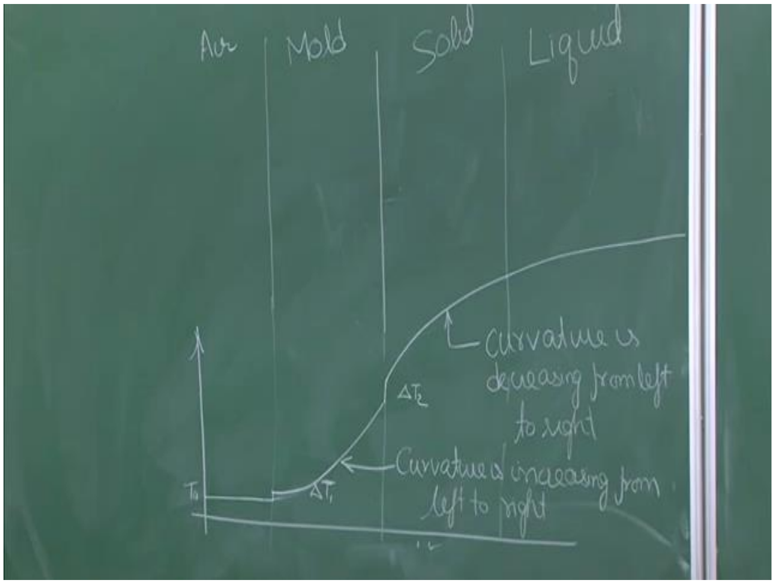

There will be liquid, and again to make things simple, we are assuming a single dimensional geometry. Thereafter you will have a layer of solid, so this is the solid which has solidified from the liquid; beyond that, there will be a mould, and outside the mould, you will have the air layer. So the heat has to go from all the way from liquid to solid to the mould and into the air; so that is the way the heat flow will take place. And if we were to draw the temperature plot, it may look something like this. So this is let us say the ambient condition T0; in the mould it will rise to some place, then there will be a interface resistance at the mould and solid; then in the solid, there will be temperature rise, and there will even be a temperature rise in the liquid. And in fact, what I forgot to draw here, let me draw it; there may even be interface resistance at the mould-air interface.

So this is how a most general condition for casting would look like. Now you can see, you have to take into account heat flow from here to here, (cond) by conductivity, then by thermal resistance, again by conductivity, then there is thermal resistance. So there is lots of conditions involved. So there is a temperature drop over here, there is a temperature drop over here, and so on; the way I have drawn, you would note a few more point. One is this, that the curvature is decreasing from left to right. the temperature profile that I have drawn, the curvature for that temperature profile is decreasing, it is higher here, it is becoming smaller and smaller as we go from left to right in the solid.

However, when I have drawn it in the mould, it is quite inverse. Here the curvature is increasing when we go from left to right. So this is something we will not talk right now, but I want you to think about it a little bit, and we will come back to it in one of the later lectures, when we discuss in more detail about the heat flow. But, you will see that this is always the case; in the mould you will always have the curvature increasing, and in the solid you will always have the curvature decreasing. So this is a complex state. Now let us look at one of the simplifying conditions, and the first of these simplifying conditions is called a insulating mould. Now as the name suggests, the mould is insulating, meaning it has much higher, you can say thermal resistance.

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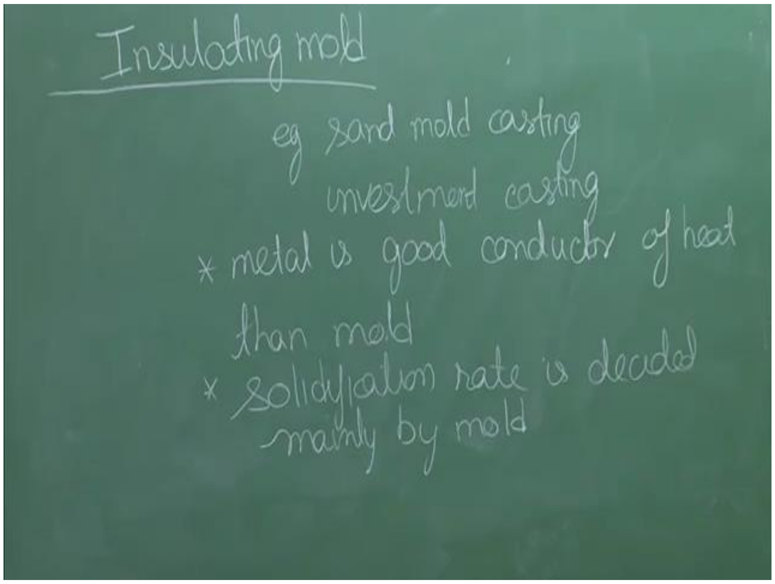

Insulating mold

eg sand mold casting
investment casting

* metal is good conductor of heat than mold
* solidification rate is decided mainly by mold

So this is the insulating mould condition; one of the simplifying assumptions, that we will first look at. Where are these conditions valid? These are valid in, for example, sand mould casting; this is also valid; investment casting. So see how we are able to relate what we know about the fundamentals to the actual processes, and this is one place you can directly see; although, even the earlier ones were that we have talked like a single crystal etc; over there also you are able to directly relate it with the real technique or the actual technique. So this is the insulating mould condition.

Now as the name suggests, metal is a good conductor. Here, and it is considered even you can say very, very good conductor in terms of these approximations, is a good conductor of heat than mould. And, because of this, solidification rate is decided by mould mainly. So, this is the implication of insulating mould, when you have a insulating mould, we are assuming that metal is a very good conductor, and much better than the mould, and therefore, the solidification rate is controlled or decided by only the mould properties, mould characteristics, and and, as you will see in the equation also that we will derive, the metal the metal characteristics, do not come into picture.

And, to and today's lecture, let me just draw a temperature profile that you would see in insulating mould condition; so this is for the most general condition. How would the temperature profile look like for a insulating mould condition?

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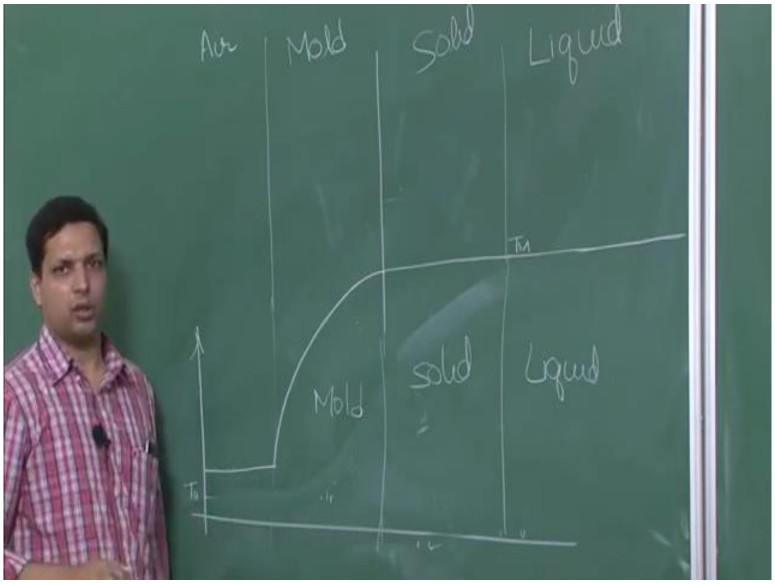

In here. So this is the temperature profile that we will have for a insulating mould condition, where this is the liquid, this is the solid, and this is the mould. Now as we have already pointed out, and we will discuss in more detail in the next lecture, that all the heat that comes into solid, gets transferred to the mould, and therefore you do not see any temperature drop over here, because solid is a very is being assumed to be a much much better conductor than mould.

All the heat will get transferred to into the mould, and all the temperature drop is taking place in mould. And, along with that assumption we are taking couple of more assumptions that we will discuss in more detail but just to give you a brief idea, we are assuming that temperature of liquid remains same, which is at the TM, so no super-heat over here, and we are assuming a semi-finite mould.

Should these are two more inherent assumptions which we have not explicitly mentioned, but it is also inherent in this that we use when we derive the analytical solution. So with that we will end today, and we will come back to this point in next lecture. Thank you.