Fundamentl of Materials Processing
Professor Shashank Shekhar
Department of Material Science and Engineering
Indian Institute of Technology Kanpur
Lecture 37
Particle Packing

So, we were discussing packing fraction of bimodal distribution. So, we have bimodal distribution of particle size and we obtained to equation depending on the two extreme conditions one we have coarse particles going into the fine once and the other one fine particles are going into the coarse one. So, we had two equation lets write down that and then will plot those equations. So, for the first case where we had fine particles going into the coarse, we had PF apparent or which is packing fraction affective it is given by this relation where (PF)c is the packing fraction in which the coarse particles are getting packed and (PF)f represent the packing fraction in which the packing the final particles are getting packed.

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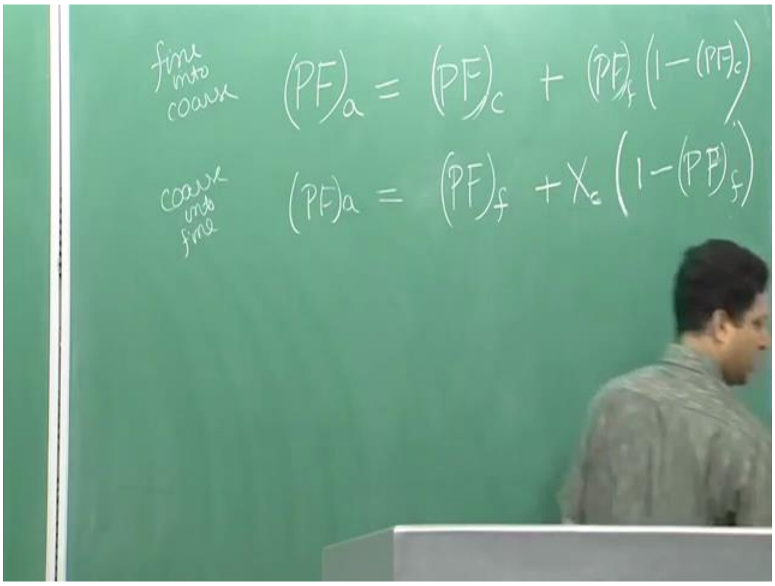

So, this is for the case where fine into coarse particles and the second one was coarse particles going into fine and for that we had a relation like this (PF)a this came out as a function of XC there X c is V c by V t. So, what percentage of coarse Crain exists with respect to fine grains or the total number of grains? So, this was (PF)f fine here X c is a variable it will change it can change from 0 to 100. So, this is the relation that we had obtained in the previous class.

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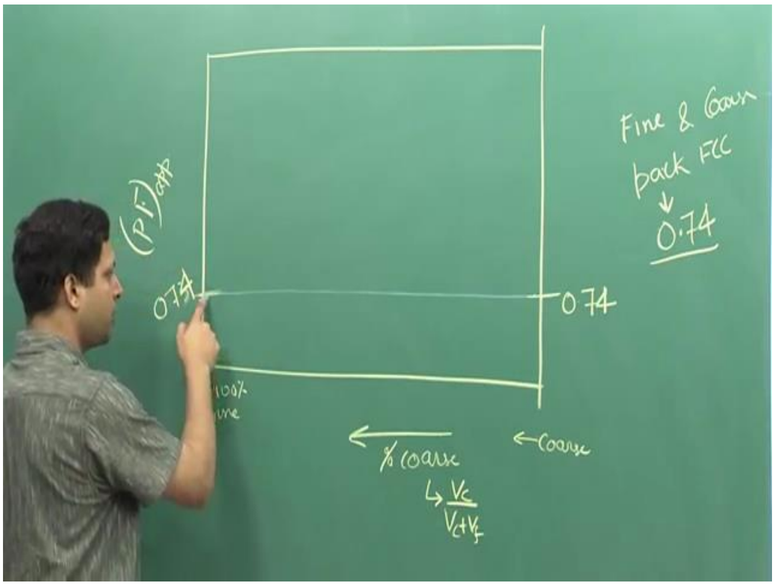

Today we will see how the plot will look like where we change on the X axis will have fraction of coarse grains and on the Y axis we will have the packing fraction. So, let me take a couple of colored chalks as I said on the Y axis we have packing fraction effective or apparent and we will make another assumption , so that we can put in exact numbers and that is that both fine and coarse pack in FCC tab structure meaning they will have 0.74 packing fraction.

When we are talking only about the packing fraction of fine or packing fraction of coarse then it will be equal to 0.74 on the X axis will have here we will have percentage coarse particles. So, here it will be coarse here it will be fine that is 100% fine and we are going from here to here, so it is percentage fine percentage coarse increasing. So, percentage coarse particle increasing now when I say percentage coarse particles you have to be careful what is this is it Vc by Vc + Vf or Vc by Vc + Vt.

So, as it turns out it has to be Vc + Vc by Vc + Vf and not Vc by Vt what is the difference between Vc, Vc + Vf and Vt? In the Vt also have the volume of the pores, so we are not taking into account the volume of the pores we are only concerned with the volume of coarse with respect to the total volume of coarse + fine and we will see again why this is a more meaning full X axis or the parameter to take on the X axis then Vc by Vt.

In the equation if you see we have Xt it is Xc which is Vc by Vt, so it is a little a different however it we will still be able to plot this. So, let say when you have only the fine particles and only the coarse particles what happens at that time. So, at this point we have only the fine particles, so here you will have a packing fraction of 0.74 we know this and in the other extreme we have only the coarse particle. So, even here will have 0.74 there is no variation in that or there is no discrepancy about that.

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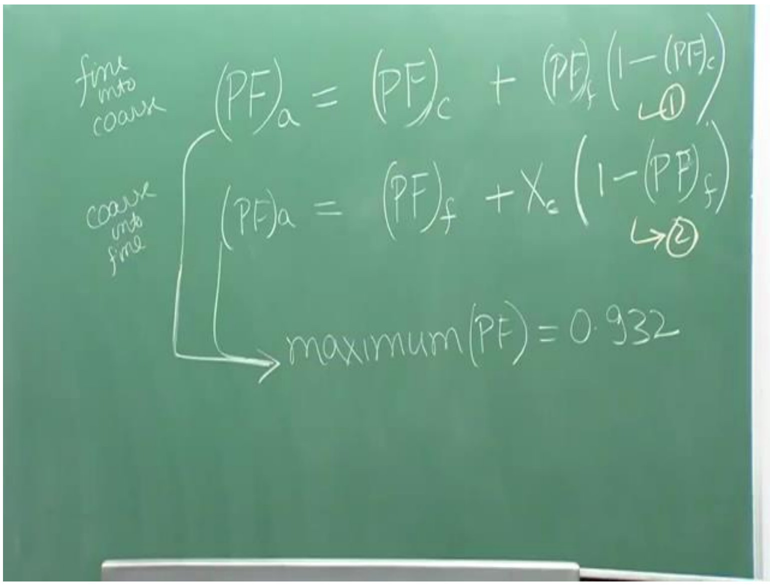

Now we know that in the fine when we keep adding the Coarse particle at particular percentage or at certain range you will see 0.932 remember we even you have all the fine particles field in the coarse then you get 0.932 and when you fill in coarse particles wherever it is possible in the fine particles then again you get 0.932. So, the maxima in both this cases maximum, so it represents the same point where you have you can say when you look at it there will be coarse particles all around and in the pores there will be fine particles.

So, you can reach it either way you have initially to begin with only the coarse particles and in the coarse, you begin filling the fine particles or the other way could be that in you start with all the fine particles and you replace you remember those this spheres of particles and pores with coarse particle but eventually you will reach earlier if you saturation where in you cannot put in any more coarse particles and therefore it will that set particular situation would be well the coarse particles are packed in their maxima efficiency and pores in effect the force are the were region were fine particles are getting field.

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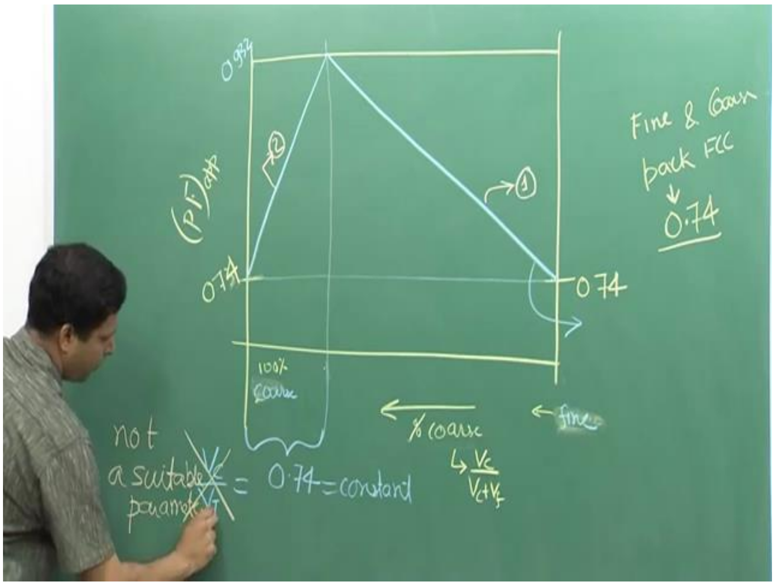

So, let us see where this would be, now in this when we are increasing coarse from the fine to the coarse. So, we are adding the coarse particle and the maximum let say this is 0.932. So, we are monotonically increasing the fraction of the coarse, so we can assume a straight line relation and it will be, you cannot add any more coarse particle to this, over here you have you started with coarse particle and in this you keep adding the small fine particles that you have the very small particles starts to fill into the pores and then eventually this will also reach a value of 0.932.

So, here this represents in effect our let me give an equation number to that, so this will if it is this is equation one and this is equation two then the first one is find into the coarse and therefore this is our equation one you are start with completely coarse and you are adding fine particles and this is when you have completely fine particles and you are adding coarse particles. So, this is equation two now let us look at why Vc over Vt would not have been a suitable choice.

Let us look at the ratio of Vc by Vt at this point, at this point we have completely field all the regions where coarse particles could have been. So, here you have coarse particles all around wherever it can be and therefore it is in the maximum efficiency which is 0.74. Now beyond that the fine particles are going only into the pores and therefore the ratio of Vc by Vt remains constant equal to 0.74. So, Vc over Vt is equal to 0.7 is equal to constant in this range and that is why Vc over Vt would not have been a proper choice for this region because the total number of the total fraction of coarse particles remains constant in this region you have increased the coarse. So, we have known the maximum amount of coarse and over here when you go over there the fine particles are decreasing.

So, there is one mistake I have done over here which is this is where you have 100% coarse and this is fine. So, just one correction take to your notes, so I have written here correctly it is percentage coarse, so this is this should have been zero percent coarse which means all fine and here it is 100% coarse because here increasing the percentage coarse. So, just a small correction over here and with that now you see that over here you have the maximum concentration of coarse that can cut into there and you are not increasing coarse particle as you keep moving over here you are only increasing or actually decreasing the fine particles you have some percent percentage of fine particles over here and it is decreasing and that is why Vc by Vc + Vf will change but Vc by Vt will not change and that is why this Vc over Vt is not a suitable parameter.

So, few more this is a very involved plot there are lot of things that we can start out of it one first of the thing is that although we have taken it for a special case where we pair the final coarse are getting packed in the FCC case or the FCC structure you can have any other you can have BCC you can have loose random packing you can have close random packing for one of them and a different one for the fine one and the fine one and that way you will have different values to begin with but the overall form of the plot would remain same that is at particular percentage of course you will reach the maxima and that maxima would also represent when you have when you have been adding fine particles into the coarse. So, both of them will eventually reach the same structure and you will have the same value at the mac.

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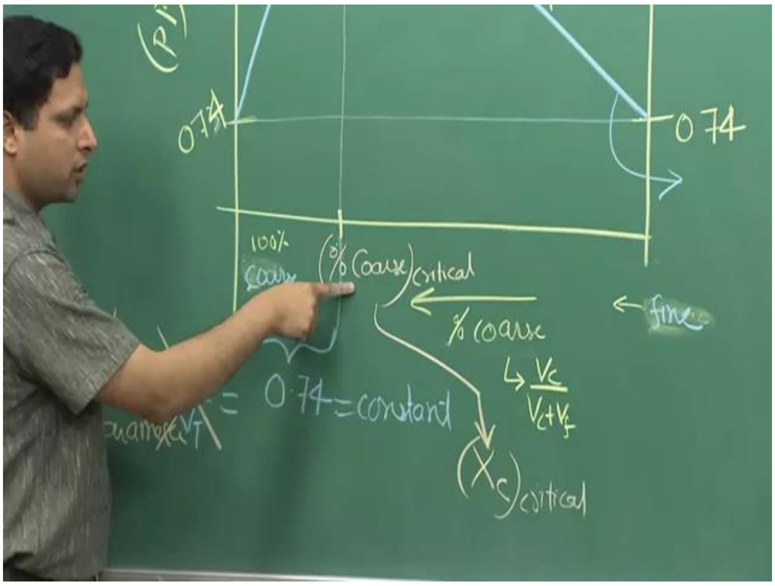

The value the exact value where change and we will actually look at one problem one one relation in the example problem to find out how to find Xc with respect to what will be the maximum value. So, this is some particular critical percentage of coarse and this corresponds to some critical Xc critical both of these are not same thing but for these percentage coarse critical you will have a critical Xc value and will see how to get this Xc value and this is how usually here plot would look like.

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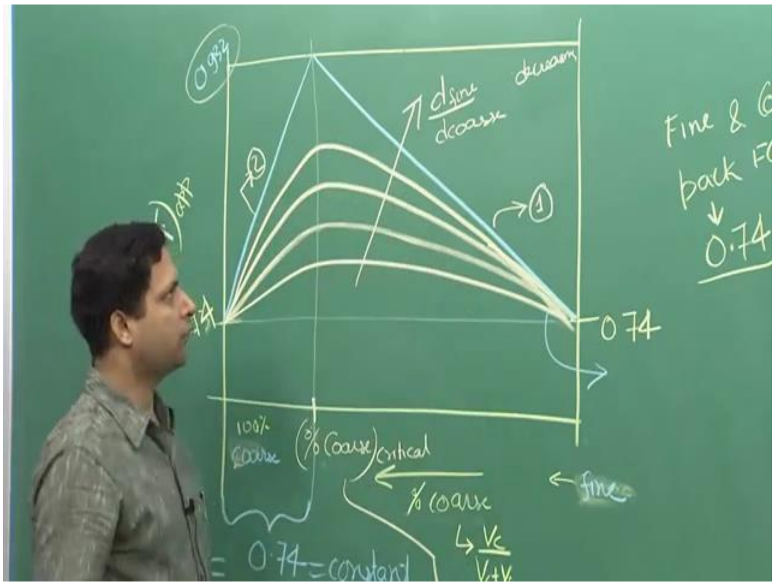

Now this is what we have assumed or we have drawn assuming a perfect packing for at each and every point in the in the whole compact but let say we are when you are actually doing it you know that the packing fraction it can be anything it can be a loose random packing and it can be a coarse random packing or it can or it can be FCC, BCC anything. However if we are actually doing it would you really see a straight line like this all the way and then coming like this actually no what you get is something like this you may get different plots and what will be the difference why you would get one, two, three or four different it will depend on the diameter fine to diameter coarse.

So, in this direction it is this value is decreasing, so as the ratio becomes much larger or basically the particle size becomes very different you will approach the ideal curve but even in the most in the case very have very large difference between the particle you would still not see a very closed match to this in fact, the greatest difference that you will see are in these regions and these the happens because of what is called as Edge effect.

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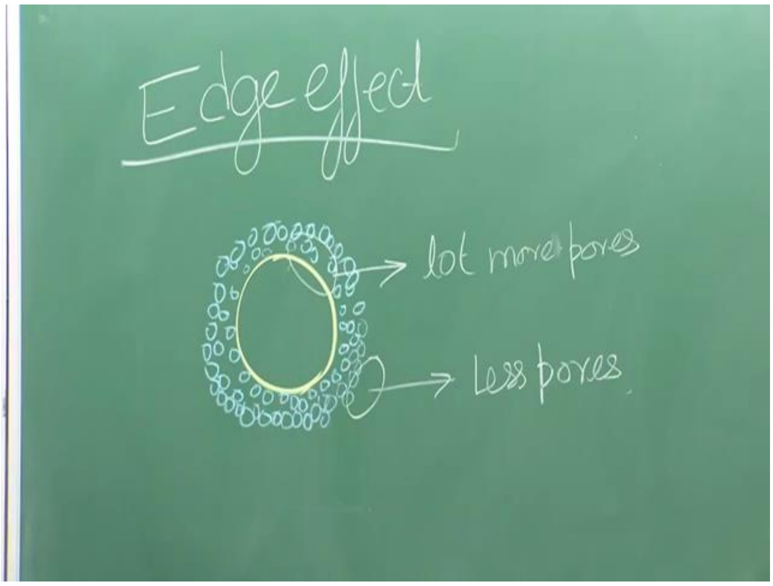

So, let see what is this Edge effect, edge effect is saying that you have some coarse particles and around it there will be some fine particles but because some of these particles you are trying to feel very closed to the coarse particle, the packing near very close to this coarse particle is not as good as it is a little distance away and this is called the edge effect. So, basically when you are much closed to the edge of the coarse particles the packing does not reach the optimum value.

So, what I am trying to convey here is that there are lot of pores as you get closer to the particle surface. So what I am trying to convey here is, if you look in this region there are lot more pores and if you go away from then less pores. So, this is called the edge effect when you have a large coarse particle and around it you want to stack fine particles then near the edge it does not it is not able to get packed as efficiently as it gets packed away from it.

Whenever there is a surface it causes that surface the existence of that surface causes the particles to not get packed as efficiently and this is why it is termed as edge effect because there is an edge or a surfaced that leads to this inefficient packing and this is why you get a larger deviation in this kind of region and that is what you get that is why it least to overall deviation from the ideal curve. So, this is how a bimodal grain size distribution or particle size distribution looks like and there packing looks like.

So, now let us move on to something a little bit more complicated, so far we are looked that bimodal now what about if we were looking at a quaternary distribution, we will not get into the detail but just qualitatively and not even qualitatively just look at some result what people have obtained for quaternary if you go with a quaternary and in the most efficient way people have shown that if you have the diameter ratios something like this, then you get a relative volumes for these pairs of these sizes something like this.

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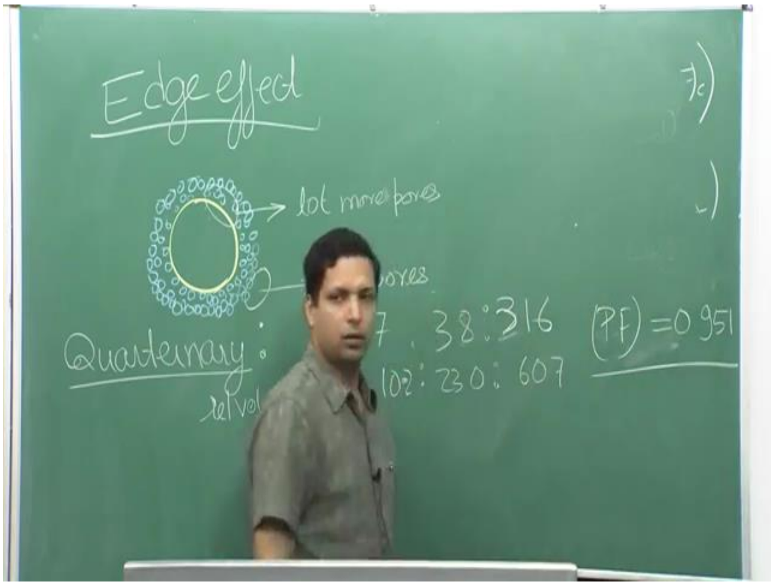

So, this is 10.2, this is 23.0 and the last one is 60.7, so this is the relative volume and these are the relative sizes. So, you let say if I take 1 millimeter, this is 7 millimeter, this is 38 millimeter and this is 316 millimeter, in this ratio if you take the four different particles and when you have a volume like this and this volume is for the optimum condition where you get maximum packing fraction. So, when you have volume ratios like this, so 6.1% of this, 10.2% of this, 23.0% of this and 60.7% of this then the best packing fraction that people have achieved is 0.951.

So, this is the apparent of effective or in the optimized packing fraction for the quaternary distribution you see is from when we had mono sized diameter mono diameter particles then will the best we can get is 0.74 when we get to bimodal we get 0.932 for the quaternary we get 0.951. So, that is how there particles change or the packing fraction efficiency changes. Now the question is if you had a continuous distribution would you expect a better packing fraction or worst packing fraction.

So, so for we have said when we say bimodal, so it is there just two grains if you have a quaternary then they have added to more peaks over here but now we are saying instead of having these distinct distribution let say if you I had a continuous distribution. So, it so happens that continuous distribution is somewhat better than your mono sized distribution but it does not yield much better then what you would get why turnery or quaternary distribution or turnery or quaternary modal by quaternary distribution of the particle sizes.

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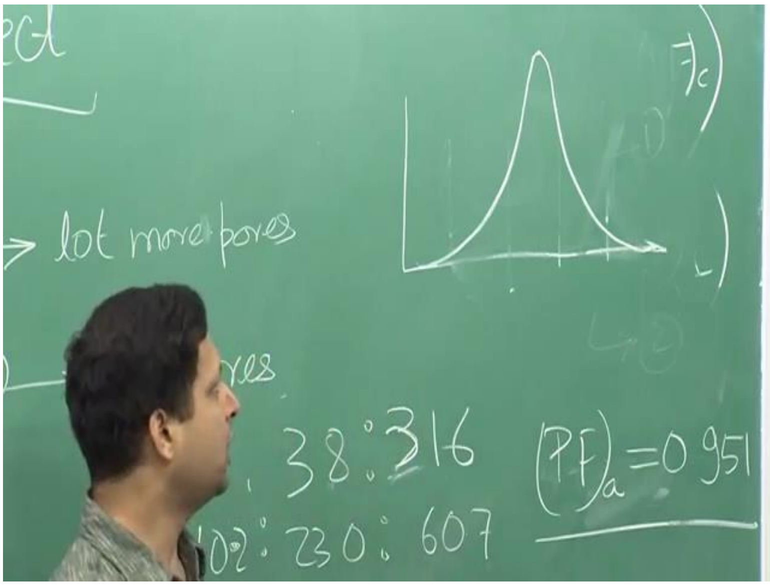

In effect what I am saying is that when you have this then the packing fraction may not achieved may not reach as high as 0.951 value it way get better than 0.74 but it may not get better than 0.951. However, there is another important expect about the continuous distribution and it that if this distribution is very wide then it is better in terms of packing fraction than if you had a much more narrower distribution. So, if you have a very narrow distribution it will start to approach something like what we know about mono distribution or the mono the singled sized particle size distribution however when it is much wider then it becomes much more efficient and gets a higher packing fraction.

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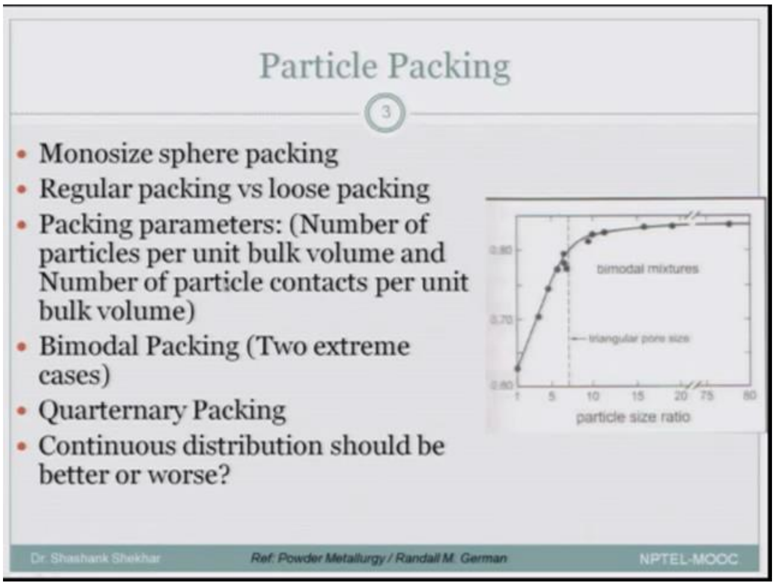

So, let us get back to that slide and see what we have learned so will we look that mono sized is fear packing we looked at what is regular packing verses loose packing we looked at packing parameters like number of particles per unit volume and number of particle contacts per bulk volume we looked at bimodal packing the two extreme cases and the plot for it is also shown over here. So, we assumed that the particle diameter ratio d large to d small should be at least greater than 10 and the reason for it becomes clear when you look at this plot on the Y axis you have a theoretical density which is equivalent to packing fraction and on the X axis you have particle size ratio.

So, here you see that this theoretical density or the packing fraction reaches it optimal or saturation value somewhere beyond 10. So, at this value it reaches it optimal value and that is why we set or we assumed that the particle size must be greater than equal to 10. So, from this plot it becomes clear the importance of that value 10 we also let that quaternary packing we just looked that the optimal condition people have obtained and we saw it was closed to 0.951 and we also discussed what will happen in continuous distribution is it better or worse. So, these are some of the aspects you have looked so for next look at what is the effect of powder shape and roughness on to the packing fraction.

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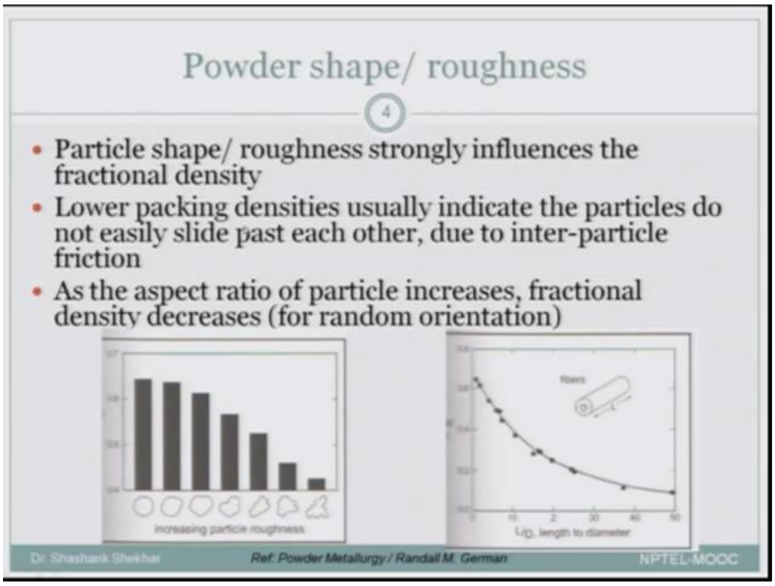

So, as you can easily aspects and guess that particle shape and roughness, roughness what we mean is not the microscopic roughness but the roughness caused by the shape change in the particle or morphology change in the particle. So, particle shape and roughness also strongly influence the fractional density and on here you are given some different kinds of particles shapes on the X axis in this plot and you can see this is a much more symmetric or in terms of the roughness related to morphology it has much lower roughness and if as you keep changing the symmetry and the in effect increasing the roughness you see this is the relative density or the packing fraction.

So, over here it is 0.7 although it is not very visible, so it is 0.7, this is 0.6, 0.5 and 0.4. So, this is decreasing and it can change very vastly or it is very sensitive to the particle shape and morphology. Not only that even the aspect ratio of particles can change the overall the packing fraction. So, again on the Y axis you have the packing fraction, so here it is 0.2, 0.4, 0.6 and 0.8 on the Y axis and for simplicity a cylindrical fibers were taking by as particles and they were try and people have trying to pack it.

So, here you are increasing the L over D meaning you are increasing the aspect ratio. So, you can see how the theoretical density is continuously decreasing. So, over here when you have almost flats or very small aspect ratio meaning they will be of the say modal in that case you get the best packing fraction over here and as you keep changing the length or increasing the aspect ratio it drops it drops to even to very as low value as 0.1. So, this is how low you can get or how low this can cause the packing fraction to decrease. Now that we are talking about it let us also look at it the sum of the simple shape and what is the powder density in the untapped and tapped condition that you will obtained, just let us look at the number to get a feel of what we are saying.

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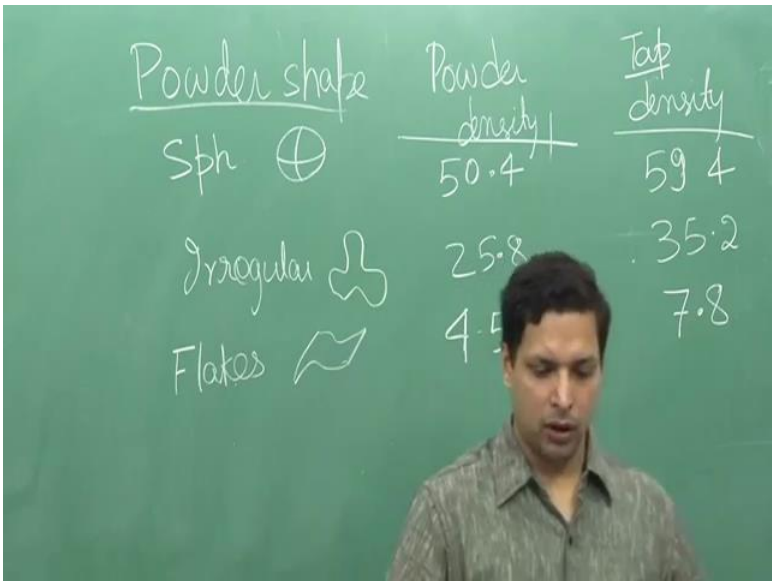

So, let say this is some powder shapes and if you take it in as it is condition will have some powder density and then you remember something this is equivalent to what will be the loose condition and if you tap it then you will get some dense condition or rearranged after rearrangement and that will be your tapped density and we are talking in relative in the relative terms.

So, here let say we take spherical powder, so something like this you have very spherical powder and if you look that the powder density 50.4% of the true density and after tapping it increases so it gets to 59.4, so tapping is always necessary to get the right condition before you start compaction. Now we let us get to some irregular shape something like this and it suddenly drops to 25.8, 25.8% of the true density. It can improve a little bit after tapping, so it gets to something like 35.2.

Now if you make it just two dimensional something like this. So, it is now two dimensional a flake kind of structure and this has the poorest packing density something like 4.5 in as it is condition and 7.8, so even after tapping it does not increased very large value and it remains at 7.8. So, this again tells you how the powder shape and morphology plays a role or is important information in determining the overall relative density. So, let us get back to again our slides and look at another important aspect agglomeration, we saw that in the some micron alumina you get 0.3 densities and that we said is because of agglomeration.

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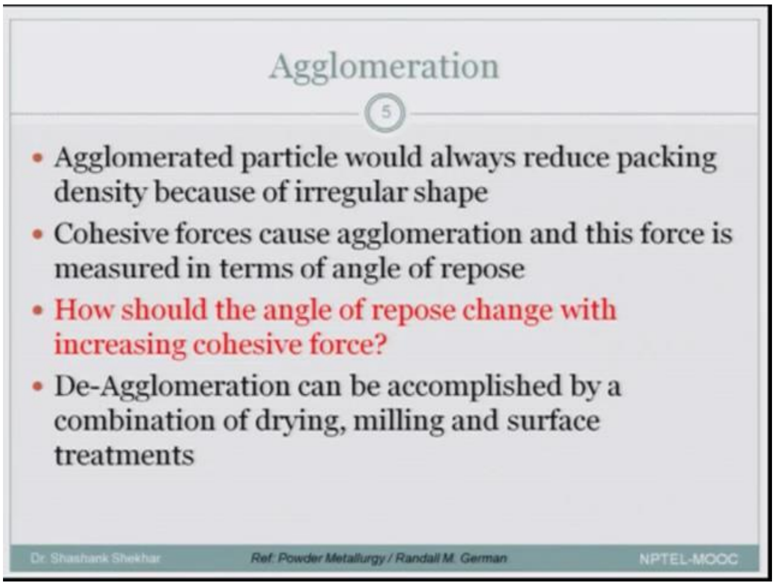

So, agglomerated particles would always reduce packing density because of irregular shape. So, although the individual particles may be spherical and uniform, in shape and size. However, after agglomeration it can become a very irregular you may have all the particles sticking to each other and giving it an overall irregular shape.

Cohesive forces cause agglomeration and this force is measure in terms of angle of repose. Now that we are talking about angle agglomeration, so this should also be aware of how to measure if there is any agglomeration. So, there is what is called as angle of repose which measures or which, which can tell you how much agglomeration is taking place. So, just a take a look at again what is an angle of repose it is not it is looking very difficult to understand it is just let say you on a surface you have powder then because of it is flow ability which again depends on the agglomeration and other characteristics it will form a heat and this angle that you formed is called the angle of repose.

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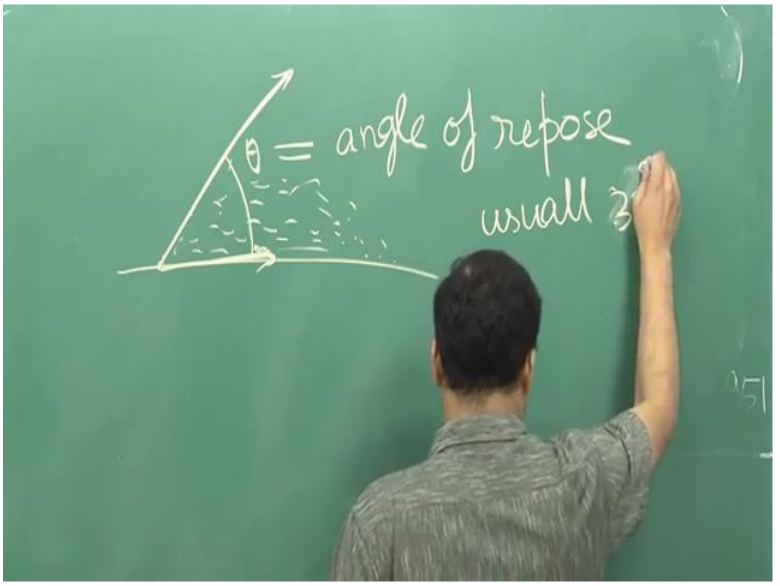

So, this decides the how much agglomeration is taking place if you have a free flowing particles then it is very small not very small but not to the lower side. So, it will be usually 38 degree for free flowing particles and if your if you keep increasing the agglomeration or it becomes cohesive, then it increases the angle of repose increases meaning it is not able to flow as smoothly as before and it increases to something in greater than 45 degrees.

If it is that if the angle of repose is greater than 45 degrees you can say there are cohesive forces and there is agglomeration taking place. So, that is the agglomeration and if ever you have agglomeration and you want to get read of one of the best way to deagglomerate is to either dry it, mill it or provide some surface treatment you there are there you can even put it in some proper solvent which can ensure that the particle get deagglomerated.

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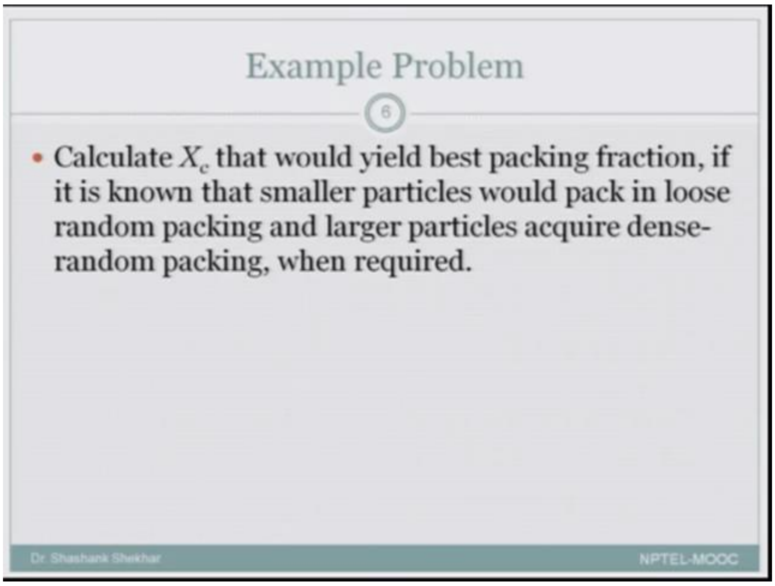

So, these are some of the methods to get deagglomeration and towards the end we have an example problem so will take a look at this example problem in the next lecture, again like I promised earlier this is about calculating Xc at the point where you have the maximum a packing fraction and that maximum packing fraction you remember is a part of both the curves we had two equations and we draw equations and we draw plot for each of these equation. So, this Xc belongs to both of the equation and the plot. So, that is the hint I am giving you try it on your own and we will solve this equation in the next class. Thank you.