Fundamentl of Materials Processing
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Lecture 36
Power Consolidation

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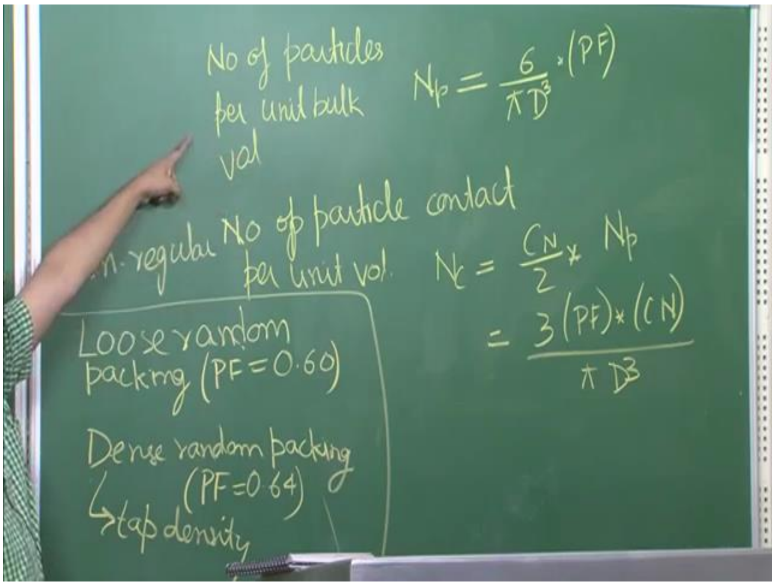

No of particles per unit bulk vol $N_p = \frac{6}{\pi D^3} * (PF)$

n-regular No of particle contact per unit vol. $N_c = \frac{C_N}{2} * N_p$

$= \frac{3 (PF) * (C_N)}{\pi D^3}$

Loose random packing (PF = 0.60)

Dense random packing (PF = 0.64)
↳ tap density

So in previous lecture, we look that some regular packing and also saw that there are some non-regular packing, of course the regular packing and not something that you would actually expect in a real material. What would you expect is a non-regular arrangement and for that we also saw that we can get relations something like number of particle per unit bulk volume and number of particle contacts per unit volume. So these were obtain directly from regular structure but then this can also be extended, the knowledge can be extended to non-regular structure and even before I proceed further, we have seen that review there are two non-regular packing we looked at one is Loose random packing and Dense random packing.

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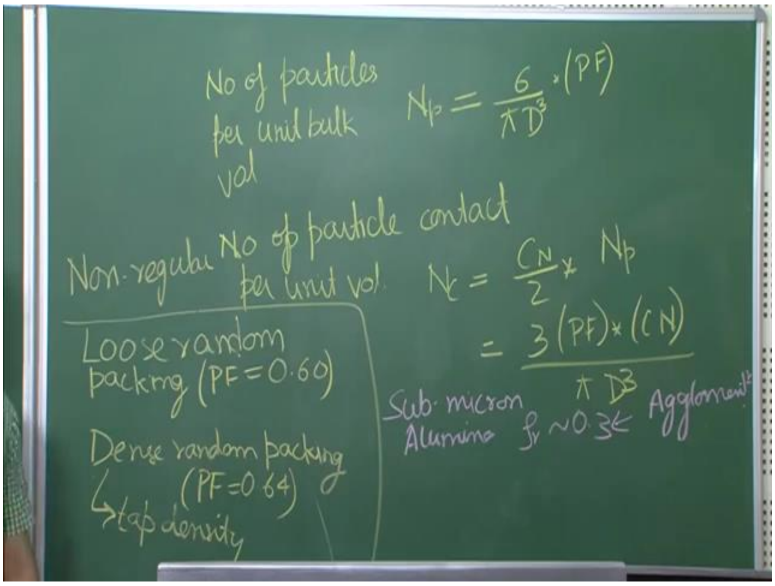

No of particles per unit bulk vol $N_p = \frac{6}{\pi D^3} \cdot (PF)$

Non-regular No of particle contact per unit vol. $N_c = \frac{C_N}{2} \times N_p$

$= \frac{3 (PF) \times (CN)}{\pi D^3}$

Loose random packing (PF = 0.60)

Dense random packing (PF = 0.64)

↳ tap density

Sub micron Alumina $\rho_r \sim 0.3 \leftarrow$ Agglomeratⁿ

However, if I tell you that for a sub-micron Alumina the rho V as that you get is of the order of 0.3. So what would your thoughts be? We said that usually you should get, when you are not tap det then 0.6 for a Loose random packing, when you have tap det then it should be Dense random packing Loose to 0.64. Then why are we getting sub - micron Alumina for us, something like rho this is again relative density, the relative density equal to 0.3. Then here we have to remember that in the when the particle size reduces Agglomeration tendency increases, so this is because of Agglomeration.

So Agglomeration can change this values, because in Agglomeration particles will come together and they will not be so tightly packed and each Agglomerate will itself be very Loosely packed and it will even if you get them to packed later on with a Dense random packing, in term the Agglomerate itself have very low density and so the average relative density would be very poor. So now have we said that, now let us again look at the Nc value that is the coordination the number of particle contacts per unit volume that we obtained. So we saw that this are Np is equal to 6 by pi D cube by PF. Now here anything we know can be obtain for even anon-regular packing, so this equation will be retain even when we are talking about non regular structure.

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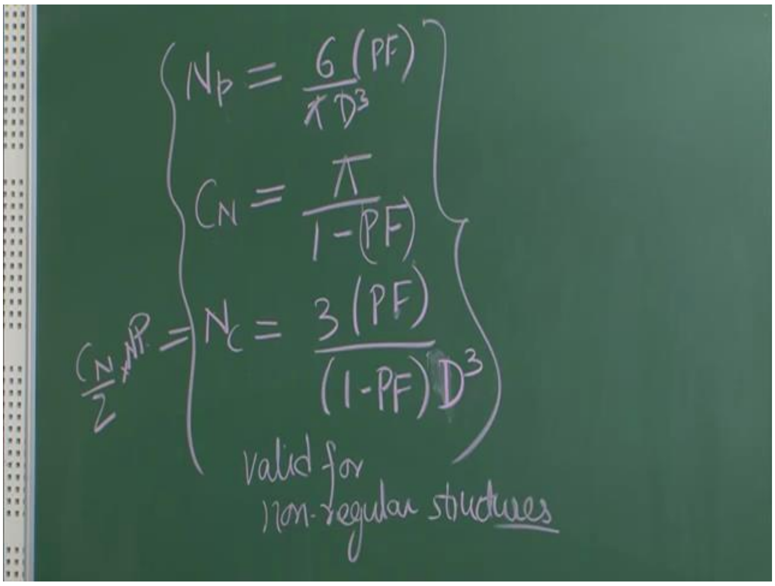

However coordination number is not something that we can obtain for a non-regular structure very easily and some researchers have shown that coordination number can actually be given by under certain condition like this. So this is now a equation or a relation that we can again utilize even for non-regular packing. So this is the coordination number and if you have the coordination number then we can also obtain number of contacts points per unit volume, which used coordination number by 12, by 2 and therefore this will come out to 3 by PF times 1 - PF cube. So this is a CN by 2 times Np and a over here you would get, the form of the relation would be like this. So this is equal to CN CN by 2 times NP.

Actually there is no cube over here. So this should be 3 PF by pi, so this is actually D cube D cube not 1 - PF cube. So this is 3 PF by D cube pi gets canceled and you also have this 1 - PF factor. So this is the relation for that is valid for non-regular structure. Here because we do not know the coordination number, so we obtain this coordination, relation for coordination number as obtained by researchers and they have given it like this in the in terms of PF, which we can calculate or we can obtain and then we have Np and Nc. These are some important parameters to understand how much densification is taking place. So this is valid for non-regular structures.

Now thus we have introduced some important parameters, which will be useful, when we talk about densification. So now let us get to next step, where, so far we used just one mono or mono size the mono size spherical particles. Next we want to move on to a pi model distribution of particles and see how much density or how much packing fraction we can get. Other once you have obtained the packing fraction then we know we can obtain other important parameters like number of particles per unit volume and number of contacts per unit volume.

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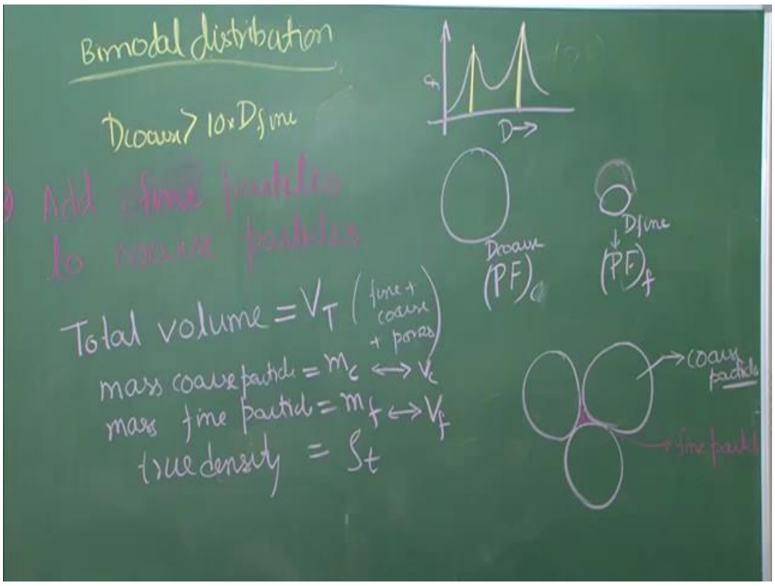

So now we will move to the next stage, which is to find out what will be the packing fraction or at least apparent packing fraction, when you have bimodal distribution. What do you mean by the Bimodal distribution? Ideally it would mean that you have two distinct diameter and spread around it. So something like this, if I were to draw it. So here you have diameter and here you have frequency. So this is one diameter this is another diameter, that in our case we are making we making the case even more simpler and we are saying we have only diameter, only particle of this diameter and particle of this diameter.

We do not have, we are not considering even a distribution. We have particles only of exactly two diameters and further make things a practical or a so that we can formulate relations around it, we will assume that D fine is less than or let say D course is greater than 10 times of D fine. So our course diameter particle are at least 10 times that of fine size particle and will see where this friction. So now we have to put it like this. We have two particles, let us say one D, we will call D course and another small particle will call D fine and one if it is getting arranged in some particular arrangement then will say, arrange assume that it has a packing fraction, which is constant given by PFc and when this is getting arranged in some fashion, will assume it is constant. Meaning it is not changing from one point to another and this is F fine.

So now that we have here, will have to stream cases. So let me write it one, when we add small particles to course particles. Meaning course particles are already arranged or already sitting inside a jar and pore in fine particles. So okay let to be again consistent; let me use fine instead of small. So add fine particles to course particles. Now what is happening over here? So again the way have drawn it is not (())(0:09:09) let make it even smaller, in fact it is even be smaller and therefore what we have is something like this, so let say there are particles arranged. There will always be some tetrahedral or triangular pore kind of structure found over here and thus alike said the drawing data shown here is a little exaggerated; particle size would be even smaller than this.

So let say this is our small particle size, so this is our and this is our course particles. What we say is that when you are adding the fine particles to the course particle, the fine particle are taking the pores they are not displacing the larger particle. It means the overall volume, the empty volume is getting reduced or the poor volume is getting reduced and therefore the packing fraction must overall increase. Now let say we have, let us define some of the terms. So total volume is equal to VT meaning it includes the volume by the large particle by the course particles sorry, the Courses particle, the fine particles and the pore.

So this is fine + course + pores, everything included is our total volume. Then we have mass of course particle. Meaning they are now only considering the mass, masses of the course particle. Similarly you will have mass for the fine particles. So you will have masses of the fine particle and corresponding to that there will be a Vc, corresponding to that within volume of the course particle, corresponding to this there will be fine particle volume, which is Vf and remember Vc + Vf is not equal to 1, because there is still some course. So Vc + Vf + V pore is equal to 1 or 2 be total not 1 here.

We are not looking at ratio. So Vc + Vf + V pore is equal to VT and we let me also define true density, meaning what is the bulk density, so will use the term rho t and therefore if you were to take mc by Vc, it will be equal to rho t true density because we are looking only at the masses of the course particle and only the volume occupied by the course particle.

So this is the true density, if we take mf by Vf that will be true density because we are talking about of the masses of the fine particle and only the volume taken by the fine particle. However, if we take mc by VT then that is low density, lower than rho t. We can now we are taking about whole volume, which includes if there is no fine particle then it includes the pores particle + the post between the course particles that volume, so mass by VT. So that densities smaller and that will be you can say the rho c are the course density.

Now let us move on and try to look at how to get the apparent density for this. So the apparent density can be, what do you mean by apparent density, meaning the total mass that we have for the course in the fine and the total volume, which is volume course volume fine + volume pore. We are not dividing it by only volume course + volume fine because that will be equal to the true density. We want average or relative density with respect to the bulk density, because course have got have got included and that is what is making a lot of difference.

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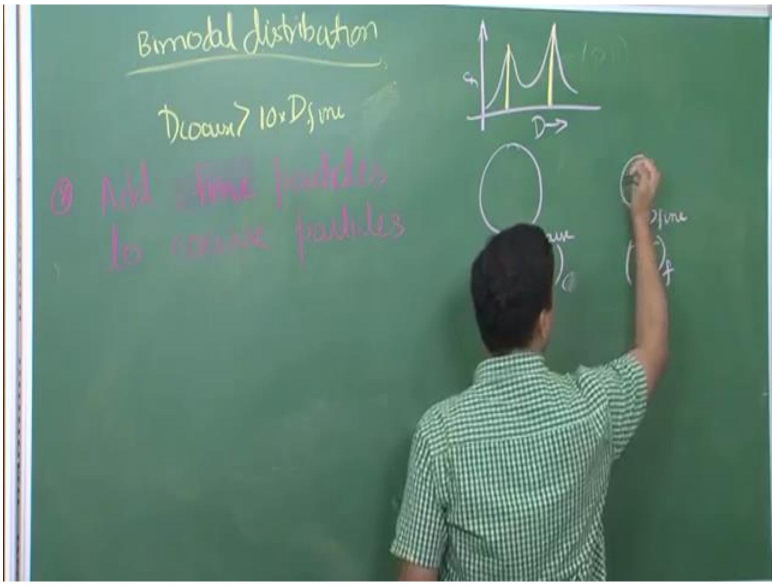

So this is what we are interested in which is going to be give us the rho apparent and this will be equal to, we can write it as mc by Vc times Vc by VT + mf by Vf into Vf by VT. So we have just multiply it by Vc by Vc, this one by multiplied by Vf by Vf. So mf by Vf we know what it is, it is rho true, mc by Vc, we know what it is, it is rho true and Vc by VT, we know it is equal to let it, we will let it be like this for now. So we have rho true in both of them and I have shorten from apparent app or we will even make it rho a, so this is rho a by rho t times Vc by VT + Vf by VT but again here what we will do is, will take Vf or multiply it by VT - Vc.

You will see in a moment why we have done this. So we have multiply it on both sides. Now Vc by VT, what it is, it is nothing but packing fraction of course. We are looking at the volume taken by the course particle and the total volume, so that is the packing fraction of course. We here we are looking at volume taken by the fine particles and the pores that where available to them overall, which is VT - Vc. So the overall pore that was available after taking a well of course particles. So this again represents PF fine and that is why we multiply it in the numerator and denominator by this quantity VT - Vc.

Now over here we have again VT - Vc by VT, which is equal to 1 - Vc by VT and we know Vc by VT is again PFc. So this becomes 1 - PF course and what is this rho apparent by rho t. You can see that this can be written as, so this is mass by density. So let say this is mc + mf still get canceled anyhow and this is by the VT. This is again Vc, mc + mf, but here we are only looking at the volume of course + volume of fine, so this translates to Vc + Vf by VT. Now this is nothing but this is equal to our rho packing fraction apparent, this is you can say the overall average packing fraction of this material because these are the volume which have been occupied by the particles but this was the total volume available.

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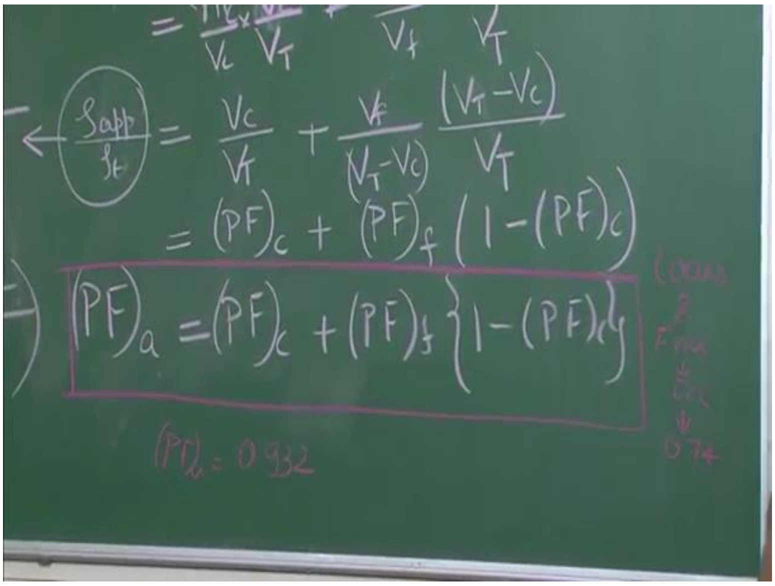

So that is what represents the packing fraction. So this is the apparent packing fraction, times. So we have now a relation for the apparent packing fraction. When we have two particle size, two diameters, one is the course one and the another one is fine one and based on that we have a packing fraction apparent, which is equal to which can be given in terms of packing fraction course + packing fraction fine times 1 - packing fraction course. So this is if you put the values for example let us assume that both course and fine packed in Fcc. Itis kind of face and the cubic type structure.

Just for the sake of arguments, we are saying this and therefore if you put this value, which is equal to 0.74. So both packing fraction course and packing fraction fine will become 0.74 and when we put it over here. You would see that packing fraction apparent is equal to 0.932. This is just one example; here you could have any other arrangement for course and any other arrangement for fine. You could as well have this as Loose random packing, Course random packing or both of the simple packing or anything in between. You can just say or you would know your example from your experiment that your pores particles get arranged and have a packing fraction of 0.4 and fine particle have a tiny fraction of 0.35.

So you will again be able to put those values over here to get, to understand what would be the overall packing fraction. So that is one extreme case, when we have added fine particles to the course particle. Now next let us move to the other extreme, which is with this time we will add fine sorry the course particles to the fine particles and why will it be different, we will see in a moment. It will be treated or you will have to treat it differently, so we will add course to fine ok.

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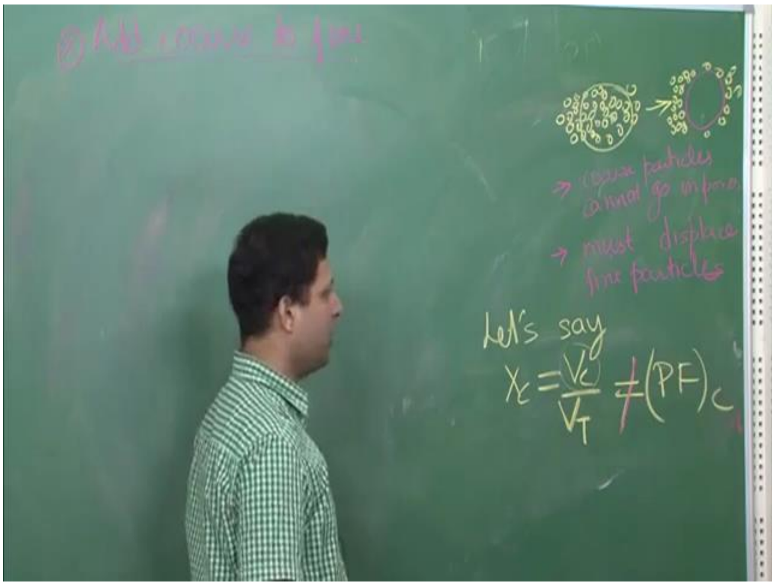

So we are not saying or basically what we are doing over here, is that this time all the smaller particles or already arranged in somewhere with some place ok. Now you know that fine particles sorry the course particle cannot go in course right. They must displace that is important part they must displaced. Earlier we did not have any displacement, must displace fine particles. So that is what makes it different. Here you here the course particles cannot get into the pores, they must displace the fine particles and then come over there.

So for example, here we were drawing like this. Now let me say there is a big region like this and when the course particle comes over here. It will take this region and around it the fine particles would still exits and so on. So again you can see that when we displace the fine particles by the course particles, there is increase in the packing fraction because in this region now, whatever pores were there, they have disappeared. Now it has been displaced by a bulk material.

Another difference here with respect to the previous case that we saw is that in the previous case we assumed that all the pores, because their fine particles are going into the pores. We assume that all the fine, all the pores would be taken by fine particles. But here we have a course particle displacing the fine particle. So we are not assuming that it will go, it will fill all the places and therefore we will have to assume some fraction. So let say Xc equal to Vc by VT is the fraction of pores particle in the overall volume. Meaning this much fraction in the total volume has been replaced by the course particles. By and another thing is that this is not equal to packing fraction of course.

In the previous case, where we assume that the overall course volume were fully packed the course particle were fully packed in the volume. Then over there Vc by VT would have meant packing fraction. But here some random places to begin with will be occupied by the course particles and therefore that Vc by VT does not represents packing fraction. So you must be aware that this is not packing fraction. This just represents section or the fraction of the volume of the total volume, which has been taken by course particles.

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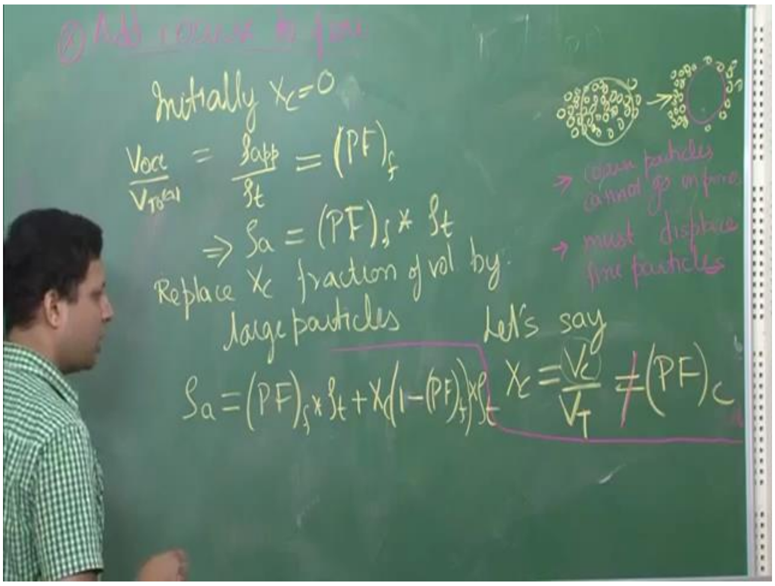

So continuing with this, initially when Xc is equal to zero. We can say that V occupied by V total is equal to we saw that rho apparent by rho total is rho sorry rho true is equivalent to saying V occupied by V total and this was, this would be equal to packing fraction fine. Because initially we are saying Xc is equal to zero, which means there is no course, there is no course particle only fine particle. So if we are looking at the initial stage, where Xc is equal to zero. Then V the packing fraction apparent which is equal to V occupied by total volume is equal to rho apparent by rho total is equal to packing fraction fine.

Now this would you also imply that rho apparent is equal to packing fraction fine times rho true. Now replace Xc fraction of volume by large particles ok. So this is at this stage we had only fine particles. Now we are saying Xc volume, which is Vc by VT, amount is being replaced by large particles. So you can say that rho apparent will be equal to this will be increased by how much, let me just put a line over here. We are replacing those regions by the large particles, which means whatever porosity their existed does not exist anymore? It is now a bulk material with bulk density. So it will increase by so this is the amount of pores that have been filled and therefore this in the increase in the rho apparent.

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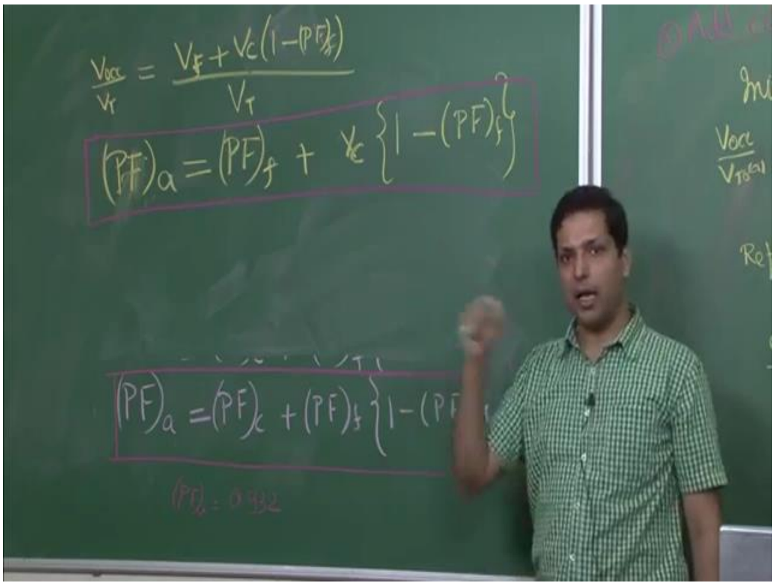

Another way we can also look at in another ways. So let me write it over here and keep the lower equation. So that we can compare it with the new equation that we will obtain. The another way to look at it would be, let say V occupied by V total is what we were looking at, so this is Vf + Vc. So initially when there were no course particles, this would have been V fine by to be I was using small f. So let me continue with small f. So initially it would have been Vf by VT. But now some of that region we said X, the Xc volume has been replaced by course particles. So that replacement is represented by this volume.

So this is again the volume which has been occupied Vc times 1 - PF f. It is reply the amount or volume which has been replaced, has the packing fraction one. Initially we were counting it as packing fraction f. So the difference have been added one - PF times Vc, which is the volume which has been replaced and therefore this represents again the rho apparent V occupied by VT is the rho apparent and therefore we have rho we can say that PF a is equal to PF fine. So this two are equation are you can see are same. What all we are doing is taking rho t to the denominator, because rho t is common on both of them and we are left with rho a by rho t which is equal to PF apparent and therefore this becomes PF a + Xc 1 - PF fine.

So this represents the apparent packing fraction, when we are replacing the larger particle, the smaller particles by larger particles, Finer particle have been moved. So when we moved then we do like this, that is the there were some very fine particles and in in that we have replaced by a bulk, it is as good as a bulk and here is the size. So this these are the course particle this have, this are come in place of this region and around it we still have those fine particles as we had earlier and in that case you will get PF apparent equal to this and the two main important difference between this are, one over here you see that Xc is present or Xc is a factor, which is a variable. So we are looking at it as a function or as of function of how much fraction has been replaced.

Remember we said not all the region can be packed with course particle unlike in the previous one. In the previous one, the fine particles where going in the pores and therefore we could have easily, we can easily say that all the pores are fill and therefore there is constant relation. When it is talking about the maximum or the point of where maximum packing has taken place. Course particle has completely packed and all the pores are bind by fine particle, which is representing that condition. We can even look for a condition where that is not the case or where it is varying, where some amount of a pores have been filled.

When again some more amount of fraction course have been filled and finally when all the courses have been filled. So that will be the region and we can also get the relation a with variable amount of fine particles. But this inherently this equation, inherently has a Xc, which shows how this relation varies with a fraction of the particles or the volume which have been exchanged by the course particles. So that is these are the two main differences. Next time what will do is, will try to or will show, how to plot this.

Not how to, basically, how it will look like when you plot this as a function of course particles in the total volume or as a function of fine particles in the total volume and we have to be careful, when we go, when we make that plot. If you do it as a function of let say volume by total volume then it will be very different from what you from what we should see when you draw it as a function of let say volume of Vc by Vc + VF. Those two would be very different, so will look at that plot in a next lecture so will come back on this slide, thanks.