Fundamentals of Materials Processing (Part–1)
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Lecture Number 25
Fluid Flow Considerations

So students we are coming close to the end of one of the module of this course which is the solidification part. So in the last lecture, we discussed about plane front solidification in polyphase or when you have more than one phase. In this particular case, we were looking at two phase system. So we looked at what are the solution condition or the liquid concentration, how it varies, and another thing that we looked at was the interface stability criterion. So what is the condition that is required for the interface to maintain that solid the plane front solidification?

We during, when we were writing the equation for the concentration of the liquid, we saw that it was a very complex and scary equation. But, I also said that promised that I will solve one equation to show that it is not really that scary. So it is time to get to one solved example. So let us get to the slide shows, one of the example problem.

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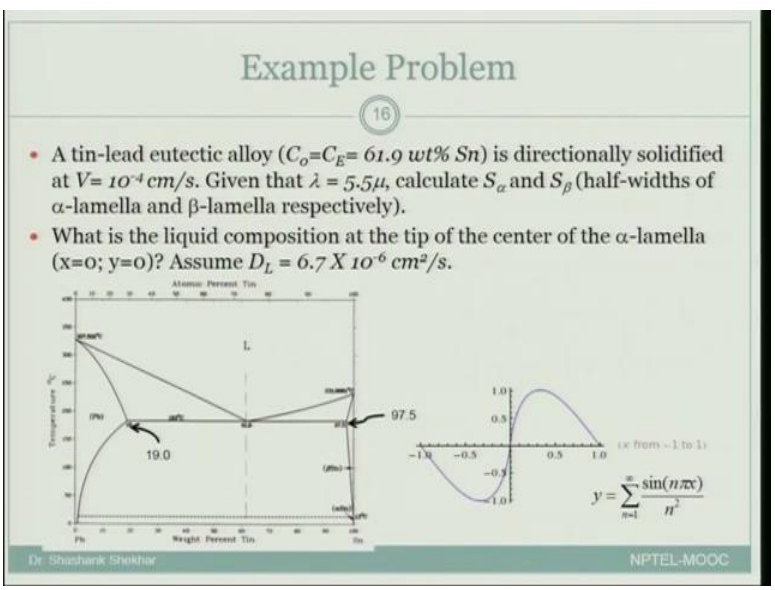

It talks about a tin-lead eutectic alloy; so you remember we are always deriving this equation about a eutectic alloy; so C0 is equal to CE; and in terms of weight percent it is given C is equal to 61 point 9 weight percent. It is directionally solidified at V equal to 10 to the minus 4 centimeter per second; it is given that lambda is equal to 5 point 5 mu; you remember lambda is equal to 2S alpha plus S beta. So it is given that lambda is equal to 5 point 5 micrometer. You have to calculate S alpha and S beta, that is the first part, and you have to also calculate what is the liquid composition at the tip of the centre of the alpha lamella, that is at x equal to 0 and y equal to 0.

You remember we take that point, the tip of the centre of the alpha lamella as the centre of the co-ordinate x equal to 0, y equal to 0. so that is our origin. So first we have to find out what is S alpha and S beta; you are given only lambda which is 2S alpha, 2 times S alpha plus beta. So you would ask how would I get from that what is the fraction of S alpha and what is the fraction of S beta; it is not really that scary as you would see; that is the first part. Now you know from the lever rule that this length will describe the weight percent of the beta phase, and this length will describe the weight percent of the alpha phase, right?

So what we need to do is find out what is the weight percent of alpha and beta, and since it is a eutectic, only those two lamellas are being formed, here is nothing else; and therefore it simplifies our equation. Now the alpha width and the beta width width be proportional to this and this respectively; so alpha width will be proportional to this, and beta width will be proportional to this. So let us get to get down to the board, and we have lambda equal to 5 point 5 micrometer, and therefore let us write it down.

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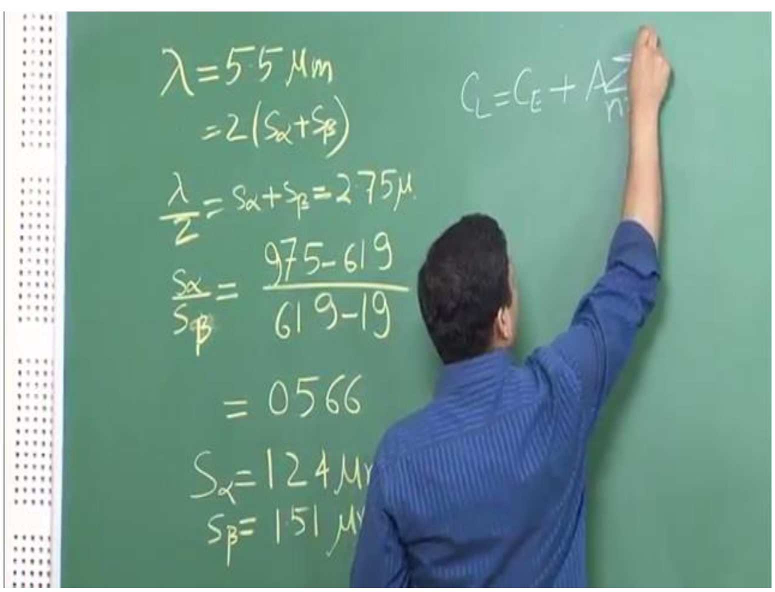

Lambda is equal to 5 point 5 micrometer, which is equal to 2 times S alpha plus S beta; and therefore you can say that lambda by 2 equal to S alpha plus S beta is equal to 2 point 75 micrometer. Now we also said that by looking at the lever rule, we can find the ratio of S alpha to S beta; so S alpha, S beta and you know the the CS alpha value, you know the CS beta value and you know the CE value; so S alpha by S beta would come down to this. And therefore, it will be equal to 0 point 566. Now if you know S alpha and by S beta, and you know S alpha plus S beta, then yes, you can calculate the values of S alpha and S beta, and we will not go through it, but you should be able to show it is just simple algebraic manipulation, and you should be able to show that S alpha is equal to 1 point 24 micrometer and S beta is equal to 1 point 51 micrometer.

So one part of the problem is solved; we have calculated the value of S alpha and S beta. But the next problem is the more complicated one, where they ask what is the liquid composition at the tip. So first we have to write down the general equation, then we have to put in the value of x and y, which is x equal to 0, y equal to 0, and then see if we can simplify that equation, because there was some really big terms. And to help us give us a hint in the slide you will see on the bottom right corner there is a plot given to us which gives a value of y for as a function of x where x is y is a summation n equal to 1 to infinity sine n pi x by n square. Okay, so this is a hint and you will see this how this would be utilized in our problem.

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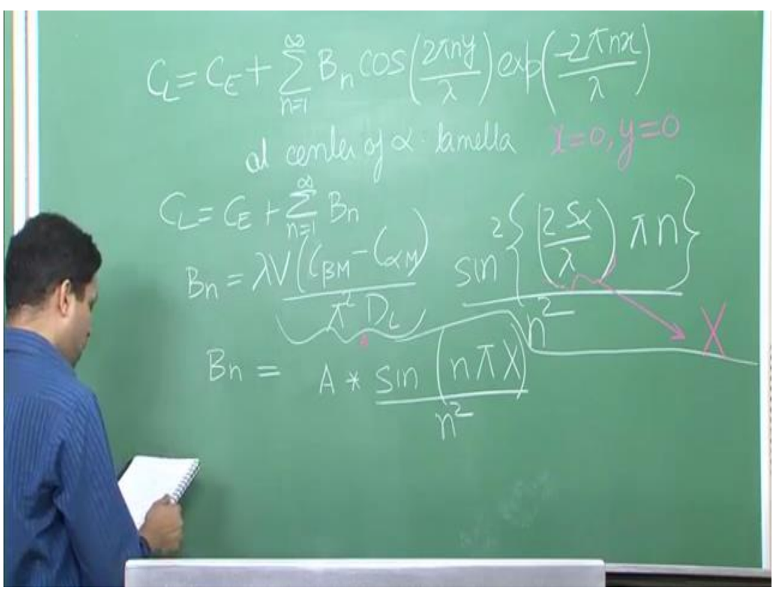

So let me take another chalk over here, and let us write down the equation we know, which is equal; it was CL minus CE, so I can write CL equal to CE, bring the CE equal to right hand side, and it becomes Bn cos 2 pi ny by lambda exp this is exponential term, that is varying with x; so remember just to revise, this is the exponential term that varies along the direction of growth, this is transverse to the growth; so this tells you the sinusoidal variation, and that tells you the exponential decrease in the concentration. Now, we are looking at centre of alpha lamella where, let me use still another chalk; I would like to use colours variation, so we have x equal to 0, and y equal to 0; or put in the values of x equal to 0 and y equal to 0 over there, and what you get is CL equal to CE plus y equal to 0 cos 0, which is 1, x equal to 0, e to the power 0 is equal to 1; so this whole, these two terms are gone.

So you see it has already been simplified to a great extent; but we are still left with one of the bigger terms which is Bn, which itself is a complicated term; but it will not be anymore, and we know what is Bn? Bn is equal to lambda V C beta M minus minus C alpha M pi square DL 2S alpha by lambda into pi into n and this whole thing is divided by n square. So this is our equation. Now you see this one this part is not very difficult; here uh we have C beta M, C alpha M, and we have pi square DL, all those things are very simple; and it will remain constant.

Only the second part, which is which involves n; it is this part that actually changes with each term. You remember we have n equal to 1 to infinity, so when you change n equal to 1, this changes n equal to 2, n equal to 3 and so on, it keeps on changing; and therefore this is a term which is, which needs summation, this will just come out of the summation. And, so let me write it like this. So this is the kind of constant, so let me put it as a A. So I am saying this is A, because A in the sense that it is not varying with n.

So let me write it like this. And, to, and to make it look or make it in a format which, for which we have been given the hint, I will use this as upper case X. So this becomes our upper case X, or the capital X. So this becomes n pi X over n square. So you can already start seeing that we have simplified so much, it is now it must be looking now something that can be solved, okay. So now at this point, let me write it over here, so that it remains visible.

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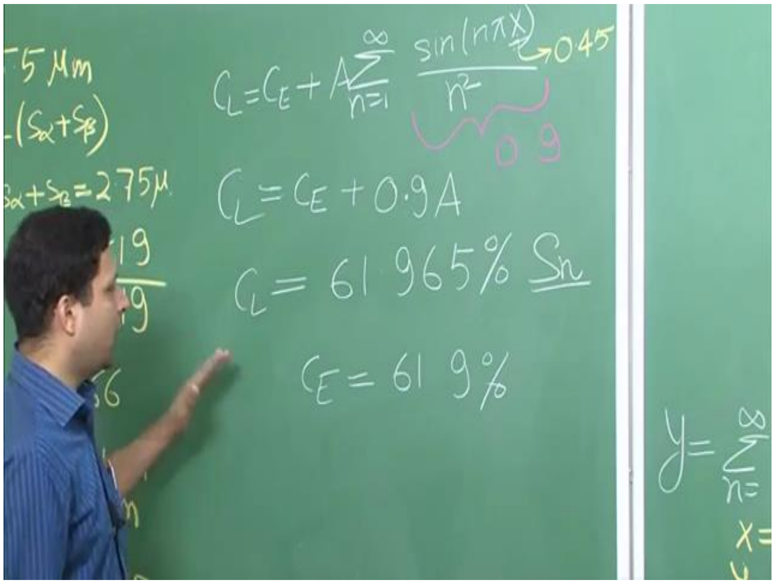

CL is equal to CE plus A times. Now this is upper case X, this capital X. What is this value of X; it is given as 2S alpha by lambda. So, we know the value of lambda, we know the value of S alpha, so we should be able to calculate 2S alpha by lambda; so X is equal to 2S alpha by lambda and you would get a 0 point 45. So now, our this X is equal to 0 point 45, units we are using, this is dimensionless, so do not get confused; there is no unit for this X, because S alpha has units, lambda has length unit, S alpha has length unit, so these two cancel out and this remains a dimensionless quantity. So no matter what unit you use, it will come out to point 45.

So now we have X equal to point 45, so this is sine n pi point 45 by n square, and it has to be summed over n equal to 1 to infinity. Now it is at this point that we will go back to our slide and see the equation that has been given to us. So this is the plot for this equation as you know, and for this is varying from x equal to minus 1 to plus 1. Our x has come out to be point 45, so what we need is to find the value of y at this value of x which is point 45. And you would see that if you take point 45, it would come somewhere over here, which is a little less than 1 point 0. So if you draw it more carefully, you would see that y for this I will write this; our y is equal to n equal to 1 to infinity sine n pi x by n square, and this is upper case X capital X.

Now when we put x equal to point 45 from the plot, for x equal to point 45, we get y equal to 0 point 9. So another thing to note is that this sum, as it is sum of the sine and still it is the sum, remember so it could have gone more than 1; but in this particular case, you see that if the y varies between, y varies between plus 1 to minus 1, and X is also varying actually from minus 1 to plus 1. The X is the upper case X which is equal to our 2S alpha by lambda. So this also as we know from this, sum over here that X, upper case X cannot vary more than, cannot go beyond plus 1.

So in our case, X will vary between 0 to plus 1. And here it varies from minus 1 to plus 1; this is the mathematical function, so it can vary beyond whatever range of interest lies. And for that particular range, for that particular value of X, which is X equal to point 45, we get y equal to point 9. So this whole quantity, let me now put it over here. So this whole quantity becomes 0 point 9. So you see, in steps we have been able to so easily simplify the problem and get a solution for it. A is something that you can have easily calculated; so now this becomes CL is equal to CE plus 0 point 9A, and if you put in all the values, you would see that CL is equal to 61 point 965 percent tin. So we were looking in terms of concentration of tin, so this is; and remember what is CE. CE is equal to 61 point 9 percent.

So you see, how small the variation is, that is another thing to look at, understand from this equation. You have the CE equal to 61 point 9 percent, and over here you see that the CL is varying by a very small magnitude; so it is close to this value A, this is the total amplitude, so the A into point 9 is coming out to be point 065 percent. So that is as small amplitude as you get for most of the conditions like this. So this is a very small amplitude, and it will remain very close to your eutectic composition, and that is why you saw that when we were talking about the interface stability, we ignored this term and there was another term for the off eutectic composition, which is based on the equation of full diffusion in liquid and no diffusion in solid, and we used that condition to find out what is the condition, what what is the growth rate or velocity that is required to maintain a planar solidification.

And, so that is this, and this is how you solve this and we get to see the, get a feel of the number on how the value should change. So that brings to end to the composition variation. Next, there are few more small topics remaining in the solidification chapter. So let us get to our some more final few topics of the of this solidification chapter. One of them is ‘Fluid Flow’. So let us look at fluid flow. First thing we will look at is fluidity. So what is the scientific definition?

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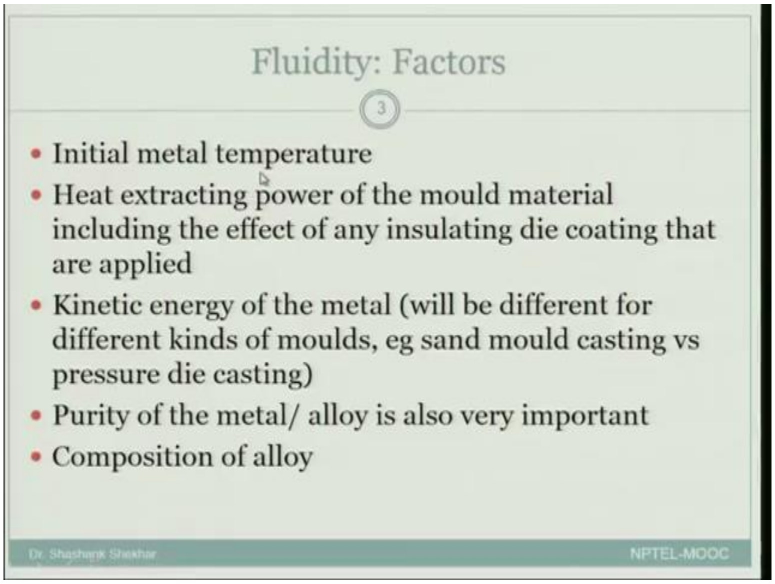

One is the initial metal temperature. So what will happen if you keep the (temperature) initial metal temperature much higher than the melting point? Let us say T0 is the or TM is the melting point; if you keep the liquid temperature TM, TM plus 50, TM plus 100, which one will be better for fluidity? So you can easily see that if it is higher the temperature, meaning if you keep it much above the melting point that is TM plus 100 in this case, then it will be able to go to a further distance, because then a lot more heat has to be extracted into the mould before the liquid will freeze.

So initial metal temperature plays a role; higher the temperature of the liquid, higher will be the fluidity. Next is the heat extracting power of the mould material, including the effect of any insulating die coating that are applied. So if you are using, for example let us say a permanent die casting or let us say a insulating mould (condi) insulating mould, these will have different ability to extract heat from the liquid metal. For example, a permanent die mould, which is made of steel, will be able to extract heat at a much faster rate and therefore the liquid will freeze much faster.

On the other hand, if you have a insulating mould, it will not be able to extract heat at a much faster rate, and therefore the liquid will be able to flow to a much larger distance. So heat extracting power of the mould also plays a role. And in case you have applied some coating on the inside surface of the die, that will also play a role because that will also lower down the heat transfer rate. So that will also play a role in determining what is the fluidity of the material.

Kinetic energy of the metal. If you pour the liquid at a higher speed, then it will obviously go to a farther distance; even if the heat is being extracted, but the amount of heat extracted per unit length is smaller, and therefore the heat will be, therefore the liquid will be able to travel to a farther distance and freeze only after travelling to a longer distance, and therefore kinetic energy of the metals also is important.

Purity of the metal. Now this is a very very important condition because in most of the cases what you will see is that as you go towards the eutectic, fluidity increases. One of the reason is that during the eutectic, the melting temperature decreases and more importantly, the liquidus and solidus has come to one point, and therefore the overall temperature of the liquid is at a higher, is much higher, and therefore, it relates to the initial metal temperature; it is at a much higher temperature than the melting point, that is also one condition. But even otherwise, the purity of the material has been shown to have a great influence, even if it is not forming an alloy, if the, if the purity is low, then in general the fluidity is also low.

Composition of the alloy, it is also related with the purity of the metal. So what is the composition, again whether you have alloy or a metal or pure metal or some impurity in it will also influence what should be the fluidity.

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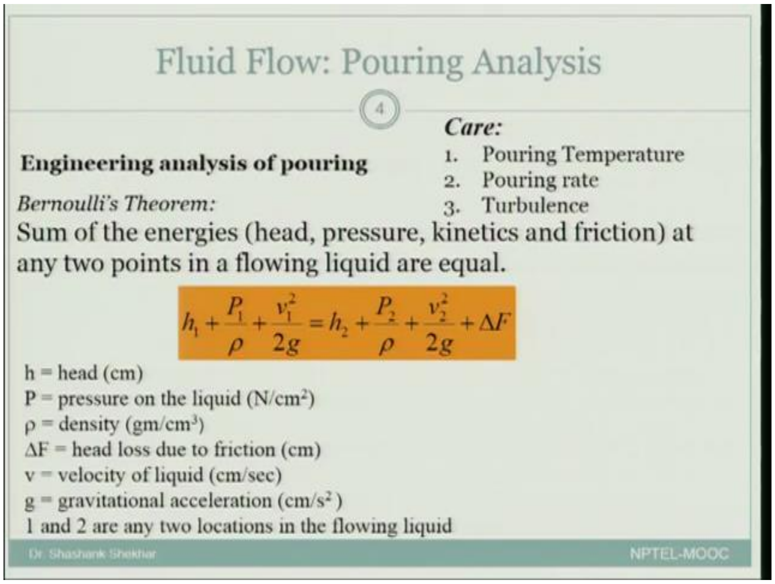

When we are talking about fluid flow, another important analysis with respect to solidification is the pouring analysis. Now when you are pouring, there are couple of things that you want to take care of: What is the pouring temperature? What is the pouring rate? How will I determine what should be the pouring rate? How will you ensure that it is not a turbulent condition? If it is turbulent, then it may start to break the mould. If the pouring rate is too fast, then the cavity through which you are pouring the liquid, it may freeze, the solid may freeze over there before it freezes into the cavity for the component that you want to make.

And for that, what we use is the Bernoulli's theorem. We know from the Bernoulli's theorem that the sum of energies had pressure, kinetics and friction at any two points in a flowing liquid should be equal. So it is given by this; this is the head, this is the pressure, this is the kinetics, and this is the term of the friction. So if we are looking at two different points in a liquid flow, then these two different points must have same amount of head. And the usual units are given in the bottom. So head is H is equal to head, and so on.

Now if we are looking at the solidification, we can get rid of some of these terms; for example, the liquid flow that we are talking about, at the such high temperature, then the friction force can be neglected. Another thing is that the pressure in both the cases will be equal; we are doing all the experiments at most likely at the atmospheric pressure, so P1 will be equal to P2, and therefore these two terms will get cancelled out, and what you will be remaining with, or what will be remaining is a equation like this.

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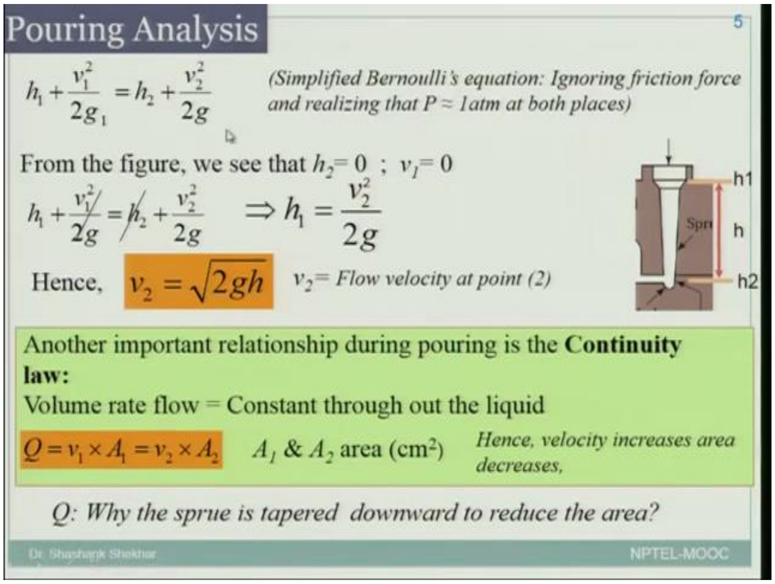

So this is the simplified Bernoulli's equation ignoring friction force and realizing that pressure is approximately equal to 1 atmosphere on both the sides. And from here, if you apply this to let us say, from where you are pouring the sprue into this sprue, so let us say you are pouring the liquid in this sprue, and this is the basin downpour basin; so let us compare this point and this point. Over here, to avoid any turbulent condition, we will assume that the velocity is 0, because the velocity will be gained thereafter when it flows through to the bottom of this cavity. And at this place we will take this as the reference, so the height h2 can be taken as 0, and therefore this height becomes h. So when we put that, we see that h is equal to v2 square by 2g. So this is the condition that we get for pouring.

Therefore this is the velocity that should be there at v2, or you can say this should be the height if this is the velocity for the pouring condition to avoid turbulence and to have a smooth flow, that is laminar flow would be maintained. Now you can use this equation in conjunction with another equation which is the continuity relation. So the continuity relation says that it is basically saying that the mass flow rate or the volume flow rate remains constant, assuming it is a incompressible liquid at those conditions. So volume flow rate remains constant and therefore q will be equal to v1 times A1, where v1 is the velocity at any cross section, A1 and v2 is the velocity at any cross section A2.

So what it is saying is total volume per unit time is same at point 1 and point 2. So we can relate these two and find out why the sprue is tapered downward to reduce the area. So when you look at these two equations in conjunction with each other, then you would be able to answer this question which is what you must have observed when you have done solidification in the labs, that the sprue is tapered; so this is this cross section is larger and this cross section is smaller; so let us look at why?

Okay so by the second part that I went to was the runner, but let us look at the sprue part. So now here, here your velocity is 0 or very small, here your velocity is larger which is equal to root 2gh. Now if the v2 is larger and v1 is smaller, then it means for constant volume flow rate, A1 must be larger and A2 must be smaller; that is why if you look at some particular cross section over here, the cross section A1 must be larger than the cross section A2 over here, and from there we get that the cross section must keep decreasing as we keep as the velocity keeps increasing which will happen automatically because the velocity is increasing as it goes down the hill in the sprue. So that is the question why the sprue is tapered downwards to reduce the area.

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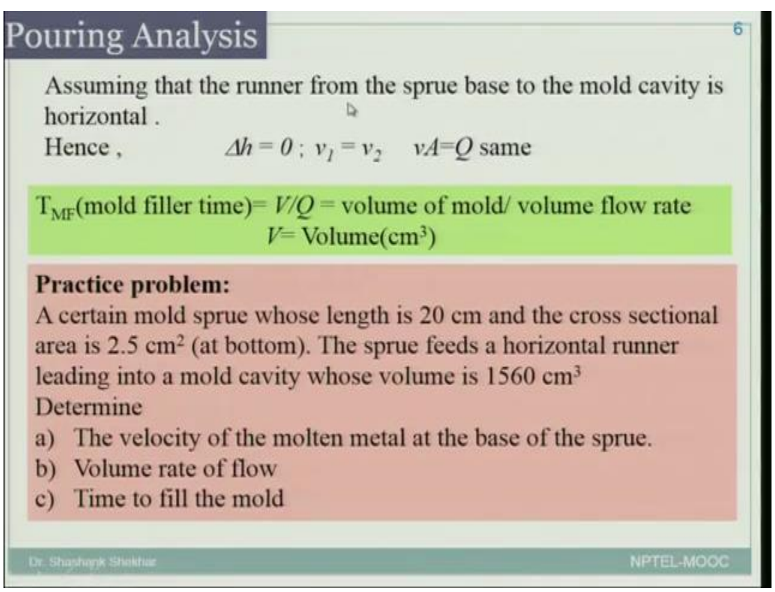

Next what we are doing is we are applying this to the horizontal runner; the same equation we will apply to the horizontal runner, and here we will see that delta h is equal to 0, both are of them, both of them has the same height, volume 1 is equal to volume 2, and therefore the velocity 1 is equal to velocity 2 because the height is 0 and that means that the Q which was the continuity equation, it says that vA must be same; it means that A must remain constant through the runner section, and that gives you why the runners have constant cross section, but the sprue have downward tapering. So that helps explain our this problem.

Now if you want to get a feel of this, you can try solving this practice problem. In here, what is given is a certain mould sprue, whose length is 20 centimeter, and the cross section area is 2 point 5 centimeter square, so you know the volume. And the sprue feeds a horizontal runner leading into a cavity whose volume is 1560 centimeter cube. So, you are also given the volume of the mould cavity; so the (mould) total mould cavity volume is given as 1560. You are also given the sprue length and you you have to determine what is the velocity of molten metal at the base of the sprue.

So that is a very simple part; we know velocity is equal to root 2gh, and height is given whose length is 20 centimeters, so that becomes the height. Find the volume rate rate flow, you know what is the cross sectional area, you know the volume, so you can find out volume velocity into area; so velocity into area is volume flow rate. Time to fill the mould. Now time to fill the mould is given by this. If you know the Q, you know the total volume of the mould. So V over Q is the time to fill the mould. So you will be able to find what is the mould filler time.

And not only this; based on this, you would also be able to determine what should be the sprue size. Let us say you assume some height to diameter ratio, and then you can find what should be the dimensions of this sprue, so that the sprue takes a longer time to cool down than the mould, because that is what you want. And , sprue and at the same time the riser, so you can design your riser and sprue in such a way that these are the last sections to freeze. Why do you want it to freeze at the last freeze the last, because they act as basin or reservoir to fill in the liquid after shrinkage.

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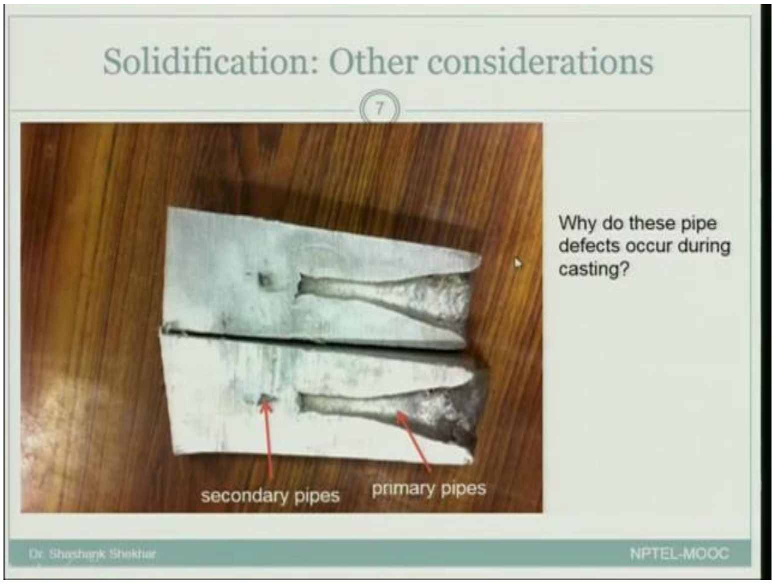

So shrinkage brings us to another aspect about solidification, which is again a very important aspect. So we will finish the solidification chapter with a small amount of introduction to shrinkage. What you are shown is what we do in our lab here; this is what is called as a primary pipe, and this is called as secondary pipe; this is a defect that is generated when you do solidification. So and this is happening why, because of because of shrinkage.

But you can explain or intuitively you would be able to understand why should there should be a primary pipe, but a secondary pipe is a little difficult to understand. Why is it enclosed by solid on the all side, and there is a cavity in between. And this is not just one off case, this is most of the time, how it will be if you do the experiment like this. So let us look at why do we have shrinkage and why do we get this kind of defect.

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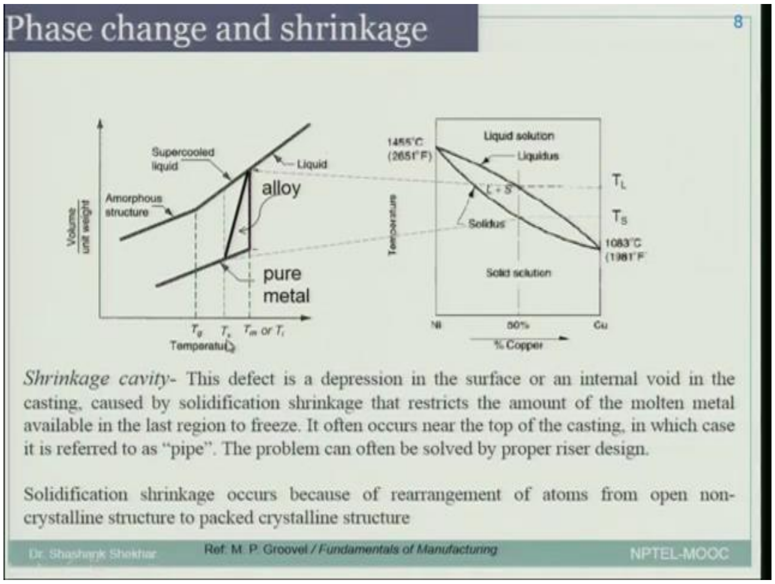

When you are cooling down the liquid; we are talking about the liquid metal, so the liquid metal, if it is pure metal, it will follow this curve. On the , on the y axis we have specific volume, that is volume per unit weight, and on the x axis, you have temperature. So this is your melting temperature, and if you keep that coming down, the specific volume is actually reducing. When you, even in the liquid. In the liquid state, some amount of specific volume is reducing, and that is contraction in the liquid.

But, when you go from liquid to solid, that is when the maximum contraction takes place or the maximum shrinkage takes place. This is for pure metal; if it were alloy, there will be a small slope in this line, because there will be a solidus and a liquidus, and this is the line it will follow, and if it were a amorphous structure, then it will follow a continuous curve which will be a very very different but we will not look into that.

We are interested in metals and alloys; so this is the curve for metal, and this is the plot for solid or alloy. So the we are reducing the temperature and some small amount of contraction is taking place over here. But the largest contraction or shrinkage takes place when it freezes from liquid state to solid state; and then again, after the solid state, again some amount of shrinkage will start to take place and if you keep lowering the temperature. And this can, you can see that this can be related to the liquidus and solidus temperature, and this is what leads to shrinkage cavity. So let us look at a specific diagram to be able to understand this.

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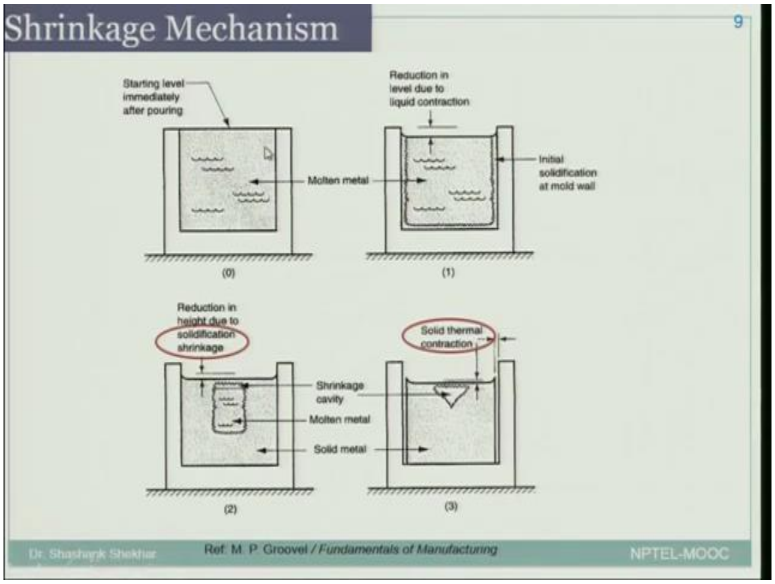

And here is the diagram. So let us say this is the liquid metal initially, and once the solidification starts to take place, the heat is being extracted from the mould surface. So these are the part which will go down the melting temperature the first. So it starts to shrink and also solidify, and once it solidifies, some amount of shrinkage has taken place, so the level of the liquid has come down. It solidifies so this line you see, everything outside it is now solid and there is liquid inside it. So there is also solid on the top layer because of the heat is being extracted even in the air.

So this part is also solid and this part is also liquid now; and this is all happening because of solidification shrinkage when you go from liquid to solid, when you lower the temperature. And towards the end, you will have a shape like this; we will not go into the details on why you get exactly a shape like this, but now what is important to see is there is solid from here all the way around it and there is liquid. Now this liquid when solidifies will also shrink. So when this liquid shrinks, it does not have any reservoir or liquid to fill in that void, and it is because of this that you find those secondary pipes that we saw earlier.

So this secondary pipe forms because there is solid formed over here and there is no reservoir liquid to fill in this. But this part, if it is continuously solidifying and they what whatever amount of reservoir of liquid was there, it went in and filled it, only because there is no more reservoir or no more liquid to fill in this reservoir, you see this primary pipe. So the formation of primary and secondary pipe is a little bit different and it is also dependent on what kind of configuration you put.

So for example in this particular case, I was just like I was saying, the particular configuration is that you see this is tapered; so this is small end up when we are solidifying it in the small end up, that is this part is on the top side and this part the bottom, the large end is on the bottom side, then you get a solidification like this. But if you were doing it the other way, that is bottom the big side up, so this side was let us say on the upper side, then what will happen is that there will be no solidification occurring over a liquid region, and you can show that just by schematic.

So there will be no solid region forming over a solidified liquid, and therefore, there will be no trapped liquid like this, and in that case you, what you will see is only a primary pipe and not a secondary pipe. So the formation of these primary and secondary pipes are very well understood, and it can be controlled, for example you can put a reservoir on top of it so that once the liquid starts to shrink that reservoir will provide the liquid, and that is another reason why you want the reservoir or riser to solidify at the end; you want that it should be able to provide the liquid all the way upto the last minute.

So these are the, some of the simple considerations about the liquid. Solidification when we are talking about the fluid flow. So we have looked at several aspects during solidification; we started with thermodynamics, then we went onto kinetics, then we looked at the heat flow condition, then we looked at the composition variation, and then we also looked at the fluid flow considerations where we talked about fluidity and we talked about shrinkage.

So that completes our module on the solidification and next we will start with powder metallurgy in the next lecture, but what I hope is that you are able to solve the assignment problem simultaneously because that is what will give you a feel of the problems and you will be able to understand the concepts in a much better way, and we are always there to help you. So please feel free to contact us and solve the assignments as soon as possible and in scheduled time. We will see you in next lecture with powder metallurgy. Thank you.