Fundamentals of Materials Processing (Part–1)

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Lecture Number 24

Plane Front Solidification of Multiphase Alloy(Continued)

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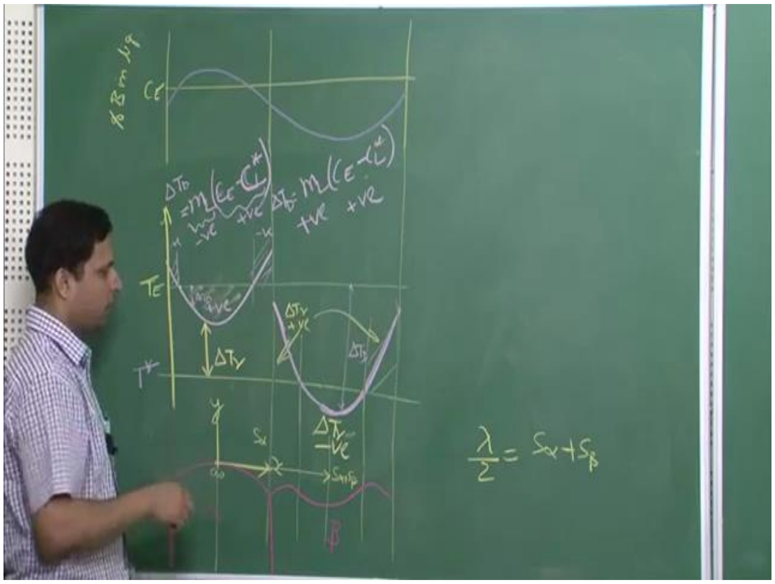

So we were discussing ‘planar front solidification in polyphase alloys’ and we have looked at how the concentration distribution would be in an polyphase when we have let us say two phases alpha and beta; so this was the alpha phase, this was the beta phase and this is a schematic of how the composition difference is there in the alpha phase and the beta phase and corresponding what is the difference, what is the temperature difference. So overall T star, which is the total super cooling or under cooling remains constant, because you remember, we have additional phase, so less degree of freedom, and so we cannot adjust or we are not externally adjusting T star; it is adjusted by the system on its own.

However delta TD and delta Tr, there are two components to this; and the delta TD component adjusts because of the change in composition, and delta Tr changes by changing the radius. So if we know the delta Tr component also changes; so we should be able to draw a rough schematic of how the radius of curvature looks like for the alpha phase and the beta phase given, this is the schematic distribution of the composition. So this is what we will look at next. So here we said delta Tr is positive throughout, and therefore if we were to draw this is a little (())(01:36), so let me redraw this plot.

So let us say this is, this is the (())(01:49) at the tip; so this part is liquid, this part is solidified, and this part is also solid. So, this is the alpha phase, that is formed on the side of it will again be another beta phase; on another side of it, there will again be a beta phase, and so on. On this side, we have another beta phase; and over here upto this point, delta Tr is a positive, and this point onwards it is delta Tr is negative. So we should have something like. This is how the radius curvature in front of the two lamellae would look like. So this is our alpha lamella, this is our beta lamella; over here, there is a sub curvature is positive throughout, leading to delta Tr positive throughout this region. Here in the beta phase, the radius of curvature is positive upto this region, leading to delta Tr positive in this and this region.

Over here, the radius of curvature is negative, leading to a negative delta Tr or negative radius of curvature in this region; and so this is how you will be able to get from schematic composition to be able to predict what should be the radius of curvature and people have done actual calculations which very nicely with observed condition regarding the shape of the lamella. So this theory, you can say is well validated even though we have taken several assumptions in this, but still we are able to predict the shape of the lamella based on these assuming these conditions. So now, once we have this, let us, the next step for us is to try to find the concentration of liquid as a function of x and y.

So now you see, the concentration of liquid is varying with x, sorry the y direction; again let me draw the co-ordinates. So this is the x, this is the y; here we have (0,0), and this is the centre of this lamella, and this is the end of alpha lamella; so this distance is S alpha, and when we get to the centre of this lamella, this width is 2S beta, so this distance is S beta, so the total distance from here to here is S alpha plus S beta, which is also called as the wavelength of this growth; and this is actually used as, denoted as lambda. So, we will see in a moment when we will write down the equation for concentration of liquid.

So the concentration of liquid ahead of the solidifying interface, assuming that the overall, remember do not forget that overall consumption assumption that the composition of the alloy is CE, or C0 that we have taken is equal to CE. So this is our overall assumption so far for the, we are doing it all for a very near eutectic composition, when the eutectic composition is there, these are all the conditions of solidification that take place; and for that, people have shown that concentration of liquid can be given by a equation like this.

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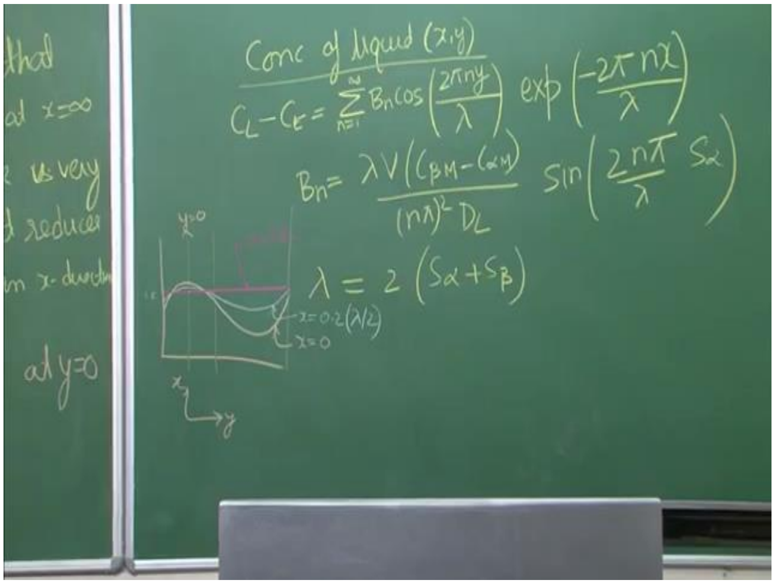

Conc of liquid (x,y)

$$C_L - C_E = \sum_{n=1}^{\infty} B_n \cos\left(\frac{2\pi n y}{\lambda}\right) \exp\left(\frac{-2\pi n x}{\lambda}\right)$$

$$B_n = \frac{\lambda V (C_{\beta M} - C_{\alpha M})}{(n\pi)^2 D_L} \sin\left(\frac{2n\pi}{\lambda} S_\alpha\right)$$

$$\lambda = 2 (S_\alpha + S_\beta)$$

So if CL is concentration of liquid at any point x and y, then CL minus CE is given by. First thing to note is that it has two components; one is this cos or sinusoidal component, and the other is the exponential component, and the sinusoidal component which is given in the term cos is a function of y; so you can see that in the y direction, the composition of liquid will vary sinusoidally and that is obvious given that what we have which we, that the (concen) or the solute is being ejected in front of alpha, getting absorbed in front of beta, so there will always be variation which will be sinusoidal in nature along the y direction.

On the other hand, in the x direction, which is away from the solidification front, it drops down drastically. So if you look at it, at very large xs, this will whole thing will become 0, and therefore CL will become equal to CE. So from this equation itself it looks like all the conditions or it looks like it is a very meaningful equation because at large xs, it does become CL equal to CE, and for y direction, we see that it follows a sinusoidal condition. So this is the equation that is given and since we have Bn over here; let us define what is Bn, and his Bn is given by this equation. And like we said, lambda is nothing but the wavelength which is equal to 2 times S alpha plus S beta.

S alpha is the half width of alpha phase lamella, beta and S beta is the half width of the beta lamella. So S alpha plus S beta times 2 is the total wavelength. Now this equation looks very intimidating, but we will soon solve a problem based on that and you will see that it is not really that scary and that difficult to handle. However just keep in mind what are those that the overall form, what does the overall form of the equation represent; like I said, this is the exponential and this is the sinusoidal form, so the sinusoidal form is representing the variation in the y direction, and the exponential form represents the variation in the x direction, and so in the x direction it in the direction of the solidification, it varies exponentially or decreases to the equilibrium concentration of CE to at a exponential rate.

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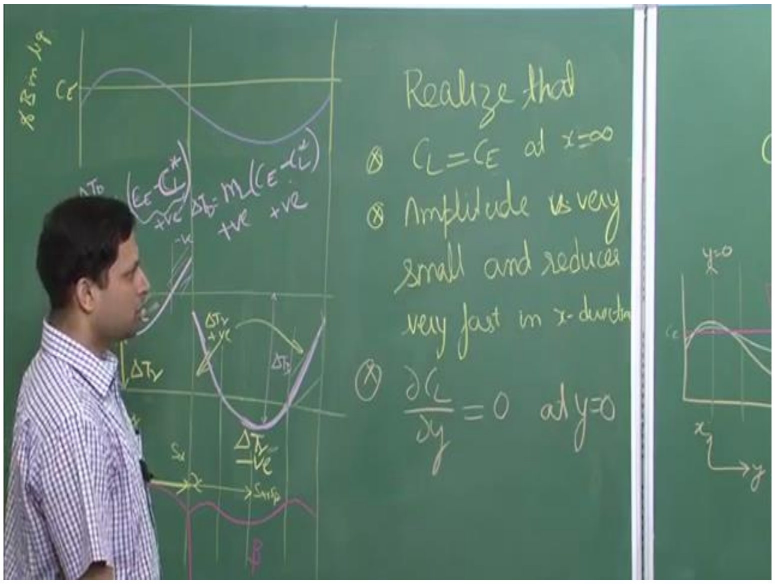

So now when we have this, so there are a few things, like I said; let me, it will be important if we write down some of those important facts that we must be aware of. So realize that CL is equal to CE at x equal to infinity, which should be obvious from it, from the equation now, that at x equal to very large values, in fact it is not even very large values; at the order of few lambdas, the concentration comes down to 0, and this value comes down to 0 which means CL becomes equal to CE.

Another fact that you must be aware of is that amplitude that we are looking at; so you see there is concentration variation in the y direction and concentration variation in the x direction; and this is the amplitude that we are looking at. So the overall amplitude that is changing, sorry this term Bn. So this amplitude variation in concentration is a very very small quantity; while this variation in the x direction is very large quantity. In fact, if we were to draw this, so let us first write it down. Amplitude is very small and reduces very fast. How fast it reduces? We will see in a moment. And it reduces very fast in x direction.

So if we were to look at the amplitude at x equal to 0, that is right at the interface; so we know how it looks like. So this is our CE and this is alpha, this is our beta. So we are looking only at one, only one wavelength; so this is how the concentration is looking like at say x equal to 0.

Now let us see how it will look like at some distance away which is not very far, in fact it is just point 2 lambda by 2; and it drops. So this distance we are representing is x equal to point 2 of half the wavelength, or point 2 of S alpha plus S beta; so it reduces very fast, the wavelength has, the amplitude has reduced. And, if I go to a distance x equal to lambda by 2, so just at half the wavelength, it has almost reduced to 0 amplitude. That is how fast the amplitude reduces. And, this also gives us understanding that this amplitude is very small in magnitude. So this is the part which we have expressed over here.

What else should we be aware of? Another thing that we should be aware of is that del CL over del y is equal to 0 at y equal to 0. Again this is obvious; we are looking at , when we say y equal to 0, this is our y equal to 0. Remember in this direction we have the y, and this direction is x; so in this direction y, and therefore the origin is this. So at this point, we have the maxima and therefore del CL over del y is equal to 0 at y equal to 0. So these are some simple facts that you should be aware of when we are dealing with this, and we will solve one equation based on this particular form of the liquid concentration variation, but before that, we should get into the, look into the fact “what are the other conditions that are required for the (stably) interface stability?”

We have said that it is planar solidification. But what condition would be required if we want the planar solidification to continue, and not to become, let us say cellular in form. So we will start with again a little bit different assumption this time, and not different assumption basically we will try to start from a more generalized solution of the liquid; and why is that? That you will get to know at the end of the relation that, when we have derived that relation. So now what is that more generalized relation?

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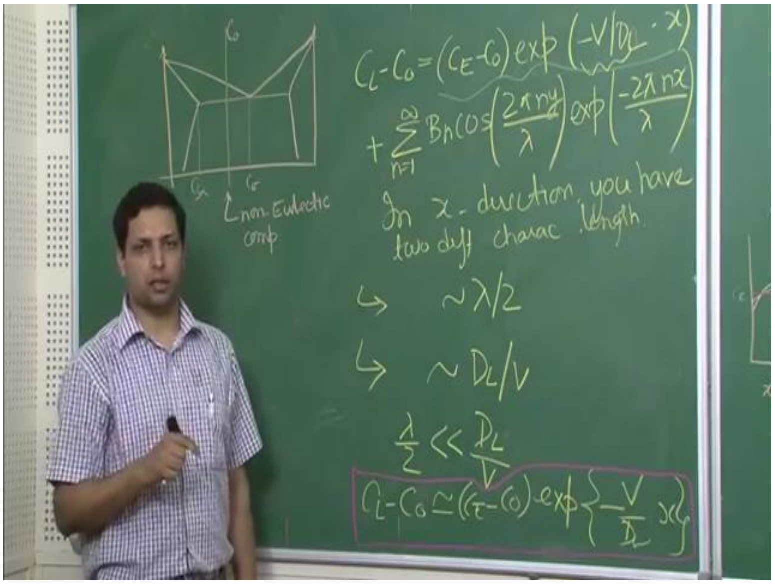

The more generalized relation is that we will now be looking at, not at CE, but some composition in that some other composition in the two phase region. So we derive the previous equation which is CL minus CE for the concentration of the liquid for the concentration of the alloy being equal to CE. Now we want to find thermal the interface stability for some other composition C0. This is a variable, this C0 can be anything from CS alpha to CE. So once we have this general form of the relation, then we can get to the more specific value of CE; we can simply put C0 equal to CE and then we will see how or what difference it makes.

So this is a non eutectic composition; and when you look at the non eutectic composition, this equation changes a little bit, which is what we want to emphasize over here. So let me write down the equation when we have the alloy composition which is a little bit different from the eutectic composition. And again, I am not deriving it over here, the derivations are a little bit more involved, but we will write down how the relation looks like. So C0 is the concentration of the overall alloy, CL is the concentration of liquid, and this will have now two different components as you will see.1

So what do we see? When we are writing the equation for the liquid concentration for of eutectic composition, we have this additional relation. Over here, we had only this relation, but here we have this component which is same as that one and additional component over here. So now when we look at this particular relation, what we see or what you would see is that there is a exponential factor over there, and therefore now if you look in the x direction, you have two different characteristics length. One of these characteristic length is the distance at which amplitude drops off; so remember, the amplitude was like this, like this and it becomes almost flat at a particular distance.

So this is the, one of the characteristics length when the y in the y distance in the y direction the composition variation becomes constant. So that distance was equal to that characteristic difference was of the order of lambda by 2. Another characteristic length is if you (look) when you look at this part particular part, first one, which describes the composition variation in the x direction. Now if your composition variation in the x direction, it becomes flat or the rate at which it is dropping is related to this value, DL over V. So this is your another characteristic length DL over V.

Now depending on which particular value is dominating, you will have or you would be able to say which is your more important parameter; let me explain what I mean. So for example, if you are very close to the surface and you want the composition variation along the surface, then this is a important parameter. Of course this is the background values, this is the overall composition. But if you are very far from the composition, if you are very far from the amplitude, then this is what the overall composition is dominated by. Or even if let us say your lambda by 2 term is much much smaller than, let us say DL over V. Now if lambda by 2 is very small, what happens? This term becomes almost 0; and if this is 0, it means this term becomes, the variation over is very small, which is equivalent to saying that we are looking where the, looking at a point where amplitude is very small.

And therefore, this particular term would become more important and we can write CL minus C0 approximately equal to. Now when you look at this equation, it should remind you of something. This is the same form of the equation that we looked when we were looking at no diffusion in solid and diffusion controlled liquid growth in single crystals or in the single component single phase alloys. So this is the kind of equation that we obtained over there, and if we this is the form of the equation that we obtained over there, we can also find the condition for thermal sorry not the thermal but the interface stability from this particular condition; so this is now the dominating condition and this can describe our interface stability criteria.

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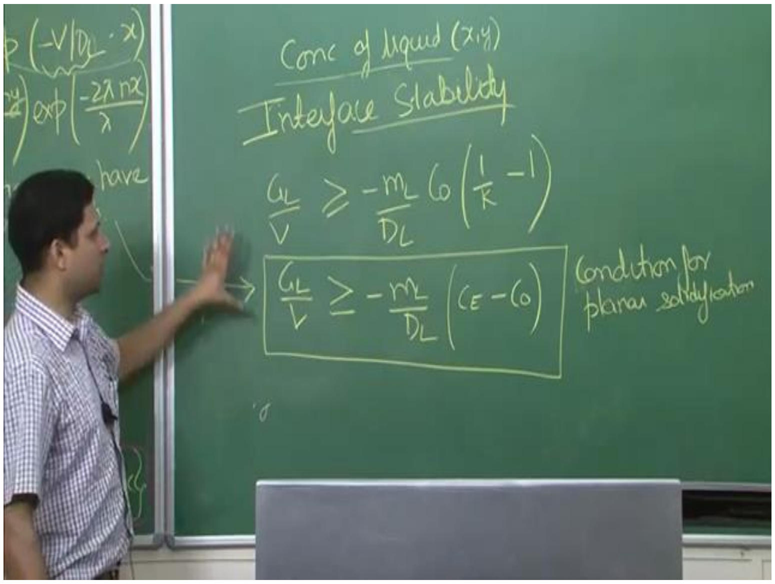

So we are looking at interface stability criteria and this is the equation that we have obtained when your lambda by 2 is much smaller than DL over V, and when we have this form of the equation, we know that it is for diffusion controlled in the liquid and no diffusion in the solid, and from there, we know what is the form of the equation for interface stability; you remember it was of the form GL over V equal to minus mL by DL. Or in the present context, this would imply, this becomes C0 by K, which is equal to CE minus C0.

So this is your condition for interface stability. So this defines when you will have a condition for planar solidification. So whenever you have this particular condition, which will always be the case because this will be the more dominating term in this equation means this particular term is a very small value; therefore this will always be the most (dominating) more dominating term, and because of that, this will be, this will how your overall composition variation will look like, and if this is how the overall variation of the composition looks like, this is how we can describe the interface stability, and in the present context, this translates to this; so C0 by K becomes CE, and C0 remains CE.

And whenever, what it is saying is whenever your GL over V is greater than this value, you will have planar solidification. If your GL over V gets below this value, then you may not get, or you will not get interface of planar interface in the, during the solidification. Now, once you have this equation, people have obviously tried to verify it experimentally and those experimental verifications have indeed shown that this is how the solidification condition goes.

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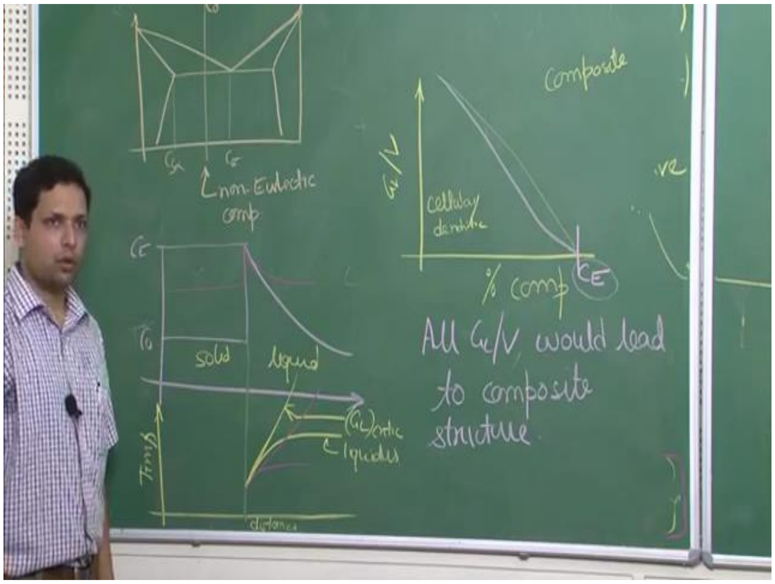

If we plot something like GL over V and percentage composition. So remember, this is varying with what is the value of C0. And if you keep increasing C0, then this value becomes smaller and smaller; and therefore the particular plot if you were to draw it theoretically, it will look like this. And under this condition over here, if you get composite or planar solidification; we are using the term composite, meaning the alpha and beta phases are growing simultaneously without interfering with each other. And over here, you have cellular kind of growth, or dendritic. So this is what is theoretically predicted and people have in fact found very close relation like this.

And what is this value at which it becomes GL over V becomes 0, this is CE. So, for compositions beyond eutectic, you would always get, not beyond eutectic, I should say at eutectic, all values of GL over V would lead to composite structure or planar solidification, and that is one reason now you can see why we went away from CE to derive or not derive but to look at the condition for interface stability, because if we had looked at C, then you would have always seen interface stable interface leading to composite microstructure; but only when we look at the general of eutectic composition, we see that it varies along with the composition, and it changes continuously with the percentage composition, and at C it becomes 0.

So beyond, at this particular value, all GL over V would lead to composite structure. And, if we were to plot this to show the condition for thermal stability, this will be your C0 solid and this is how your liquid composition is varying; somewhere over here you have CE; and for corresponding to that you will have a liquidus temperature. So we are looking at the liquidus temperature over here; the liquidus temperature varies like this, and that GL over V that GL that we are talking about over here is this critical value. This is the liquidus, this part is solid, this is liquid, this is composition, this is temperature, and this is distance. Both; so this is composition versus distance; solid has this composition and the liquid ahead of it is forming is has a composition distribution like this, which leads to liquidus variation like this, and because of that you can get a critical temperature gradient which is like this.

So this is our critical temperature gradient, and what else can we say about this from the plot itself. Now you see if you change CE minus C0, so our CE, if you keep reducing CE minus C0, let us say we now make it smaller. So it becomes like this, which means correspondingly, if we look over here, it becomes like this. So there is now much larger critical value that much smaller critical value is allowed, so anything below this critical, anything above this critical value would lead to composite microstructure; and beyond a certain point let us say, at any particular point, it becomes, CE becomes C0.

So this is like this, theoretically speaking; there will of course be some variation in the real case scenario, but theoretically speaking, if C0 becomes CE, this is the composition of solid, this is the composition of liquid, and which means the liquidus is over here like this plot; so any thermal gradient is going to lead to a composite microstructure. So that is what we have about the plane front solidification of polyphase alloys. Next time, we will look at the one of the equation or one of the problem related to this, and you will see that the very complicated equation that we have seen are not really so intimidating and complicated and we will be able to get solutions from that. So with that, we leave here and we will meet in next lecture. Thank you.