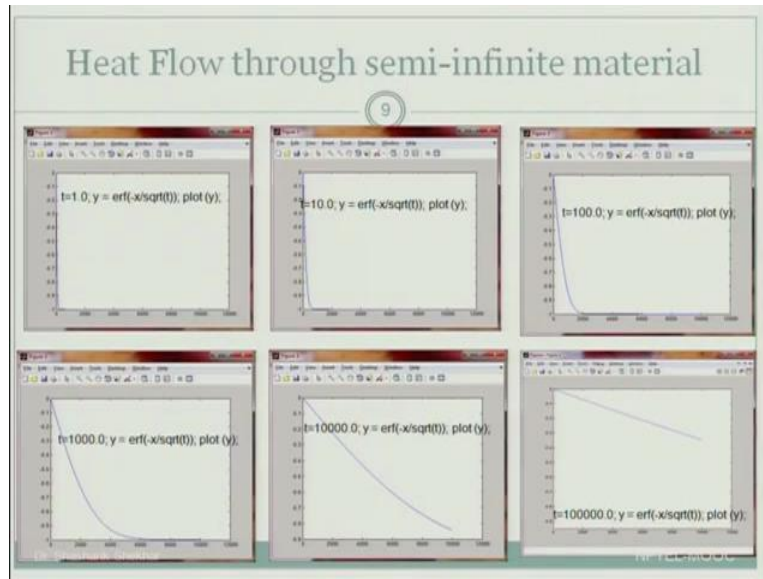


Fundamentals of Materials Processing (Part-1)
Professor Shashank Shekhar
Department of Materials Science and Engineering
Indian Institute of Technology, Kanpur
Lecture Number 14
Composition Variation

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So we were showing you the plots, the simple MATLAB plots that you can obtain, to understand what is insulating mold condition, and which is now you can clearly see is different from the condition where you impose the external temperature, and this is a simple MATLAB program where the solution has been, the analytical solution has been used, but actually you can also like I have already mentioned in my, some of my previous lectures, that you can use MATLAB to write or get the solution from first principle, that is using the thermal heat equations. So you can directly use the heat equations, solve those differential equations and get a plot like this by putting initial and boundary conditions.

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The slide is titled "Unidirectional Heat flow: Insulating Moulds" and contains a list of six bullet points. The first five points are in black text, and the sixth point is in red text. The slide also includes a slide number "10" in a circle at the top center, and the names "Dr. Shashank Shekhar" and "NPTEL-MOOC" at the bottom.

Unidirectional Heat flow: Insulating Moulds

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- It is assumed that all heat flows in only one direction
- Metal has much better thermal characteristics than the mould and hence solidification rate depends primarily on thermal properties of the mould
- Moulds can be considered to be semi-infinite, i.e., the outside of the mold is far enough that it does not heat up during solidification
- Heat flow problem can be further simplified by assuming that the metal is poured with no superheat, that is, exactly at its melting point, T_M
- Particularly true for sand casting and investment casting
- How should the temperature plot look like after making these assumptions?

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And then we had looked at the insulating mold condition, which was our, and all of these we discussed in uni-directional heat flow conditions. So we looked at the insulating mold conditions, where we assumed that the heat flow is in only one direction, and would assumed that metal has much better characteristics over here, and because of which the conductivity of metal is very high, and all the temperature thermal drop takes place only in the mold. And assuming this and taking some boundary conditions, we were able to get solution for this using the thermal diffusivity equation.

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Unidirectional Heat flow: Insulating Moulds

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- Metal is poured exactly at its melting point against a thick, flat mould wall initially at room temperature T_0 .
- The mould wall is suddenly heated to T_M at time $t=0$
[How should the temp plot change with time?]
- Thermal Diffusivity Equation can be applied
- This relation is more accurate for sand castings of high conductivity such as nonferrous metals than for iron and steel
- High melting temperature and low heat of fusion favor rapid solidification
- The product $K_m \rho_m c_m$ is a measure of the rate at which the mould can absorb heat

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Unidirectional Heat flow: Insulating Moulds

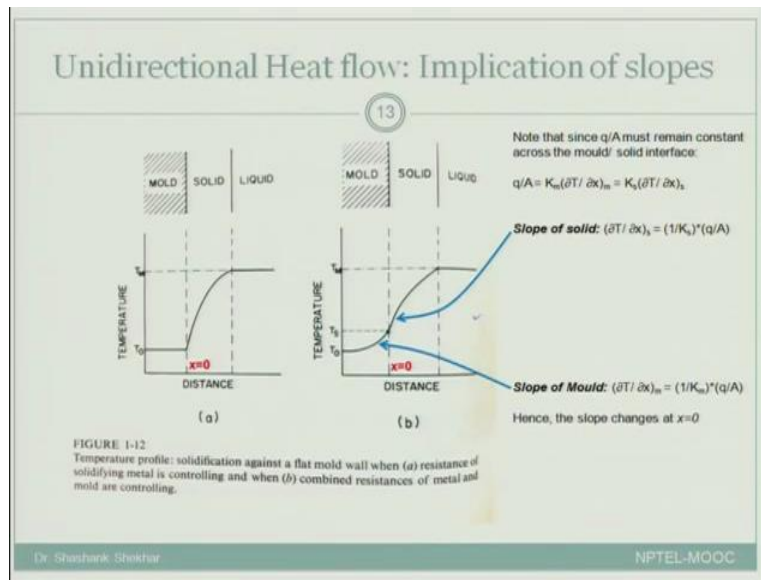
12

- Contour of mould wall is also expected to play role in absorbing heat
- For example, it can be expected that concave shapes will be more divergent and hence will allow higher rate of heat flow
- However, for simple shapes, one can make approximations that a given area of mould surface would have a fixed ability to absorb heat regardless of its contour or location on castings
- So, we can replace S with V_s/A , where V_s is volume solidified at time t and A is area of mould-metal interface
- And replacing t by t_f where t_f is the total solidification time for a casting of volume V , we get *Chvorinivo's Rule*

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And then we also obtained S as a function of t , and S as a and $T t_f$, which where t_f represents the total time for solidification. For that we assumed that the 3-dimensional object that we are taking into consideration are simple geometries, and based on those assumption, we were able to get t_f as a function of V and A ; A is being the inter-(mold) metal-mold interface area. So we were able to get these equations using the insulating mold condition.

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And then we also discussed the implication of slope; you remember we always draw the slope with increasing slope if we go from in mold, from air-mold interface to the mold-metal interface, and this is always increasing while the mold-(metal), while the slope for the solid going from mold-metal interface to the solid-liquid interface is always decreasing; and we found that what is the reason. The reason being that solid is actually cooling down from T_M , while mold is getting heated up from T_0 , the room temperature. And, if you assume that there is no interface resistance, there is also a relation between the slopes of the mold-metal interface on the two sides, the solid and the mold side. And these are the, some of the implications or some other important things that we looked at.

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Unidirectional Heat flow: Interface resistance dominated heat flow

14

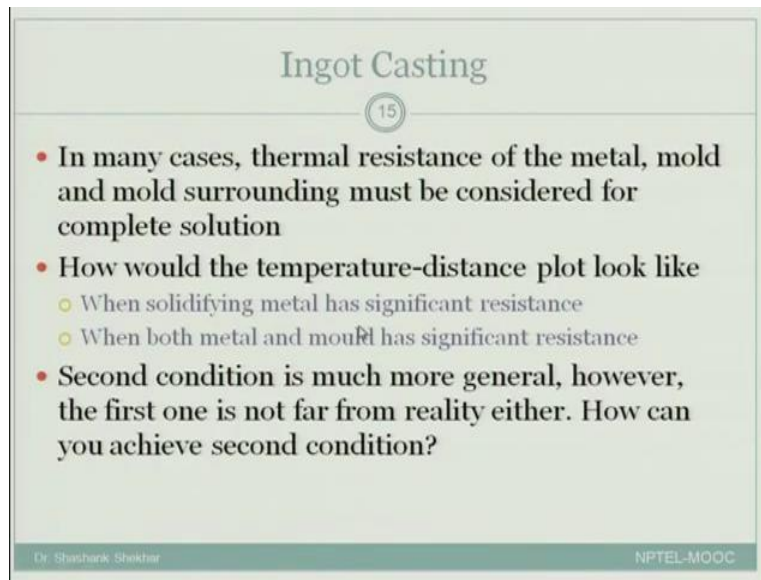
- In many a casting processes, heat flow is controlled to significant extent by resistance at the mould-metal interface [How should the temperature plot look like?]
- These processes include the permanent-mould-casting, die-casting, powder manufacturing processes
- The mould being assumed infinite in extent, remains at its original temperature
- Compare the time of solidification for this condition, with 'Chvorinov's Rule'

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Then we also looked at another simple case, or the where we can where we took some simplifications, some assumptions, and that particular case was interface resistance, meaning a temperature drop is completely taking place at the metal-mold interface instead of the solid or the mold part. And, we also saw how the temperature plot should look like, it actually looks like a for all time, it will always have the temperature drop only at the metal-mold interface, and it is particularly useful when we are looking into applications like permanent-mold-casting, die-casting, powder manufacturing processes.

But when we are talking about these kind of processes, we should also talk about the processes that we are involved in the previous one which was simple sand mold casting. And, in the another assumption in here was that the mold is assumed to be infinite in extent, that it; it means that it remains at it's original temperature, which is also the semi-infinite condition that we invoked for the previous case, and we were able to get in the previous case we obtained 'Chvorinov's rule', and here we got a solidification time, S versus time, which was different, where we saw that S is proportional to t . In the previous case, we obtained S proportional to root t .

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The slide is titled "Ingot Casting" and contains a list of three bullet points. The first bullet point states that thermal resistance of the metal, mold, and mold surrounding must be considered for a complete solution. The second bullet point asks how a temperature-distance plot would look like under two conditions: when solidifying metal has significant resistance, and when both metal and mold have significant resistance. The third bullet point notes that the second condition is much more general, though the first is not far from reality either, and asks how to achieve the second condition. The slide footer includes the name "Dr. Shashank Shekhar" and "NPTEL-MOOC".

Ingot Casting

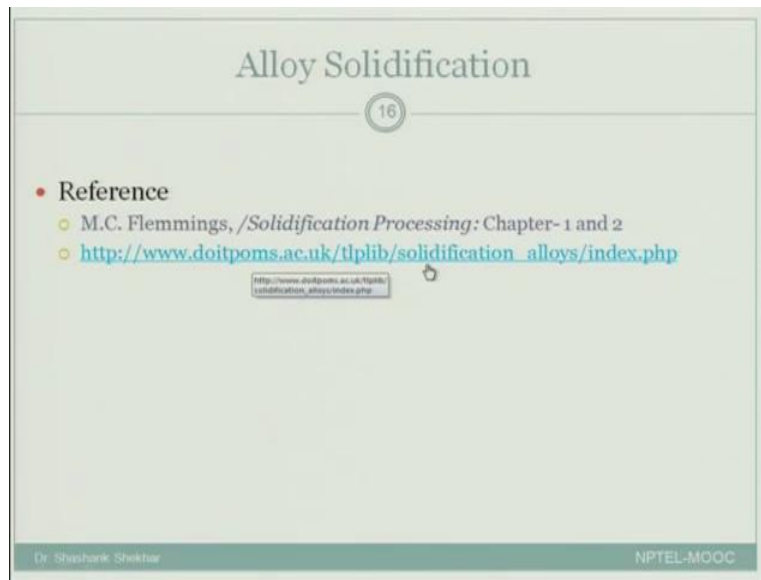
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- In many cases, thermal resistance of the metal, mold and mold surrounding must be considered for complete solution
- How would the temperature-distance plot look like
 - When solidifying metal has significant resistance
 - When both metal and mold has significant resistance
- Second condition is much more general, however, the first one is not far from reality either. How can you achieve second condition?

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And then we looked at some realistic conditions, two of these, when the mold is chilled, and another when the mold is not chilled. So when solidifying metal has significant resistance, and when both metal and mold have significant resistance. So, in one of them, the temperature drop was taking place in solid, and in the other case, temperature drop was taking place in mold as well as in solid. These are a little bit, if you remember we said that these are a little bit more complicated conditions, because the temperature drop is taking place in solid, and the length extent of the solid is actually changing. So the overall temperature profile is changing, not only because of the change of the heat flow, but also because of the length of the solid region. And second condition is much more general, where you have no chilled mold, meaning you are not doing anything extra to keep the mold at particular temperature.

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Alloy Solidification

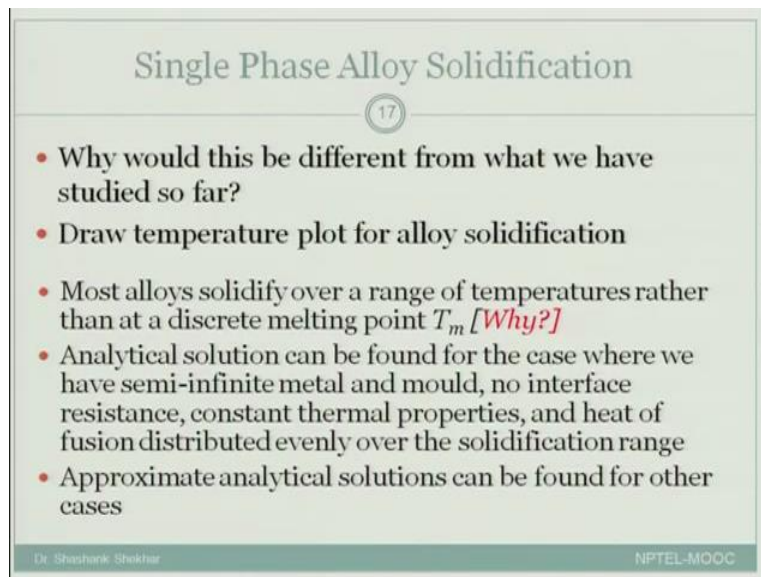
16

- Reference
 - M.C. Flemmings, /Solidification Processing: Chapter- 1 and 2
 - http://www.doitpoms.ac.uk/tlplib/solidification_alloys/index.php

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And we looked at solution, although we have not derived it, but we looked at the solutions that have been obtained. And then we went on to ‘alloy solidification’ that we discussed in just the previous lecture, and there are some very interesting why I am showing this slide is that there are some very interesting websites for example, this website where educational website where you can see animations which explain some of the phenomena in very vivid way. So it may be very helpful to get a visual understanding of this phenomena.

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Single Phase Alloy Solidification

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- Why would this be different from what we have studied so far?
- Draw temperature plot for alloy solidification
- Most alloys solidify over a range of temperatures rather than at a discrete melting point T_m [Why?]
- Analytical solution can be found for the case where we have semi-infinite metal and mould, no interface resistance, constant thermal properties, and heat of fusion distributed evenly over the solidification range
- Approximate analytical solutions can be found for other cases

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We saw that single phase alloy solidification is different, and it is different because there is a range of temperature for solidification. And we obtained and we did not obtain solution, but we looked at the S versus t plot and looked at the different, for different conditions; for example when there is no interface resistance, when there is interface resistance, and when there is when there is a when there is a insulating mold. So for some of these conditions, we looked at how the S versus t plot should look like, and what are their implications.

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Example Problem

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- Aluminum is splat-cooled to room temperature, on a copper substrate with a mold-metal-interface heat-transfer coefficient of $1 \text{ cal/cm}^2\cdot\text{C}\cdot\text{s}$. Assuming heat transfer is interface-controlled, how thin must the sample be to achieve a cooling rate (in the liquid at just above the melting point) of 10^6 C/s ? (Assume an ambient temperature $T_o = 20^\circ\text{C}$)

(Data: **Solid Aluminum**: specific heat = $0.20 \text{ cal/gm}\cdot\text{C}$; Density = 2.7 g/cm^3 ; Thermal Conductivity = $0.53 \text{ cal/cm}\cdot^\circ\text{C}\cdot\text{s}$; **Molten Aluminum**: Melting Point = 660°C ; Heat of Fusion = 95 cal/gm ; specific heat = $0.26 \text{ cal/gm}\cdot^\circ\text{C}$; Thermal Conductivity = $0.20 \text{ cal/cm}\cdot^\circ\text{C}\cdot\text{s}$; **Solid Copper**: specific heat = $0.09 \text{ cal/gm}\cdot\text{C}$; Density = 9.0 g/cm^3 ; Thermal Conductivity = $0.94 \text{ cal/cm}\cdot^\circ\text{C}\cdot\text{s}$)

Dr. Shashank Shekhar

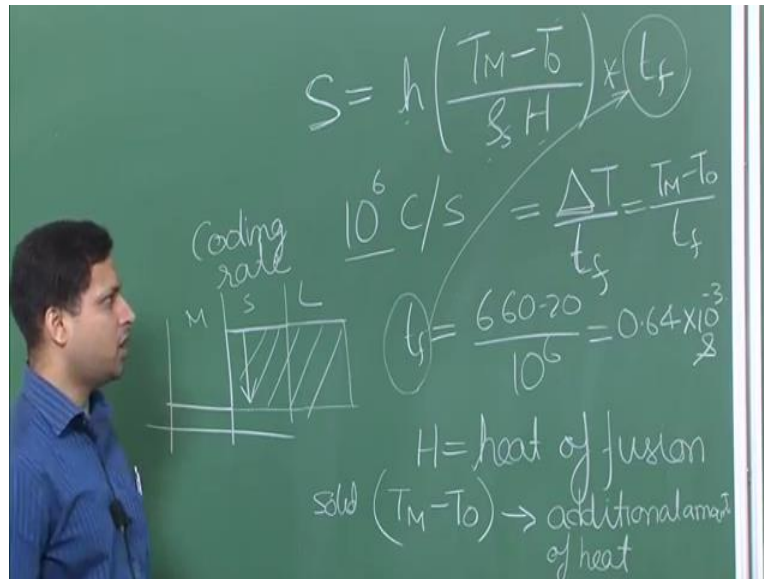
NPTEL-MOOC

Now that we have gone through all this, it is time for us to look at some example problem, that will be, that will help you understand exactly what we have been discussing in last few lectures. So to begin with, let us look at this example problem. Aluminium is splat cooled; remember splat cooling that we discussed about, and we are talking about aluminium. It is splat cooled to room temperature on a copper substrate. So if it is splat cooled, meaning it is started from melting point, and if its temperature is not given, so you can assume that it has started with its melting point, which is 660 degree celsius, and then it cool down to a room temperature on a copper substrate, so it heat a copper substrate with a mold-metal interface heat transfer co-efficient. So it is also given to you, that what is the phenomena that is taking place; it is mold-metal interface controlled heat transfer.

Assuming heat transfer is interface controlled, how this, how thin must the sample be to achieve a cooling rate of 10^6 celsius per second? So you are given that the it is interface

controlled, but now you have to find out, what should be the thickness of this of the droplet that has formed a thin layer? How thin this layer should be, so that you can get a cooling rate of 10^6 celsius per second? Now that looks a little bit interesting problem, because we have never talked about cooling rate so far. What so let us look at what do we know and try to solve this problem.

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We know, for interface resistance, S is equal to $h (T_M - T_0) / (\rho S H) \times t_f$. Now if you look at, look at over there, H is given, ρ S is given, heat of fusion is given, T_M is given, T_0 is given, t_f is something that we do not know. And this will give us the total length which has solidified. So we do not know this, we do not know this. And, what we are given, or what we have been asked to relate it, is with cooling rate. Cooling rate is given as 10^6 celsius per second. So now, you know what are the data that has been provided to you, you know that cooling rate is given, although we have never talked in terms of cooling rate. But then, what is cooling rate? It is only the, in the simplest form, it is the total temperature change taking place in the time. And what is his time? This is the time for solidification.

So now, we know the ΔT , we know cooling rate, which is this; and from here, we are in a position to get t_f . And, if you put this equation over here, ΔT is equal to $T_M - T_0$; because we are cooling it all the way to the room temperature. So if you put this over here, you will get t_f equal to. So it is the total time for solidification, as you would expect in splat cooling

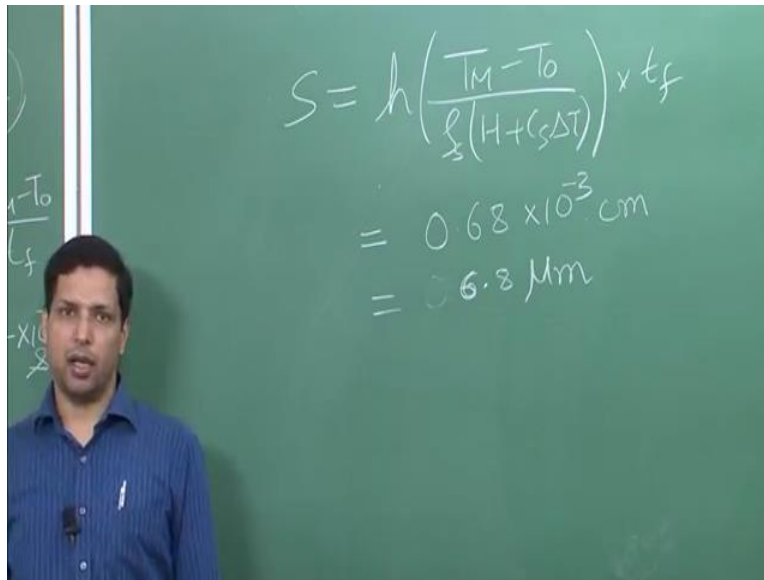
is coming to point 64 into 10 to power minus 3 second point 64 milliseconds. So it is the very small time; that is also, that will also give you an idea, what is the time frame that we are looking at when we are talking about splat cooling kind of phenomena. So this is the total time for solidification.

Now the question is can we put this directly in over here to get S? This S will be the thickness that has solidified in the $(t)(10:50)$, and if that is the thickness of the material, then it would mean that it is indeed, this is this time this is the time it takes for cooling and therefore it gets or it achieves this kind of cooling rate, which is 10 to power 6 celsius per second. But remember, when we were talking about the total solidification over here, we were talking, we were saying that H over there that we were taking, it was only equal to heat of fusion.

But over here, we have some additional amount of heat that is taking place, and that is, when we are cooling all the liquid, all the way from T_M to T_0 ; and therefore some additional amount of; not the liquid, sorry the solid; the solid is being cooled from T_M to T_0 ; additional amount of heat has to be absorbed. Because you remember how was the plot, over there interface resistance, this is the mold, this is the solid, this is liquid. So we were only concerned with solid becoming liquid, but we were never concerned with decreasing the temperature of the solid to the room temperature. So there is some additional amount of heat that has to be absorbed to bring the whole solidified splat to room temperature. So that amount of heat has not been taken into account in this original equation, and that is something we need to take into account.

Therefore, what we will do, or how do we take into account that additional amount of heat, where there is no in the original equation, there was no way where we could have included; but since we are taking some approximations, we can say this H can be replaced by this H plus delta heat; and what is that delta heat? $C_{metal} \Delta T$. So the specific heat of metal, solid part, the specific heat of the solid times the temperature. So the this will become the total amount of heat that is needs to be emitted by the solid or and that needs to be absorbed by the copper to achieve this kind of cooling rate that we have talked about, that 10 to power 6 celsius per second. So the major difference that we are doing here is we are changing H to a new quantity H plus delta heat, which is $H + C_S \Delta T$.

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$$S = h \left(\frac{T_M - T_0}{\rho_s (H + c_s \Delta T)} \right) \times t_f$$
$$= 0.68 \times 10^{-3} \text{ cm}$$
$$= 6.8 \text{ } \mu\text{m}$$

Therefore your equation becomes. So this is a simple simplification that we do or a simple assumption we do to get to the solution of this problem; and when you put it over here, all the values, you would see that S comes out to; or equal to 6 point 8 micrometer. So we are talking about a very very thin layer and that is only when you can get a cooling rate of this order, 10 to the power 6 celsius per second. So you need the droplets must be small enough so that once it splashes onto the substrate, it becomes as thin as 6 point 8 micrometer, and then you will be able to get cooling rate like this, 10 to the power 6 celsius per second. So that is the solution for this problem.

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Example Problem

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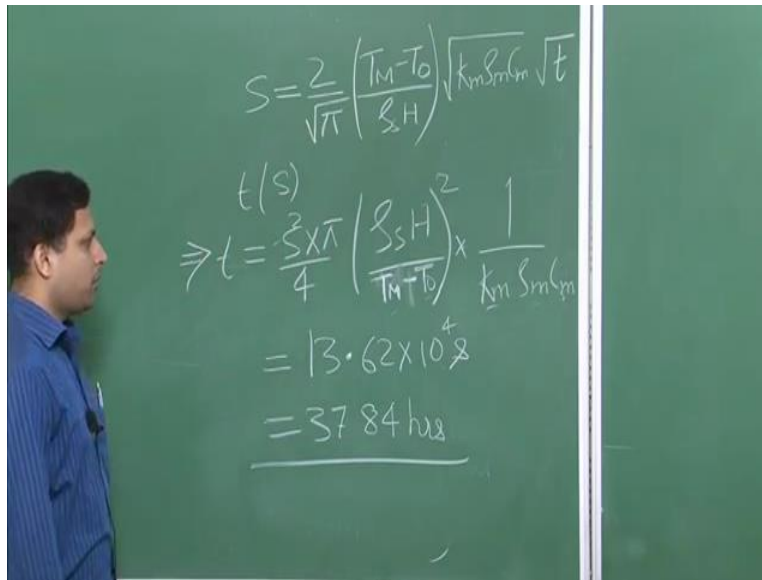
- A large slab of aluminum, 25 cm thick, is poured in a sand mould with no superheat. How long will it take for the slab to solidify? (Assume negligible resistance to heat flow at the mold metal interface and within the solidified metal. Take room temperature = 25°C)

[Data: Aluminum: $\rho_s = 2.7 \text{ gm/cm}^3$, $H = 95 \text{ cal/gm}$, $T_M = 660^\circ\text{C}$; Sand mould: $K_m = 14.5 \times 10^{-4} \text{ cal/cm.C.s}$, $\rho_m = 1.5 \text{ gm/cm}^3$, $C_m = 0.27 \text{ cal/gm.C}$]

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Now let us solve another simple problem quickly. And this problem is about a large slab of aluminium, 25 centimeter thick; it is, when I say large slab, meaning and it is poured mean that it is in liquid state; so it has to be made into a slab this much thickness, but to begin with, it is in liquid state, is poured in a sand mold with no superheat. How long will it take for the slab to solidify? So we are given that it is a sand mold condition; so you can, so you can understand what particular model you need to use. The model that we need to use is the insulating mold condition. It is given that there is no superheat, and from there we have to (form) find how long will it take for the slab to solidify.

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The image shows a man in a blue shirt standing to the left of a green chalkboard. The chalkboard contains the following handwritten equations:

$$S = \frac{2}{\sqrt{\pi}} \left(\frac{T_M - T_0}{\rho_s H} \right) \sqrt{k_m \rho_m c_m \sqrt{t}}$$
$$\Rightarrow t = \frac{3 \times \pi}{4} \left(\frac{\rho_s H}{T_M - T_0} \right)^2 \times \frac{1}{k_m \rho_m c_m}$$
$$= 13.62 \times 10^4$$
$$= \underline{\underline{3784 \text{ hrs}}}$$

So let us try to solve this problem; again it is this one is not, does not involve much trick over here, this should be straight forward. So we have a aluminium slab, meaning at to begin with, it is in the liquid state. And since it is the insulating mold condition, so let us first write down the equation we have. So what you need to find according to the question is how long will it take, and this is the equation where we have S related to T. We need to find t in terms of S; so it is not very difficult, we just write it like this, t is equal to. So S is a function of root t. If you want to take find out t, you have to square the, square everything, and then you are able to get t in terms of S.

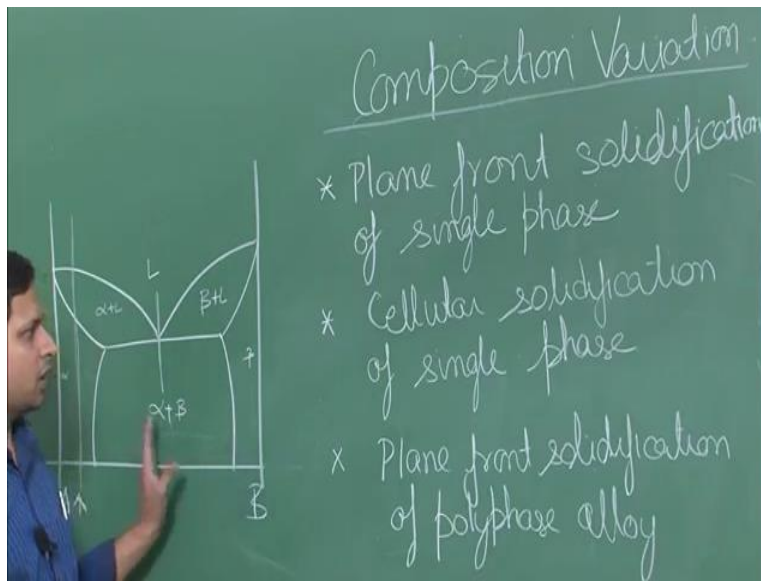
Now here, you know all the parameters; you know S, you know rho S, you know H, TM, T0, all the parameters are given; and therefore you can just simply plug in the values and you would see that the time comes out to. Just when you are, just you have to be careful in about one thing, when you are plugging in the values, make sure that the dimensions remain same; for example, you cannot put a unit per minute somewhere over here and then expect to calculate in seconds.

So you have to be careful in selecting the units properly; you will have to convert it to the right unit, so that the dimensions match properly, or the units match properly. So this was a simple problem. We had no manipulation to be done over here, we just had to use or we have to realize that it is a insulating mold condition, so from the insulating mold condition, we take the S versus

t equation, and then you write t in terms of S, plug in the values, and you get the total time for solidification.

So these are the two simple problems that we have looked at, and with that, we come to end for this module which is the heat flow. Next, we look at the compositional variation. So now, we are, so far, we have looked at three important aspects of solidification; first was thermodynamics, then kinetics, then heat flow, and now, we come to composition variation; and when I say composition variation, you can understand that we are talking only about multi-component systems. A single component system cannot have variation in composition.

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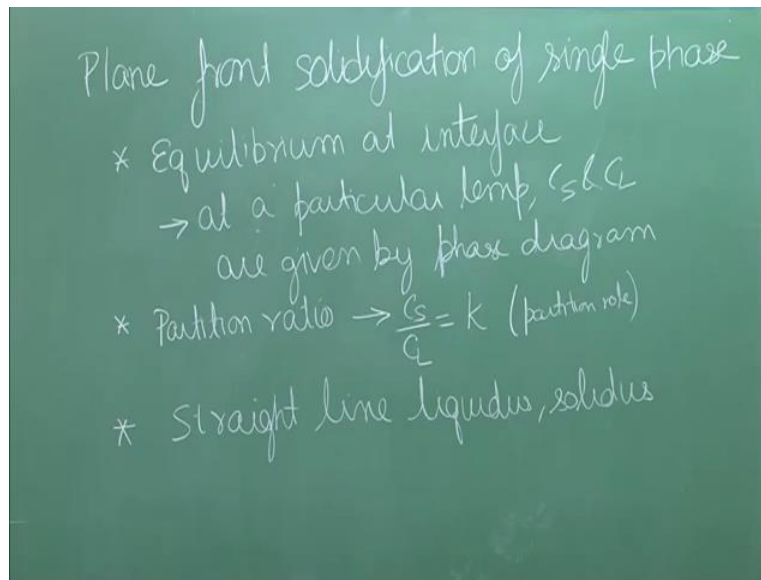


Now in multi-component system also let us draw a simple, the simplest multi-component would be two component system. So, let me draw one of the simple diagrams for multi-component. Now over here, even in a multi-component, you can get single phase; for example, if we call it alpha, this is a single phase, this is a single phase, there here you have two phases alpha plus beta; over here also you have two phases, alpha plus liquid, this is all liquid, this is beta plus liquid. So now, we will divide our problem into two parts; one, where we get single phase, another, where we get double phase. So, we will talk about a double phase near the eutectic that will be the simpler problem to deal with. And single phase region, where we are getting only one phase; although the components are two, A and B, but we are getting only one phase.

Now even for this single phase that we get, there can be some complications or let us not say complications, but there can be variations; one would be where you have plain front solidification. So let me define the overall broad class of composition variation that we will discuss in. So plane front solidification of single phase; this is one. Second is, instead of plane front, you may get cellular solidification, or where you are not getting plane front of solidification. So we will have cellular. So this is these two are related with this single phase. Another broad class would be the multi-phase, a polyphase. So here, we will look at plane front.

We will not get into, even in the plane front you will see that you will have more than one phase is getting formed; but we will not get into the cellular solidification of polyphase alloys, as of now we will keep it simple, and we will discuss only these three. So these are the three broad classes, and even amongst these, you would see there are several some subclasses you can see, and we will touch upon them as and when we get to them. So first we will start with the plane front solidification of single phase alloys.

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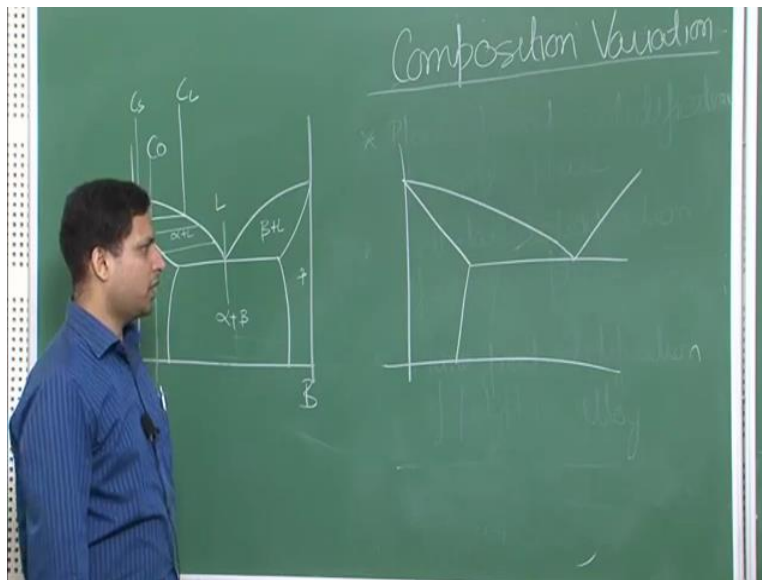
Now whenever we are trying to consider solidification of such kind of alloys, then we it is appropriate to make some assumptions. Otherwise if you do not take any assumptions, then it will be very difficult to formulate any kind of understanding. So some of the assumptions that inherently take when we are talking about these, one of them is 'equilibrium at interface'. What do we mean by 'equilibrium at interface'? So in the interface is here it is the solid and the liquid interface. So when we are doing the solidification, at the just at the interface, we assume, we always assume in these cases that there is equilibrium, meaning the solid and the liquid atoms can easily join or detach, basically based on what is, what is decided by the phase diagram.

So if we have taken a composition like this, and we are at a temperature like this, T , then this is the composition of the solid part, c_s , this is the composition of the liquid part, which is c_L . So when we say equilibrium, what we are saying is that the solid is at a composition c_s , and the liquid will always be at the composition c_L . So for a given temperature, c_s and c_L will get fixed, or the three combinations will be always, the three will always have fixed relation with each other. If you fix T , you would know c_s and c_L ; if you fix c_s and c_L , you can say what is the temperature. So the, in effect, equilibrium at interface is saying, that at a particular temperature, c_s and c_L are given by phase diagram. So this is a assumption that we make when we are talking about plane front solidification.

Another assumption that would take is regarding the partition ratio. What is partition ratio? The term C_S by C_L ; so you see we have drawn C_S or this particular temperature, you have a C_S value and a C_L value. Then for the ratio of this C_S by C_L is called, I have given a term K which is the partition ratio, and it has a important role to play. It determines that for a particular temperature what will be the solute concentration in the solid part and the liquid part. So that is why it is called partition; what fraction of solute is in the solid, and what fraction of solute is in the liquid, that is determined by K .

But as you can see, if you can go from one temperature to another, this is C_S by C_L , over here, it will be different, here it will be different, the C_S by C_L ratio would be very very different. And therefore, this is not easy to again mathematically understand, or mathematically you can say formulate. And therefore another (equa) another assumption is taken is that of straight line liquidus solidus. Now this is a very very interesting assumption, as you will see. Instead of taking these as curved, as they actually appear in the phase diagram.

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For the purpose of understanding, we take these as straight lines like these. Can you think of a reason why we take it as a straight line? We will think about it until the next class and we will come back at this point in the next lecture. Thanks.