Fundamentals of Materials Processing (Part–1)

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Lecture Number 12

Heat Flow (Effect of Superheat)

Okay so we get back to our problem of superheat. Like I said earlier in the previous class that so far, for simplification purpose, we were assuming that temperature of the liquid or the liquid metal that you pour in is always at melting point; but that is not the case. There will always be some amount of or you want it to be a little higher than the melting point, because then the fluidity increases. If you are very close to the melting point, the fluidity is very poor. So now, how will the things change? We have always talked about the temperature profile with respect to assuming that the liquid temperature is constant.

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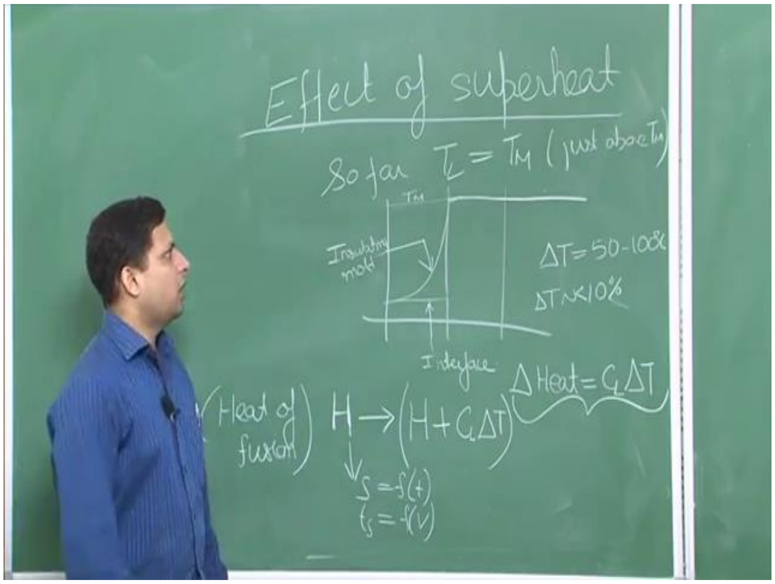

So if; somewhere say the temperature was like this, and solid was like this, or it was something like this, in the two condition; this is the insulating mold, and later on we will see other conditions. These are the two different conditions, but what what was common in all these was that, solid will of course never go beyond TM, but the liquid is usually higher than TM. But then how much higher that is the question. (The question) The the answer is it is usually of the order of 50 to 100 degrees celsius.

Now if we look at the temperature in Kelvin that is less than 10 percent. So in terms of absolute temperature, the temperature delta T is approximately 10 percent, less than 10 percent actually. So the temperature that we are talking about, the increase in the temperature, the liquid is not a very large quantity to look at the temperature profile, or to change the temperature profile by a much different value. But what will be different is that because there is a temperature rise, or the temperature is higher, and the liquid quantity may be very high; so, because of the specific heat of the material, when the liquid cools down, there will also be some amount of heat being generated or being given out because of that.

So what is that amount of heat? Delta, let me just write it at heat. So that will be equal to Cp, or, since we are talking about liquid, so CL delta T. Now this amount of heat is additional, and wherever we have used the term (ener) wherever we have talked talked in terms of heat energy, over there, we can make corrections by adding this term, because this is the only thing that is changing in a major way, otherwise, you see that the total temperature drop is less than 10 percent; but even that 10 percent temperature drop can lead to very large amount of change in the energy, heat energy.

So what we usually do is you (have) what was the place that we actually looked at the heat, or the total amount of energy, and that was heat of fusion H. So this heat of fusion H can now be changed to H plus CL delta T. Now, once wherever we had that those values, if we replace those values by H plus CL delta T, we will be able to take into account the effect of superheat. So, because the liquid is at higher temperature, it will only change the total amount of heat and because of that the solidification rate may be a little different, time required for total solidification may be a little different. And, if you remember the temperature profile equation does not have the H value, which means it will not get changed.

So this is what is changing here is only the H which will change the S as a function of t, and tf as a function of total volume. These are the places which are getting changed when we change the superheat of the liquid. So that is the effect of superheat. Next let us look at some of the realistic situations. We have looked at two very simple situation, two simplified situations, which although have application in real life, but if you look at it closely you would see that the those do not very accurately follow the temperature profile that we described; it is just too simple. What are the realistic situations? One of them is when the temperature drop is also (taking) or in fact is taking place mostly in the solid. And therefore, when; so we will call it case 3. So our case 1 was insulating mold, case 2 was interface dominated conditions, and case 3 is when solidifying metal has significant thermal resistance.

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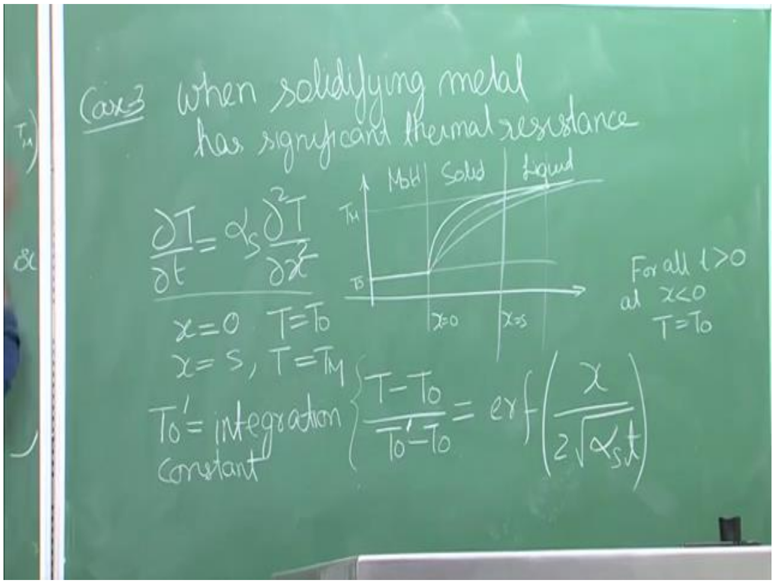

Case 3 when solidifying metal has significant thermal resistance

$$\frac{\partial T}{\partial t} = \alpha_s \frac{\partial^2 T}{\partial x^2}$$

$x=0 \quad T=T_0$
$x=s, \quad T=T_M$

For all $t>0$
at $x<0$
$T=T_0$

$T_0' =$ integration constant

$$\frac{T-T_0}{T_0'-T_0} = erf\left(\frac{x}{2\sqrt{\alpha_s t}}\right)$$

Now, how will the temperature profile look like over here? So we always ask that question first. How would the temperature profile look like over here? We said that solidifying metal has significant thermal resistance. Again, let us draw our three regions, mold, solid and liquid. Now we said that solid is where the thermal resistance lies. So again we are making a small simplification over here, which is that the mold remains at T0 temperature, and solid temperature profile will look like this. So now, the temperature drop as you can see, is taking place in the solid, and this is our x equal to 0 condition.

Now although it looks like a very simple temperature profile plot, instead of having the temperature drop in mold, we are having a temperature drop in solid. But it makes the overall situation much more complicated, the reason being, that here, the total length of solid is not fixed, and hence, this temperature profile will keep changing with time, even though things may be in a steady state, and I am saying steady state within codes, it is not really steady state, even then the temperature profile will keep changing. For example, if the solid the solid-liquid interface was over here, this is how the temperature profile would look like. Now if the solid-liquid interface moves over here, the temperature profile would look like this and so on; so it will keep changing.

So just because of the solidification taking place, there will be change in the temperature profile; along with the change in the temperature profile taking place because of the solid cooling down and heat passing through from the liquid to the solid to the mold. So there are so many different ways the profile can change, which makes this situation a little bit more complicated. Now again, like I said, we will not show would the analytical solution, but if you were to solve the analytical solution, you will have to again, solve these this this fundamental equation, this governing equation that we have, and our conditions are, at x equal to 0, T is equal to T0; x equal to S, T is equal to TM.

What is x equal to S? This is the x equal to S, and it is like we said, it is changing, it is a variable quantity. This is changing from as the solidification is progressing. And this is the initial condition, the boundary condition would be that for all T greater than 0, at x less than 0, T is equal to T0. Therefore, these are our boundary conditions, and initial conditions and boundary conditions; from these, the equations have been obtained by Carlson and Eager, and it is given like this. So this is a analytical solution of, the other solution, the other way that you can obtain this solution is through MATLAB, or not MATLAB, but any finite-element method in the class that I teach at IIT Kanpur, the students have been able to obtain the profile by using first principle, which is using finite-element method technique, and you need not always depend on the analytical solutions provided.

So that is another way you can always start from the fundamental equations, which are your this, which is this equation and use the finite-element method or some or other techniques, computational techniques to actually obtain the temperature profile. So this equation gives you the temperature profile for this condition where T0 prime is integration constant. Now having obtained the temperature profile, we always want to go the next step, which is to find the S as a function of t. So we will look at that too, and you can show that S as a function of t is given by this relation.

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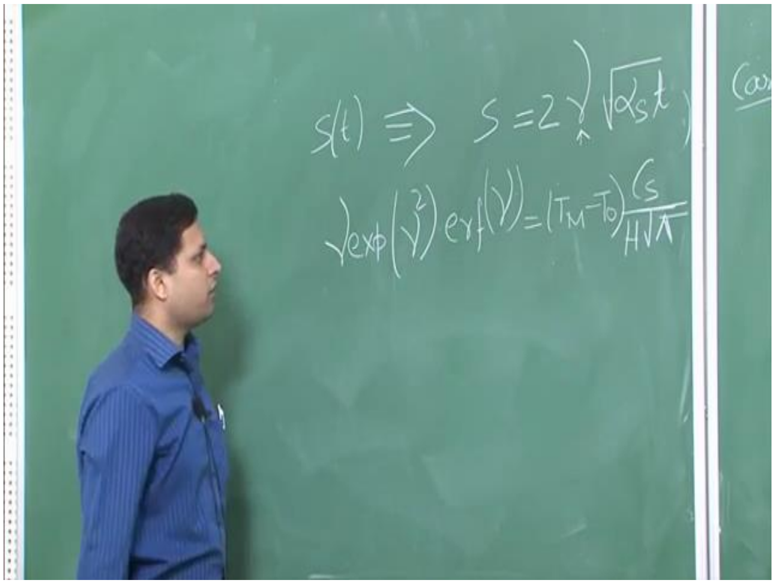

Although this may look like a very simple equation, but it is not so simple because again we like I said, there is this interface is keeps moving, and therefore, the values that you obtain are changing with, continuously changing with time. And therefore this factor, gamma factor that we have over here, it is obtained by a very complicated equation which looks like this. So this, gamma value that you have, you will obtain that gamma value from this equation. Somehow you have S as a function of t, and you have T as a function of x and t, which is x and small t, which is the time.

So this is a condition when you have solidifying metal has significant thermal resistance. Again, you will the question is where would you think this kind of condition would prevail? The answer is, in a water chilled mold, sand mold, water-chilled. What does that mean? In the mold itself, you let us say, you have water connections or water chiller, chilled water going in through, let us say water pipes, and, that keeps the temperature of the mold constant at ambient temperature. So there are this layer that you see over here, it is kept constant because, inside the mold itself, you have some mechanism to keep the whole mold at low temperature.

So this is and through, for example like a water chilled system. Even more realistic scenario is when, you (also) you do not have even a water chilled sand mold condition. Now that is getting as close as you can get to a very real sand mold casting that you would have, most of you would have seen in your institute. So, this is a even more realistic scenario; over here, you do not have the water chilling, and therefore, the mold may also get heated up. So this is our case 4.

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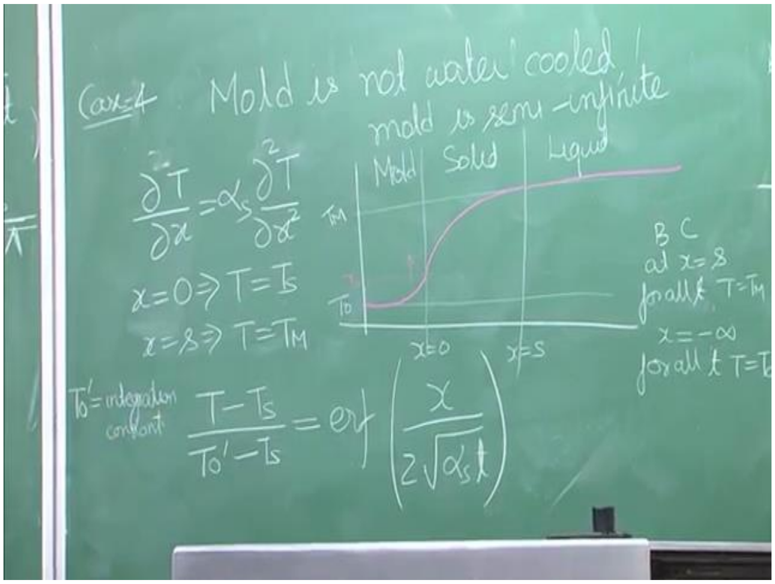

Mold is not water cooled. But, and again we are, we have to make some simplification to get to the analytical solution that we have over here; and that simplification is that ‘mold is semi-infinite’. This is not a very unreasonable assumption, we can always, like we said earlier also; as long as the solidification is fast enough, then you can get this this assumption is valid that mold is semi-infinite. And therefore in , you can say that most of the real case sand cast, sand mold casting that you see, does fall in this category; and mold is not water cooled in this condition. So mold temperature can also rise.

Again we have the three layers as usual; and we have the two important temperatures; the T0 and the TM. Now here, we have assumed a semi-infinite mold, but the mold temperature can rise, and it will rise near the mold-solid interface. So this part will look like this. Here the temperature, outside temperature of the mold is T0. By virtue of being very semi-infinite, meaning, long enough that before it starts to get get heated up at the outside, the solidification is complete; and, this is the temperature profile in the solid, and the liquid of course remains like this. Again, this is we are assuming, that there is no superheating, meaning, it is at TM temperature.

And, this temperature is a variable value; it will keep changing with time. Initially this TS will be close to T0, and as the progress, as as the solidification progresses, this TS will rise. However, this T0, the temperature we have over here or the outside, owing to the assumption that we have already made which is the semi-infinite mold, it will remain constant. So now it is time to look at the solution; again, like I said, you are not (going) going to derive the solution, it is very detailed work done by Carlson and Eager, and you can refer to their work for more detailed study, but we will look at the final solution. And again just for sake of completion, let me write the equation that (you) that will be the fundamental equation that needs to be solved to obtain these analytical solutions, or even the computational solutions.

You can like like I said, in my class, students have written MATLAB codes to get the temperature profile from first principle, meaning just using this, and not using the analytical solution. So here, we have, at x equal to 0, T is equal to TS, and so this is our x equal to 0, this is our x equal to S. So at x equal to 0, T is equal to TS at some time in between, and at x equal to S, T is equal to TM. And if you were to look at boundary condition, the boundary condition would be that at x equal to S, for all t, T will be equal to TM; and, at x equal to minus infinity, for all t, all t meaning time, this small t is for time, T is equal to T0.

So taking these conditions, this is the equation that has been obtained. So remember TS is the temperature at the interface, which is again changing. So this is for a given temperature given condition, and T0 prime is your integration constant. So we have obtained T as a function of x and t for as close to a realistic condition as you can hope to get using analytical solution. And, the analytical solution has been obtained by some by researchers, Carlson and Eager and this is how the temperature profile looks like for this realistic condition.

Now that we have the temperature profile, again, the question is, what will be the S as a function of t; that is, what length of solidification will take place, assuming it is a uni-directional heat flow? So assuming that it is a 1D object, we will like to know how the S looks like, and you will be surprised to see that this equation can also be written like, in this form.

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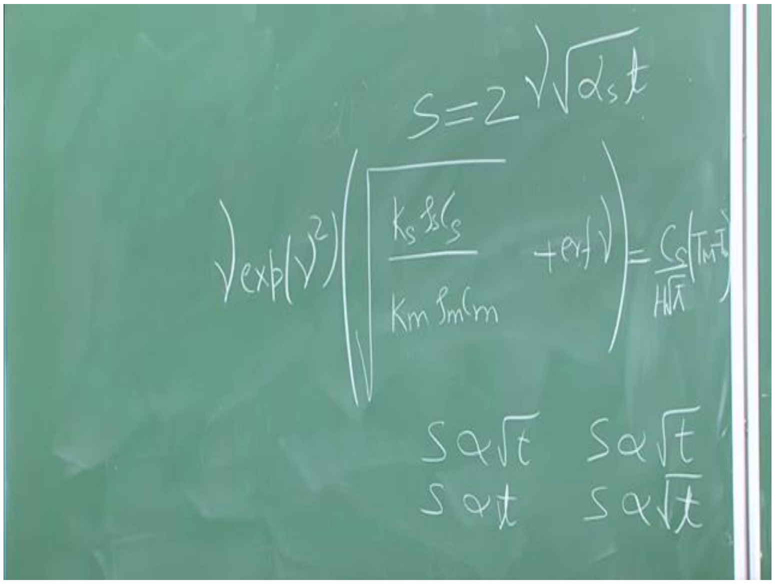

So S is still in the same form. But, what is different here, is this gamma; this gamma becomes even more complicated, and this time you would see, it will be written like. So the right hand is CS, divided by H root pi, times TM minus T0. But, the beauty of this is that the form of the equation for S in terms of t remains the same as in the previous case when we had water chilled mold condition. So now we have looked at two (simpli) very simplified case, and two more realistic cases. We will derive (eve) even derived the equation for temperature in terms of x and t for the simple cases, along with S and S as a function of t and the total time for solidification, for those two simple cases; and in that two (difficult) difficult cases, we have just given the solution.

And we have seen that there is, there was a big difference when we were looking at the two simple cases; one where there was insulating mold, versus one there was the interface resistance dominated solidification. When we had the insulating mold condition, we saw that S was proportional to root t, while when we had interface dominated, we saw that S was proportional to t. Now, let us compare this with what we obtained for the two two realistic condition that we have seen here. And over here also we see that S is proportional to root t.

So this is for the chilled mold condition and this is when the mold is not chilled. So in chilled mold condition, not chilled mold condition, and the insulating mold condition, we always, we all get S proportional to root t; while in the interface dominated condition, we get S proportional to t. Now in the experiments that people have carried out, not for this interface dominated condition, but for sand mold casting tower regions; sand mold kind of conditions, what they have found is that, S is actually proportional to t, but it is shifted. What does that mean? In mathematically, that means that people have found that S is equal to a root t minus b; of this form.

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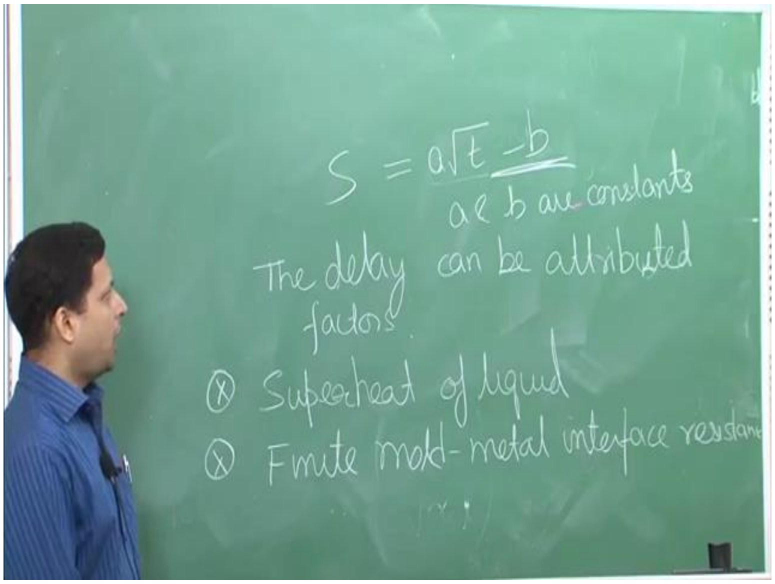

So the solidification does take place proportional to root t, but it is shifted in time by a small factor b, where ‘a’ and ‘b’ are some constants. Now why does this shift in b take place? There are various theories, but the most common explanation are that the shift, the delay can be attributed to two factors; to mainly two factors, depending on what is the condition; it is not necessary that both of them will play simultaneously. So we are talking about why this extra term b appears, meaning there is always some delay in the time, or delay in the solidification curve that you see, as a function of time. And it is mainly attributed to two factors; one is the superheat of liquid.

Now whenever your liquid is at a higher temperature, it will take some time for it to cool down, and therefore, at that particular time, even though solidification is taking place, it will actually be taking place in a linear or in the interface dominated condition. And therefore (inter) in the interface dominated condition, what do you have; S proportional to t. So, there is a (delay) actually what you what you see is not really a delay but actually it is very small rate of solidification taking place and in a linear condition, not in the a root t form.

Another cause for this delay is that finite mold metal interface resistance. You remember, except for the interface dominated condition, we have always assumed in all these three cases, that the resistance between mold and metal is negligible, and that is why we saw that continuous curve. If there was a interface resistance, there will be a temperature drop over there. But, that may not always be the case; there may be some amount of finite interface resistance. And because of that finite resistance, you may still get a delay in the again, what you are seeing is a very small solidification rate, and that rate will be proportional to t, that is linear (for) it will be in the linearly proportional to time.

So these are the two main conditions why you see a delay in the solidification rate, or the form of the equation like this a root t minus b. And this is what people have observed universally for most of these realistic sand mold (conditi) type of solidification conditions. So we will leave at that, and in the next class, what we will look at is the solidification of alloys; and why is it different? It is time for you to think about it a little bit, and we will get onto it in the next class. Okay, thank you.