Fundamentals of Materials Processing (Part–1)
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Lecture Number 11
Heat Flow (Interface Resistance controlled Solidification)

Okay, so we are back, and we will get started where we left last time, which was discussing what is the implication of the slope when we whenever we draw this temperature profile.

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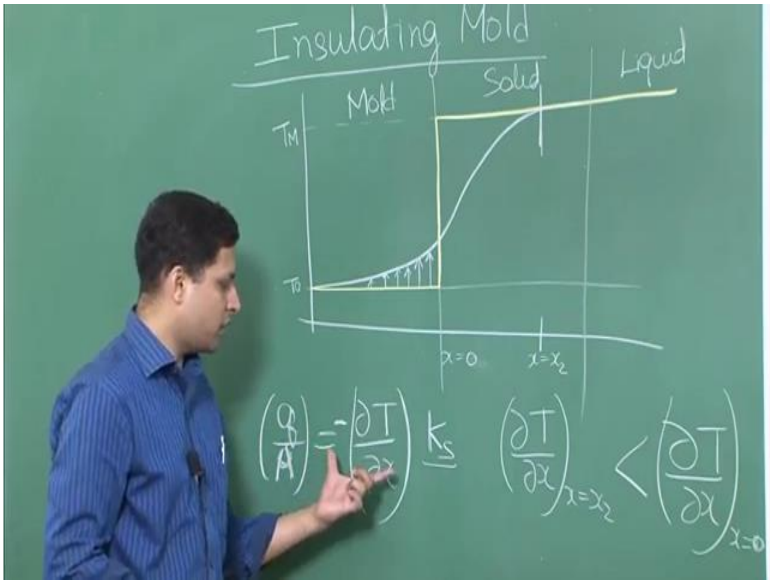

So you remember we talked last time about the rising slope for the mold part, and the decreasing slope when we are going from mold-solid interface towards the solid-liquid interface; there is a decrease in slope for the solid part. So this one, we showed that even you have a increasing slope like this, it means that the amount of heat that goes in from here is higher than the amount of heat that is coming out, and the implication is that that is because the mold is getting heated up. So the some amount of heat is getting absorbed.

Now what about this point? Let us say, we are talking about somewhere over here, which is x equal to x2, and this is our usual x equal to x0. So let us look at what happens over here. So, q over A is given by del T over del x times Km, and this is not Km, this is solid, so Ks. Okay, so it is the same thing I have written Ks towards the second part, but the equation is same. Now over here, what do you see? q over A, which is the amount of heat going in per unit area, per unit time, into a particular interface. So let us say we are talking about this, comparing this versus this.

Now this is at x equal to x2, what is different at x2 versus x0, what is different; this is not different, this is the material property, so it is constant. This is different; this is del T over del x is small over here. So, if you will look at the quantity del T over del x, at x equal to x2, this is less than del T over del x at x equal to 0. This is okay, let we have been using this as 0, not x0. This is x equal to 0. So, what is changing is the slope, and we see that the slope over here is less than the slope over here, which means if we look in terms of q over A, and there was a minus sign, okay, messing messing up a little bit; but there is a minus sign, but as far as we are concerned with the magnitude, you would see that the magnitudewise, the heat that is going in is small over here, and the heat that is coming out is larger over here.

So is not that very surprising that, the amount of heat that is going into the solid at this point is less, but the amount of heat that is coming out at this point is higher. So why is that happening? Here we said that the mold is heating up, so can we say anything similar about this part; and yes, we can actually say something similar about this part. So you remember this was your original temperature profile, meaning the solid was all the way up to TM temperature, just meaning just a little bit less than TM, after solidification. So what is happening is that, the solid is cooling down a little bit.

So when it is cooling down, amount of energy equal to Cp delta T or Cs delta T will also get added into the heat that has gone in here. So the total amount of heat of this Cp delta T; so the delta T for each and every point is different, and so you will calculate the Cp delta T for each small incremental amount of the solid, and calculate the total additional energy, so that will be equal to the total energy that is getting transmitted at this point. Now few more things. What can we say about the slopes at this point versus this point? Should they be equal? Well, let us look at it.

One thing we know, if we are just talking about one interface, then the amount of heat coming out should be equal to the amount of heat going in, as far as the heat, the resistance of the interface is negligible, that is, if we assume that no heat or no loss is taking place because of interface, which is also called as interface resistance, we will see next, then, the amount of heat coming out from this (so) solid-mold interface should be the equal to the amount of heat going into the solid-mold interface towards the mold side; meaning q over A should be equal to minus Km. So this is the total amount of heat, both of them should be equal.

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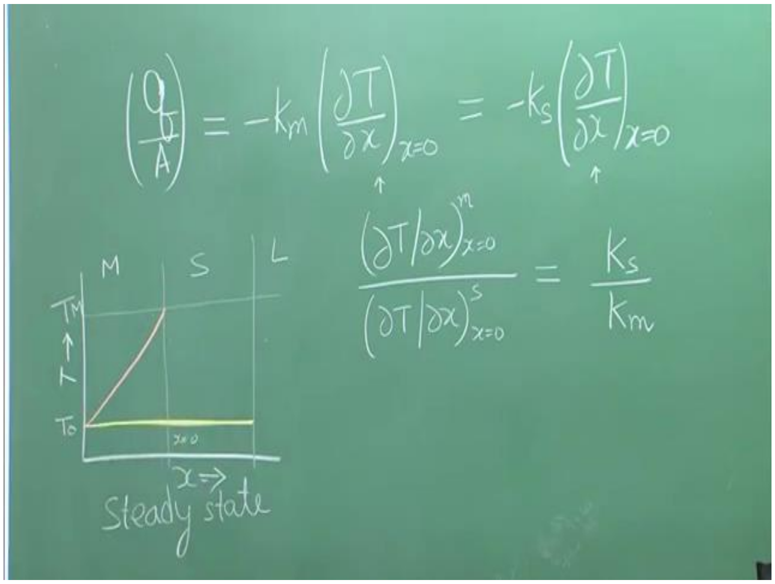

This is the amount of heat going into the mold; this is the amount of heat coming out from the solid, and, since there is no interface resistance, so these two quantities are equal. So now, we can see that the two slopes at the interface are actually related, but they are not equal. So what is their value equal to? Or how are they related? Del T over del x at x equal to 0, and now let me put a subscript over here, which is mold. So the slopes of mold versus solid is equal to the ratio of the thermal conductivity of solid to thermal conductivity of mold. So these two are related; meaning, this is the slope we are talking about, and this this is the slope we are talking about. So this is the slope of del T over del x, for solid at x equal to 0, and this is the slope in the mold part.

So these need not be equal, although the, there is no interface resistance and the amount of heat coming out is equal to (going) the amount of heat going in. There is no loss over there, but still the thermal gradient will not be same at the interface, it will be related by the ratio of thermal conductivities. So that is another interesting information that we get from here. Now whenever another thing that we are that we should be aware of when we are talking about the temperature profiles, is that whenever there is a slope, it means that it is in a transient state, it is not in a steady state. Now if we had fixed this temperature T0, then what will be the what will the steady state look like?

So this is our mold, this is our solid, this is our liquid, this is our x, and here is our x equal to 0, this is temperature. So like I said, this is a transient state or non-steady state in the in the steady state condition what will happen; so this is T0 or the ambient temperature, this is melting point. Now let us make another assumption because without that assumption, it will be a little complicated to draw the temperature profile; the assumption is that again the temperature drop is taking place only in the mold, that is we are again back to insulating mold condition. So this will be the steady state.

Now you see there is no change in the slope, throughout the length of the mold. What does this mean? That no whatever amount of heat is going in over there, is the same amount of heat that is going out, and that is called the steady state. So the mold has gotten heated up upto whatever point it had to, and now, there is no additional absorption of heat or giving out of heat, and that is why I have excluded a solid part, because if you draw the two parts, then it will not be in steady state, it will come to steady state in a very different way, which will be, when the whole system cools down to the room temperature, which is or the ambient temperature.

So that will be the, another steady state when for example, where you have not fixed the outside temperature, and let us say, you are eventually allowing the solid also to cool down. So here we have not allowed the solid to cool down; so solid was fixed at TM; but if we allow the solid also to cool down, then obviously we all know, from experience that everything will come down to the ambient temperature. So this will be the condition in the steady state for that scenario. So we have learnt enough about these temperature profiles; it is time to move on to another case that we were looking at, and this case see earlier we were talking about insulating mold condition with semi-infinite mold; that was one you can say, simplification that we were talking about.

Now we need to talk about another case, which is ‘interface resistance controlled solidification’. And again we are looking only in the uni-directional condition, because like you saw earlier condition, it is very easy to draw implication or draw equations or derive analytical solutions for a uni-directional condition, and thereafter, we can extend that uni-directional solution, 1-

dimensional solution to some simple geometries, even though they may be 2D or 3D. And that is why we start with again 1-dimensional condition.

So over here, when we talk about interface resistance, what we are saying is that, at the interface there is a temperature drop. So far you have seen, whenever we were talking about, for example mold and solid, and solid and liquid, we assumed there was no temperature drop at the interface. But, when we are talking about interface resistance dominated solidification, we are saying that it is at the interface that the resistance, not only that the temperature drop is taking place, but also the maximum temperature drop or in fact, for simplification, all the temperature drop is taking place at the interface. Now there is analogy of these kind of condition to the current flow; and let us look at it in that way.

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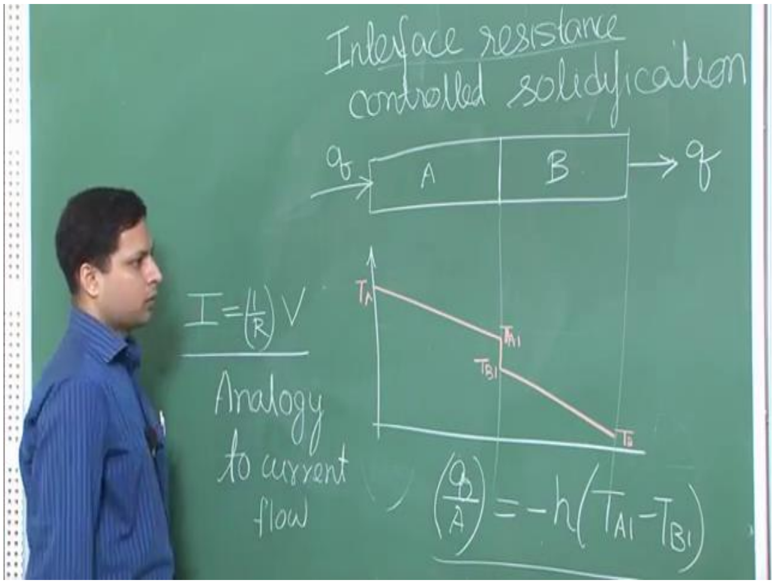

So let us say these are two different materials A and B; and they are connected at this point AB. Now if you draw the temperature profile for this, what you would see is that, the temperature would look something like this. So this is Tata, let us say this is TB, and over here, this is TA1, this is TB1. Now this (re) drop in temperature is what we are referring to as interface resistance. Because there is some resistance, because the conductivity between the A and B material is not taking place as smoothly as it as you would like to, there is a resistance created; and this resistance leads to temperature drop.

But this does not mean that the total amount of heat going in will change. If it is in a steady state, which (we) which we have drawn, because you see this is again a straight line; when I have drawn a straight line like this, it means it was, it is in a steady state. So when we have drawn a steady state, then in the steady state the amount of heat going in will be equal to the heat going out. We could not have said that, remember, if we were talking about transient state, remember in the mold and the solid, where the mold gets heated up, so it absorbs some amount of the heat, and when the solid is cooling down, it gives out some amount of extra heat. So the heat going in was not equal to heat going out.

But here if, like I said, it is in steady state; so amount of heat going in is equal to the amount of heat going out. And, therefore, we can write, q over A equal to minus h. Now next you will see why it is compared with, or the analogy is given with current. This is like the current; this is like the (temp) voltage drop, and this is like the conductivity. You remember we have I equal to 1 over R times V, where ‘1 over R' is the conductivity, because R is the resistance. So, the current flowing through any (conduc) if you put a resistance; let us say you have a current flowing through a resistance, then the current flowing throughout the resistance is same, it is constant. And, R is the resistance, then the relation between the voltage drop across the resistance is given by this relation, and this is similar to what we are getting over here, and that is why, there is analogy to this, to current flow.

Okay so now coming back to our problem, which is, we are talking about solidification; now we have seen how the, what is the meaning of interface resistance.

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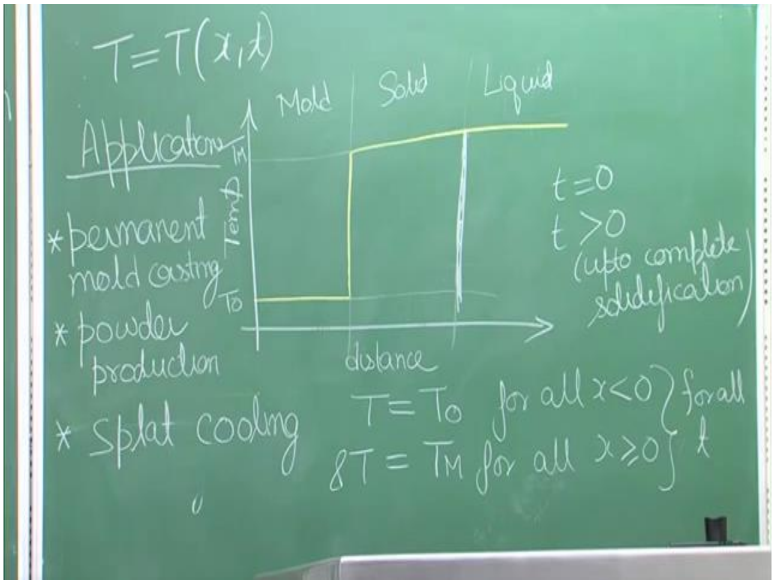

Let us look at our simplification that we have been talking about. So again, we will draw the temperature profile, and this is distance; and as usual we have three layers, mold, solid, and liquid. And we will again define how the (temp), the important temperatures one of which is the ambient temperature, the other is the melting point. So to begin with, we know that the temperature is like this. T0 is the temperature of the mold, to begin with, and, we are poured in liquid, which is at just about the melting point or just (abou) little bit above melting point, and when it, once it starts to solidify, what we are saying here is that the temperature drop only takes place because of the resistance between the solid and the mold; meaning there will be no other change in the temperature, or there will be no nowhere else where drop in temperature will take place.

And this will remain not only for T equal to 0; so this is valid for T equal to 0 as well as for t greater than 0, up to solidification, up to complete solidification. Beyond solidification, yes the whole thing would become solid, there will be no liquid, and the temperature of the solid would start to drop from TM to T0. But until the point that solidification is taking place, this scenario will hold. And this is why it is called interface resistance because the temperature drop is taking place only in the interface over here. So now, let us try to formulate the relations for this, and even before that let me say what are the places where we can, although this is like we have already mentioned it several times, there is a little bit of simplification, but there will be cases where these conditions or these equations can actually still be applied. So what are those applications, where we can say that interface is the dominating mechanism and temperature drop is only taking place because of interface resistance, and not because of anything else.

So, one thing you can realize is that the mold again has to be conducting; it cannot be non-conducting mold, because in the non-conducting or insulating mold, you will (have) start to see temperature drop in the mold, so that will become our previous case. So over here, it has to be conducting, which means we can have something like permanent mold casting. One of the most important applications for this, because in permanent mold casting you have something like a steel mold, into which you cast a high conducting material like aluminium. Another important case can be for 'powder production'. You remember, in the powder, the liquid is broken into small droplets, and then these small droplets fly at very high speed and get in touch with the substrate.

Now when they get in touch with the substrate, they will cool very quickly, and since the size is very small, we can assume that the total temperature drop is taking place only at the interface, and only after solidification the whole solid may start to cool down. So, during powder production; and another very important scenario that we discussed when we were talking about different kinds of casting was 'splat cooling', which is used in applications like coating, thermal coating barriers; splat cooling. So these are the important places where you would see the application for this kind of interface resistance dominated solidification.

Now, let us get back to some to getting to the equations, to show that we can find our equation for T in terms of x and t. We know the differential form of the equation, but now that we have the initial and the boundary condition, where we can get the analytical equation in terms of x and t.

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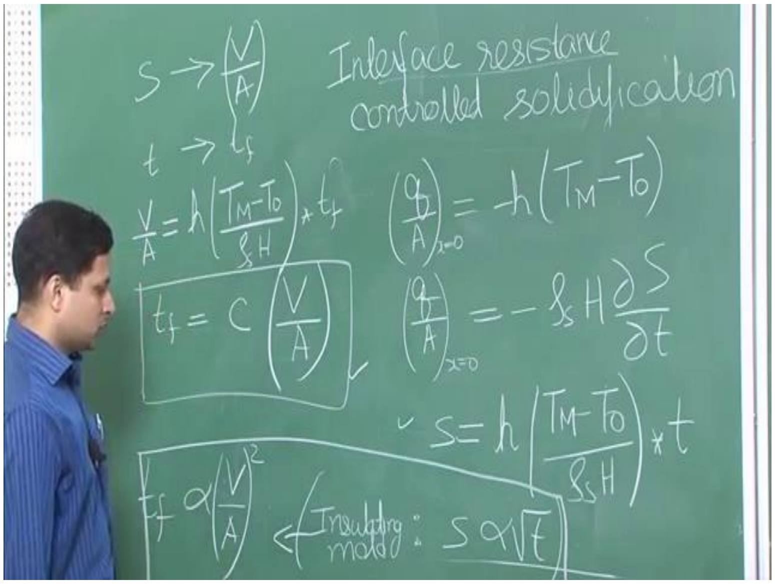

Now since it is a interface dominated, we said we know that q over A can be written as. So the amount of heat that is going into the mold is equal to this term minus h TM minus T0. So this is again, remember in analogy with the current flow. But at the same time, we also know from our previous discussion, that q over A and let me put the subscript x equal to 0, because we are talking at the interface. We also know from our previous discussion that q over A at x equal to 0 is what, what is the (amou) what is the heat that is coming out?

The heat that is coming out is the thin layer of liquid that has solidified; the thin layer of solid that has formed just recently. So it is this heat per unit time that is getting transferred all the way to the mold. And therefore, we can write this equation also as; so if del S is the small thin section of liquid that has solidified in time del t, then this is the total amount of heat that is being given out, and it is being transferred all the way to the interface; and the interface resistance equation we know, is like this; and therefore, we can get the equation for S by integrating.

Now what do you see here? This S is the amount of length that has solidified as a function of t. Now this equation as you see, is very different from what we obtained earlier. What did we obtain earlier? We obtained earlier that S is a (funct) is proportional to root t. That was for; so for S for insulating mold, S is proportional to root t; but here what we are getting? S is proportional to t. So the two conditions lead to very different solidification rates. Not only that, if you now look at the tf that we earlier obtained, which is tf was the total time for solidification; and again we will make the similar assumptions. Now that we have described everything about 1-dimensional condition, why not try to find out what will happen when we are talking about 2D and 3D.

So to begin with, we will talk about, let us say simple geometries like a rectangular bellets or rods or some simple geometry where the heat absorption rate is or the heat is not diverging. Then, in those cases we can say that S or replace S by V over A; where V is the total volume and A is the interface metal mold interface area. And if that is the case, then t becomes tf, because we are talking about the final solidification, only at the final stage do we get the total volume.

So t, we are talking about only of the total solidification time. And therefore we can replace it by. Again, you see we have obtained a total solidification time, which says that total solidification time is proportional to V over A. And how was it, when we were talking about insulating mold? Over there, we had tf proportional to; so these are for the insulating mold condition, and you see these are for the current condition which are the interface resistance dominated solidification process.

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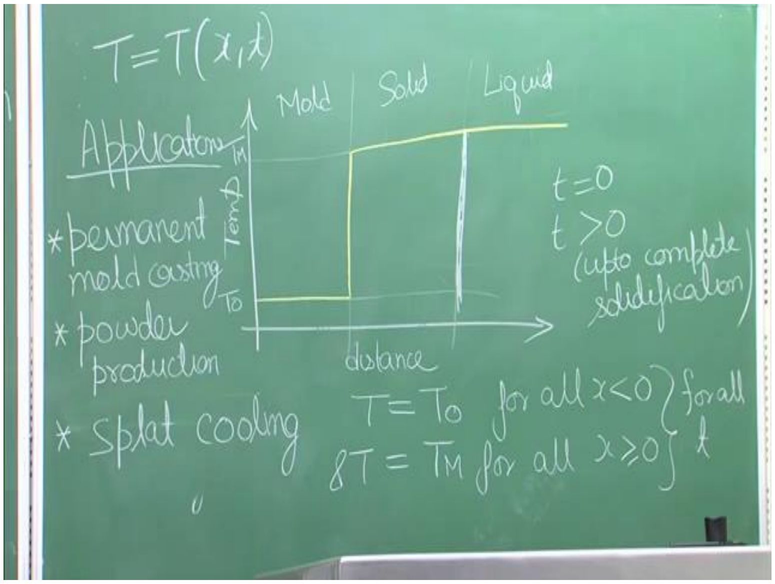

So we get very different condition when we are talking about interface resistance, or when we are talking about insulating mold. So we have these equations. One thing we missed or you can say we skipped was the equation to give t in terms of x and small t, which is time. And why are we not doing that? It is very simple. You look at this, this is how the temperature profile would remain throughout the solidification process. It is not changing, we are not getting any variation over here. So, we the simple equation for T as a function of x and t would be that t would be that T is equal to TM, for all x greater than 0, greater than equal to 0, and T is equal to T0 for all x less than 0.

So, we are getting the (rel) this is the relation T equal to T0 and and for all t, this is true for all t. So if you want a explicit function of T in terms of x and t, this is how it will look like, T is equal to T0, for all x less than 0, and T is equal to TM, for all x greater than equal to 0. Now all this we have discussed, we have always you would have seen, we have always taken liquid to be equal to the melting point. Now it is time that we understand and look into this parameter. So this is, so this is these are the two important solidification simplifications; one is the insulating mold, the other is the interface resistance dominated solidification.

We will also look at two other conditions, or two more realistic conditions but we will not derive those; we will only look at the final solutions that are, the that have been found, the analytical solution that have been found. But at this point what we want to do is understand the effect of superheat.

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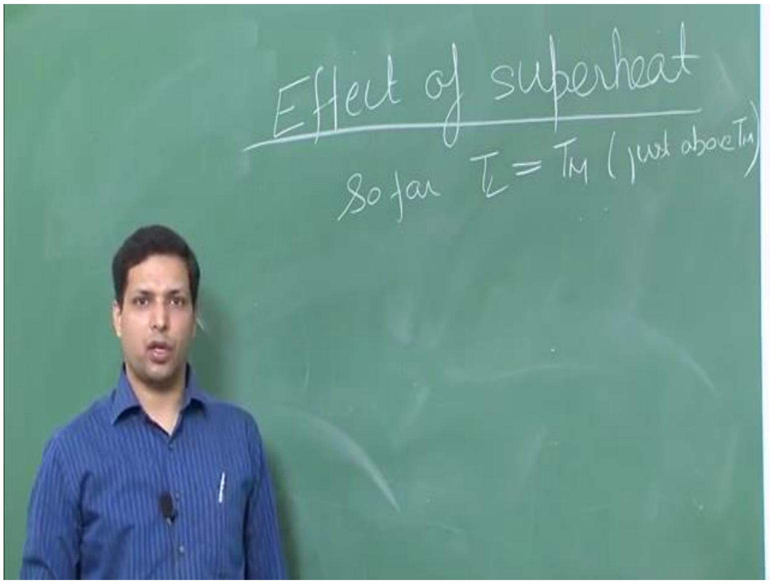

First of all, what is this superheat that we are talking about? So far we have assumed that TL is equal to TM, meaning that temperature of liquid was equal to TM, or may be you can say just above TM because we want the liquid to be in liquid state. But that is not always true, and what is different, you will have to look at that, and we will continue with this discussion in the next class – “What is the effect of superheat”, because we we do not always have the condition that T will be equal to TM; so I will come back to this in the next class.