Fundamentals of Materials Processing (Part–1)
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Lecture Number 10
Heat Flow (Insulating Mold Condition) (Continued)

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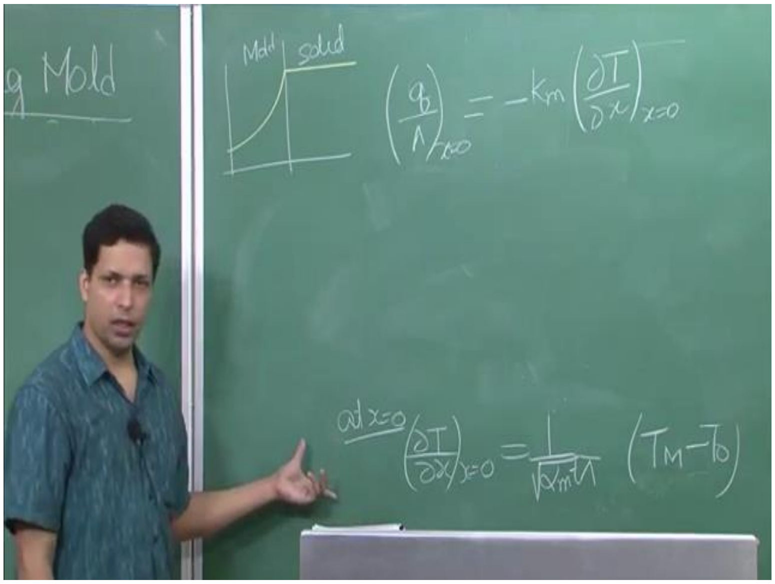

Okay so let us get back to the equation that we left with in the (last) previous lecture; so this is the del T over del x, at x equal to 0, and this is the form of the equation. Now if we want the heat flow rate which is q over A at x equal to 0, that is the heat going in from the solid into the mold, that is what q over A at x equal to 0 means.

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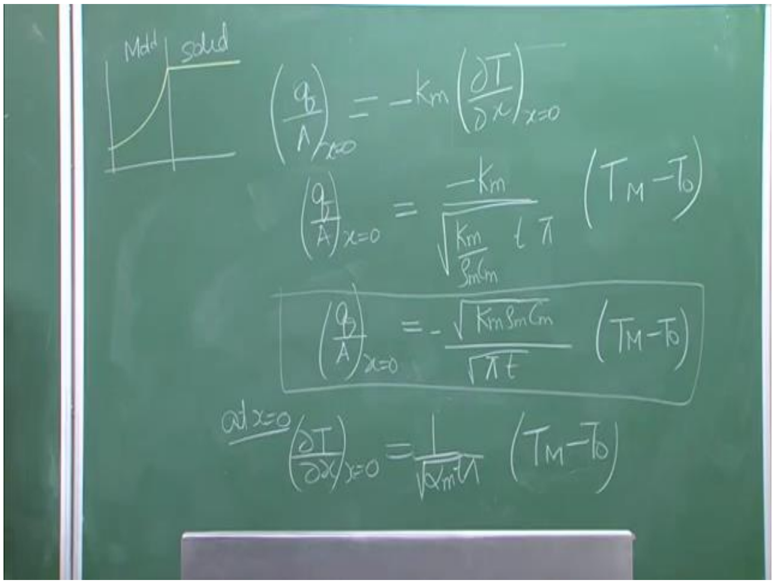

So this q over A at x equal to 0 turns out to be, and since alpha m also has the Km factor and rho m Cm, so we will break it into those smaller ones, which is Km by rho m Cm into t into pi times TM minus T0. So this becomes root; so this is our q over A, heat flow rate into the mold at the interface. We have invoked, remember we have invoked x equal to 0; if we had not invoked x equal to 0, then it would have been valid throughout the mold that equation of del T over del x, but we invoked x equal to 0 to get to this equation, because you will see we will get something substantial out of it. So this is our equation that we will carry forward from here, will the heat flow rate. Now that is one form of heat flow rate. We can also get by (doing) q over A in another or you can say using another logic.

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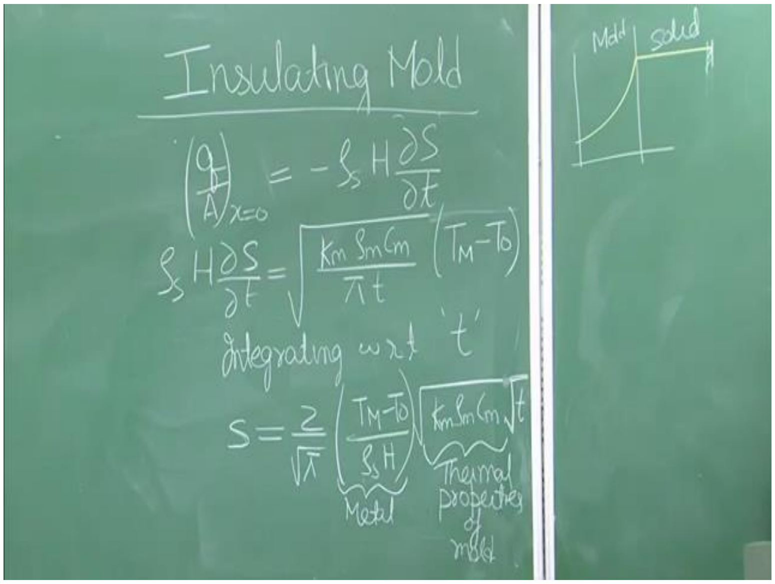

What is actually heat flow rate? That is going into the interface. If you think about it, what it is is whatever small layer of a liquid that solidifies. So let us say this is the liquid that solidifies in time sometime del t. So this is the del S length that has been formed, or del S in time del t. Now whenever this solid forms, (amount) some amount of heat is released. What is that heat? It is heat of fusion; so depending upon the amount of solid liquid that has solidified, that will be the equivalent amount of heat that is released. And therefore, q over A can also be written as; meaning in del t, this del x length has solidified and proportional to that, whatever is the amount of heat that has been generated because the when the liquid solidifies, then some amount of heat is released.

So whatever heat has been released, is what is being transferred all the way from the liquid to the solid to the mold. So that is the real q over A or the heat, actually that is going it. So now you see, we have q over A at x equal to 0 in this form, and q over A at x equal to 0 in this form. So we can write, equate these two and get a equation, something like this. And, if you want to find what is the length that has solidified as a function of time t, all you need to do is integrate this equation with respect to t. So integrating with respect to time, well you can (so) the show that or you can easily see or obtain from this equation, that S will be equal to square root of t.

So this S, the length that has solidified, can be obtained in terms of this time t. This is proportional to square root of t. Not only that, there are few more thing; we see that it is it is dependent on metal properties. So these are the metal properties; the rho S which is the density; TM and T0; TM is actually the melting point of the solid, H is the heat of fusion, and these are the thermal properties of the mold. Note how the Km Ks etc have not come into the picture; why, because we have assumed that conductivity of mold is so small that it will only play the role of or that will be the limiting condition. That will determine the rate of solidification, and that is what we are actually getting.

So you see that S is dependent on Km but not on Ks. Similarly it is dependent on Cm but not on Cs. But overall that is the form of the equation S, or the length of solidification as a function of time t. Now this , as you can clearly see, that S is dependent both on metal properties, not not the (thermal) not exactly the conductivity and specific heat, but still some of the metal properties and on some thermal properties of the mold.

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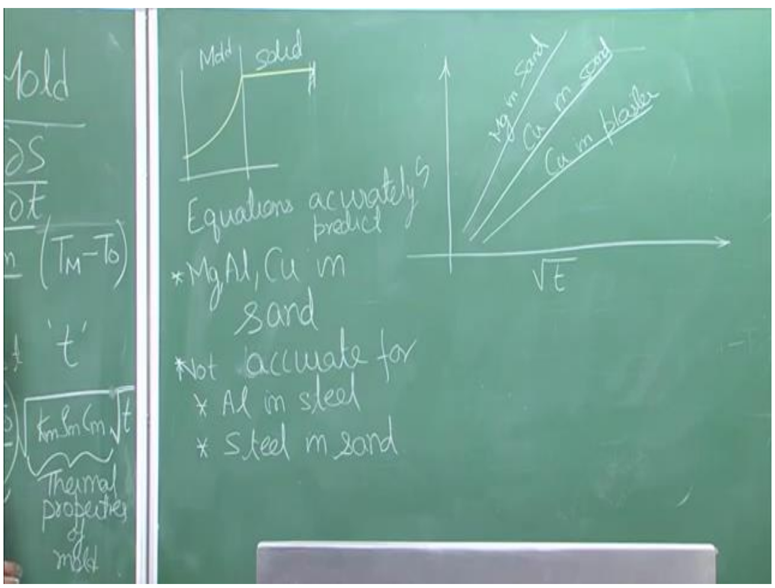

Therefore, if you change the metal or the mold, you would be getting different curves; so if we let us say you plotted S as a function of root t, what would you expect? You would expect a straight line. Let me draw it again to make, to show that it is indeed a straight line, it is a schematic. So this is a straight line that you would obtain because S is a proportional to root t, and all other parameters over there in that equation are fixed, or they are material properties. Now if, let us say this we say that this is for one particular material which is magnesium or one particular combination of metal and mold; so magnesium in sand. So it will come out like this.

Now let us say if I change the metal but keep the same mold material, you may get a different curve, and this is, these are just schematic, I am not trying to say which one is has higher conductivity and lower conductivity, this is just to show that it will remain a straight line but it will still be dependent on (well) what combination of metal and mold material you select. So for example this can be copper in sand; and let us this time let us keep the metal constant and change the mold material; so let us say (this) we have copper in plaster. So you see you get straight line curves; meaning, S will always be proportional to root t, but it is still change depending on what is the metal and mold combination. So this is our ‘insulating mold condition’.

Now just last bit of knowledge that we need to look into when we are talking about ‘insulating mold condition’; so (what) what kind of materials we said or what kind of (metal) metal-mold combination this would be suitable for. We said that it is insulating mold. So this will be suitable when you have a, like I said, like we said earlier at the beginning, for a highly conducting metal and a very insulating mold. So for example, if you are talking about aluminium, copper in sand, or even magnesium; so this will be these equation will be accurately, they will accurately predict for some combination like this, magnesium, aluminium which are high conducting materials, but in a insulating mold like sand.

But if you are talking about let us say, aluminium in steel, you do have a permanent mold casting, so, aluminium is very conductive too but steel is not orders of magnitude different from aluminium. So in that case, mold material, which is the steel in this case, will not act will not act like a insulating condition. And therefore, this will these equation not true, or not accurate, it may be combinations like aluminium in steel; because steel is not very different in conductivity, it is smaller than conductivity of aluminium but not by several orders of magnitude. Similarly, steel in sand, steel is low in conductivity and sand is not so bad. So they are again, not orders not several orders of magnitude different. So again steel in sand is not a good combination that can be predicted using this ‘insulating mold condition’.

Okay I said this is the last bit of thing, but actually I missed one more thing. We have another important thing that we can get from here, which is what is called as a Chvorinov’s principle, which is to find out what will be the total time for solidification. And again, what we all we need is equation that we have already obtained, which is S equal to 2 over root pi, TM minus T0 by rho SH times Km rho m Cm root t. So now what is how do we get to that? Remember we said that these equation are being derived for 1-dimensional objects.

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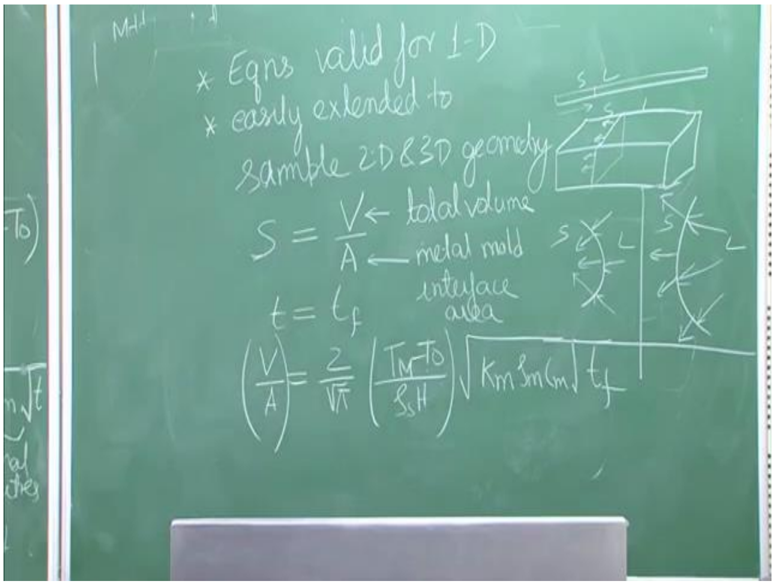

So for example a rod, very thin rod if you are looking, and we were looking at the solidification of this, then it is as good as , so this is solid, this is liquid, and then you can say okay, this question was valid. But, what will change if we are talking about some simple geometry, but 3-dimensional, let us say, a cuboid. The heat flow directions do not change by much, because these are still simple geometry, so if this was the solid part, this was the liquid part; still the heat flow directions remain similar as we had in a 1-dimensional rod. And therefore, these equations, the equation that we obtained for 1D can be easily extended to simple 2D and 3D geometry.

Okay? So, although we derived it for 1D, like I said, but if the geometry is very simple, like these, we can easily extend that equation which was our, all the equations that we described about. For example, our S as a function of t and q over A and del T over del x, all those things would be equally valid for these simple geometries. Now what will be different when we are talking about a little bit more complex geometry; let us say, if you are talking about a geometry where there are convex shapes. This is the way heat is going in. Or concave geometries. So this is the liquid part, this is the solid part; here this is the liquid part, this is solid part; these are two different conditions we are talking about.

So what you see is changing here is that here the heat is converging. Over here, the heat is diverging. So yes, there is some amount of difference, and because the heat is diverging, it is no more 1-dimensional, like the like what we assumed earlier for the previous case. It becomes 2-dimensional at the least, and even 3-dimensional depending on if it has curvature in the third direction, what is the z-direction. So this these things can cause a difference to the solidification times the total length that we have talked about S as a function of t.

But assume, let us assume that we are talking about simple geometries like these, then, we can replace this S by V over A. V is the total volume, and A is the metal-mold interface area. So if we talk about S, S is the total length, as you see, if you keep going all the way to the end, then this becomes your whole volume, which is the total volume that has solidified, and this is the interface area. So this interface area remains almost constant. And therefore in simple geometry, it will always remain constant.

So under that assumption, that the heat flow for unit area remains constant, we can say that S will be equal to V over A, which is the S that we obtained here, but in the final time, that is when the complete solidification has taken place; just at the point where complete solidification has taken place. So at that point, S can be said as V divided by A. So see the dimensions are the same, and at the same time, V over A, if as far as simple geometries are concerned, it remains a same quantity as what we derived earlier. And this will also imply that t will have to be equal to tf, so it is not a variable any more, it is a total time that has been taken for solidification.

So now, you can replace these over here, and we can get; where V is the total volume we are talking about. Whatever is the total volume of the cast that has been solidified is the metal surface contact area, the metal-mold contact area. So V over A will be proportional to tf, or in other words, let me erase this part to write what will be called as the Chvorinov’s equation.

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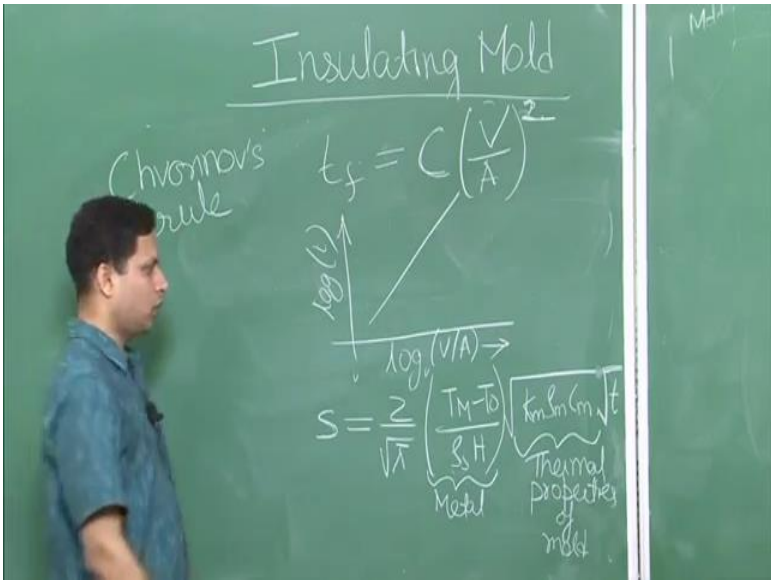

So this is what is called as Chvorinov's rule or equation. This is giving you the total time that will take for solidification in terms of the volume and the area contact area of metal and mold. So tf is proportional to square of V over A, and C is a constant; you can get C from this for simple geometries, and in fact even if the geometry is not very simple, you can obtain a C for that particular (geom) kind of geometry, and still get a relation like this where tf is equal to C times V over A square. So if you were to plot this as log (t), you will get a straight line again. So this is volume by area ratio for different molds, for the same metal-mold combination, and the total time it takes.

You take a, you plot the log log plot of this these values, and you will get a straight line, which shows that Chvorinov's rule is valid, and it holds true for this when when you have insulating mold condition and remember what were the insulating mold condition; when you have aluminium and copper in say sand mold, or even magnesium, any other conducting material like those, but not something like aluminium in steel or steel in sand cast mold. So those will not aluminium in steel or steel in sand cast will not be considered as insulating mold.

Okay so, at one point of time earlier, we talked about the form of the curve that we see; the temperature profile that we see, and we said that we will look back into it later on. So now is the time because we are again and again plotting it, and even before we move on without getting into the proper understanding, let us have a look and try to understand.

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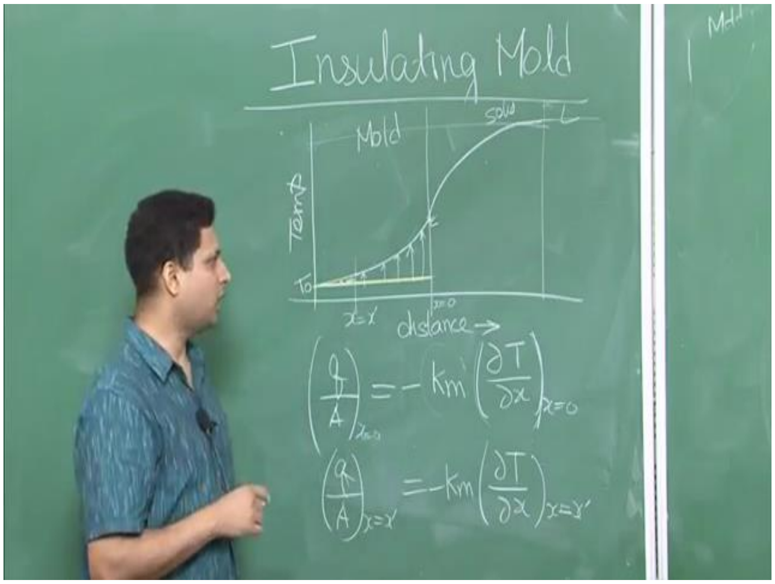

So this is your mold; this is a liquid, this is a solid, and not in let us say we are talking about some condition where there is also temperature drop in solid, but there is no interface resistance. Again, we are just trying to understand why the plot looked like these; that is the whole intention. You remember I said that when you go from left to right, you see a increasing slope for the mold, while you when you go from left to right for solid, you see a decreasing slope for the temperature profile. So this is again temperature, this is distance. So now it is time to take a look into it. Why do we always draw it like this, , you can say a concave here, and a convex over here, and not the other way round?

So again let us go back to our equation for the heat flow, q over A; let us say we are talking about the heat going into the mold, which was at x equal to 0, is equal to minus Km. So the heat that is going into the mold is q over A, by at x equal to 0 minus Km times del T over del x at x equal to 0. So, what we want to understand is what is happening or what, how is it changing when we, let us say when we go from this point to this point. Now this is a parameter, do you think that can change when we go from here to here in the mold? No it will not. What will change is this. It is the slope, or the thermal gradient.

So this thermal gradient, when we go from x equal to 0 to some minus values of in the mold, we will see that this becomes smaller and smaller. So what will that mean? So if you are talking about q x equal to 0 and let us say, x x equal to x prime, which is lower than x equal to 0. So in here, this is this quantity is larger, and this quantity is smaller, and therefore, this whole quantity becomes in magnitudewise, this is smaller than this. So what does this mean? It looks like, by the time, this this represents remember the total heat rate flow. So total amount of heat that went into the mold at this point, is higher and the total amount of heat that goes into this point onward, is smaller.

Now that is that looks like some magic is going on over here. But it is not really magic. What you have to realize here, is that what was the initial condition of the mold, and let me again draw this. So this was the initial condition of the mold, that is, it was all at temperature T0. When we say that the temperature from the heat that went into this is higher and that the than the heat that went into this point, what we are saying is that some amount of heat has been absorbed, and, you can see the fact that the temperature profile has raised. Initially it was a flat line; now it has went upto this. What is this? This is the, this increase represents that some amount of heat has been absorbed in a mold, and that is why the total amount of heat that goes into this point, which was q over A at x equal to 0, which in magnitude is larger than q over A at x equal to x prime, which in which is in which in magnitude is a smaller quantity. So a smaller amount of heat is coming over here.

And in an insulating mold, what you expect is that the length of the mold is such that the by the time the outside of the mold starts to get heated up, your solidification has taken place; so that is what the semi-infinite insulating mold condition means. We will look at some more at this profile, temperature profile (al) in an all for the solid part again in the next class, but for the time being, just look at this part, and also try to understand what is going on in the other part. In the other part, if that is the case, why is the other part showing an inverse curve? So we will look at that in the next class. So we will finish off over here, and come back to this in the next class. Thanks.