**Structure of Materials**
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**Lecture - 5**
**Geometry of Crystals: Symmetry, Lattices**

The next question we ask ourselves is that, what is the shortest lattice translation vector for this cantered rectangular lattice.

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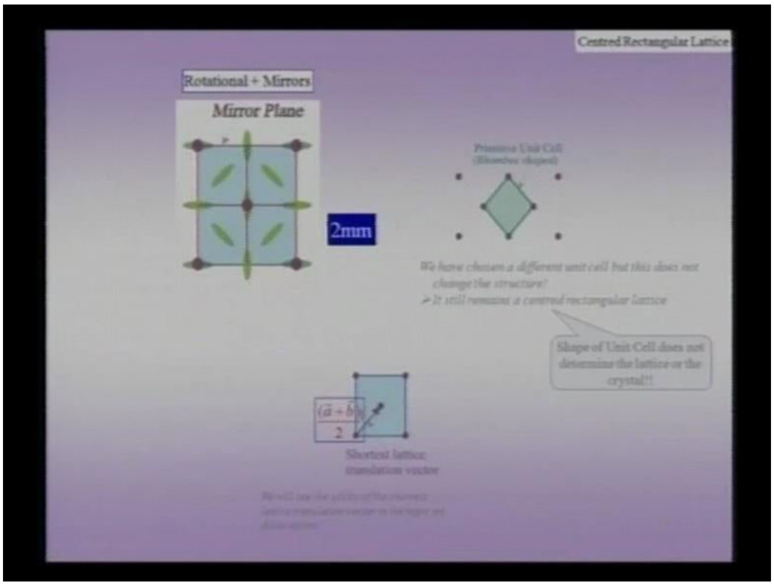


So, we have the unit cell of the cantered rectangular lattice, and clearly neither a nor b is the shortest lattice translation vector, but it is actually half the diagonal of the unit cell, which is a plus b by 2, which is the shortest lattice translation vector. The shortest lattice translation vector is an important quantity, and we will deal with it later when especially in the context of dislocations, wherein this shall play the role the Burgers vectors.

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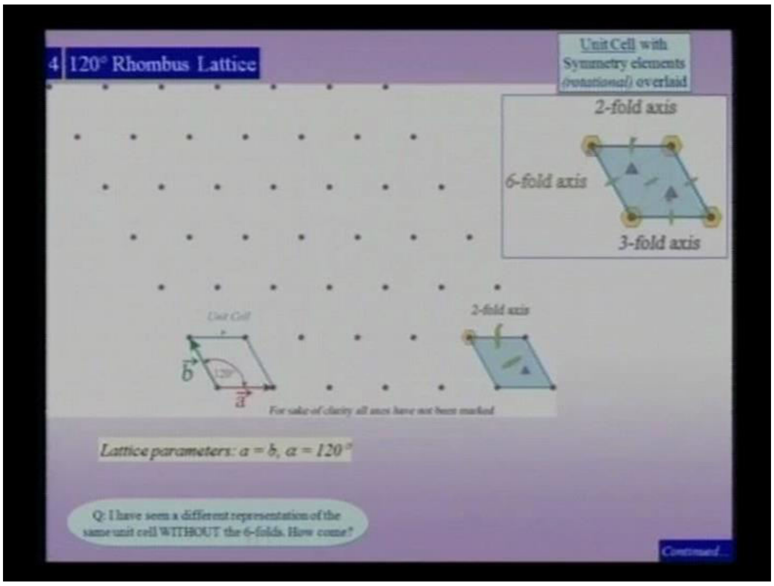


The fourth lattice we consider is the 120 degree rhombus lattice, as you can see it is not just any rhombus, but a rhombus with an included angle of 120 degrees. In this picture we can see the 120 degree rhombus lattice, also overlaid on the lattice is the lattice translation vectors a and b, with an included angle which is 120 degrees, and also the unit cell of the lattice. The lattice parameters for this lattice are a equal to b which is why it is called a rhombus, and the included angle alpha is equal to 120 degrees.

On the right hand side of this lattice is the same unit cell, which some symmetry elements overlaid on it, as before we have not overlaid all the symmetry elements, and that is for the sake of clarity, for you to understand the essentials of this lattice. Clearly there is a 6-fold axis, there is also a 3-fold axis which is at the centroid of the triangle, which is half the rhombus unit cell; there is also a 2-fold which is at half the distance of the diagonal.

Let us look at this lattice unit cell a little more carefully, looking at the figure on the right hand side, here in we have overlaid all the rotational symmetry operators on to the units cell. So, there are 6-folds at each vertex of the unit cell, and inside the unit cell at the centroid of the triangles are 2, 3-fold axis and at every edge centre, there is a 2-fold axis and additionally along the diagonal centre, there is a 2-fold axis. One question which sometimes people have read some textbooks, might ask is that, sometimes there is a representation of the same lattice without the 6-folds, how is this possible.

Clearly we can see from the lattice and from the symmetry of the lattice that, this lattice has a 6-fold symmetry, yes there additionally 3-fold axis. But, this 3-fold axis is in addition to that 6-fold and not at the expense of the 6-fold, we will try to answer this question very soon.

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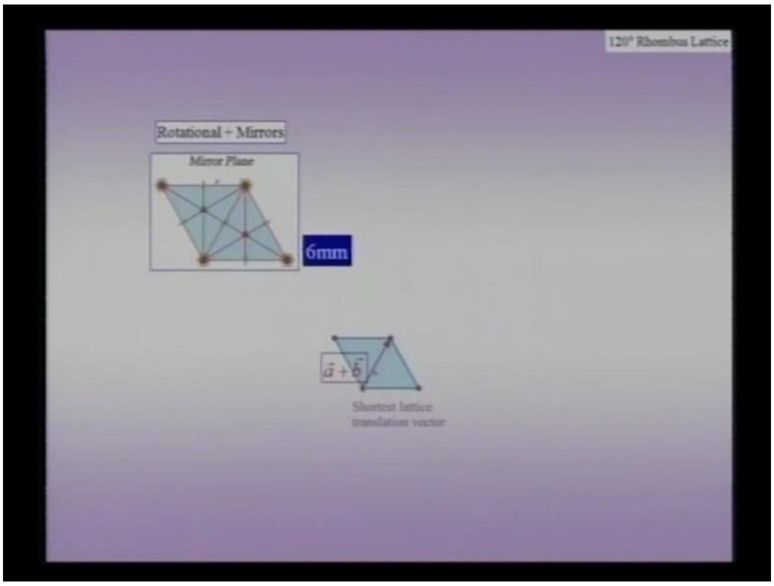


Now, on the same unit cell, suppose I overlay the mirror symmetry operators, that is in addition to the rotational symmetry operators, I am overlaying the mirror symmetry operators and that is what has been shown on the figure on the left hand side. So, you have original 6-fold axis, the 3-fold axis, the 2-fold axis and additionally there are these red lines which are mirrors in this lattice. So, you can see this edges are mirrors, there are these vertical mirrors and also which are equivalent to these inclined mirrors also.

The long diagonal and the short diagonal are also mirror planes, to write a short hand notation for the symmetry of this kind of a lattice, we write it as 6 mm, wherein the first mirror can be considered as this mirror. And the other mirror is the diagonal mirror, which is inclined at an angle of 30 degrees to the original mirror, the other mirror which is at 60 degrees to the original mirror need not be considered or written separately. Because, that will automatically be generated by the rotation of this original mirror say for instance this horizontal mirror, by the 6-fold axis.

In other words, the 6-fold axis will take the horizontal mirror to the mirror which is inclined at the angle of 60 degrees, but this mirror is not related to that mirror and has to be listed separately. Say in other word, this would constitute a mirror 1 and this mirror at 30 degrees to the original mirror would constitute the mirror 2; and the shorthand notation for that would be a 6 m 1 m 2 or even more concisely a 6 mm symmetry. And just to reemphasise the fact, I have redrawn the lattice here with the shortest lattice translation vector.

As you can see also in this case, the shortest lattice translation vector is neither the a vector nor the b vector, as we already know the modulus of theta vector is same as the modulus of the b vector, but they are inclined at the angle of 120 degree to each other. And the shortest lattice translation vector is the vectorial sum of a and b, and this along the diagonal of this rhombus, which is shown here in this figure. So, we have identified the shortest lattice translation vector, we have looked at the mirror symmetries and the rotational symmetries of the 120 degree rhombus lattice.

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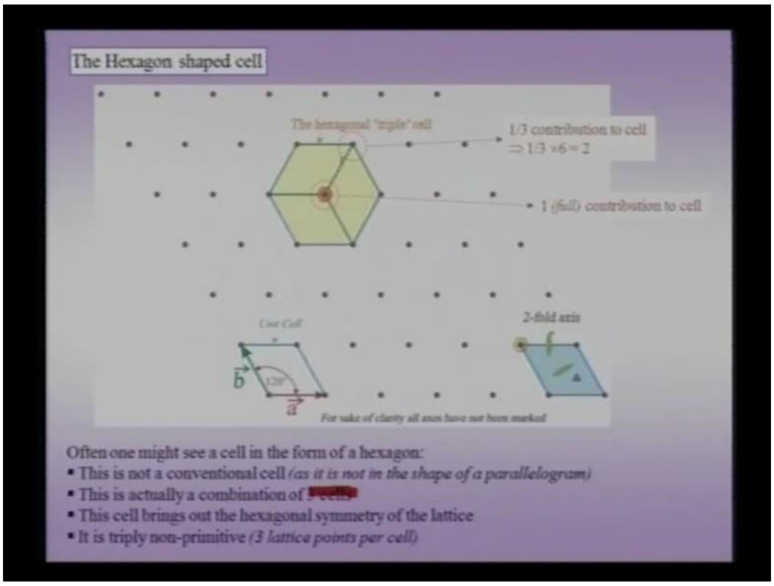


An important question which comes to mind, when we are dealing with such lattices is the fact that, when we consider a 120 degree rhombus unit cell, the rhombus unit cell itself does not have any 6-fold symmetry. And often when you are reading textbooks you might have come across the unit cell, which looks like this yellow unit cell which has got 6-fold symmetry. So, the question which comes to mind is that, why are we using this as the cell for this lattice, and more importantly what is the role of this yellow shape cell for this hexagon shape lattice.

So, let us consider this hexagon shaped lattice, this hexagon shaped lattice is in fact, a composite of 3 unit cells or 3 cells of the original type, but they are all not along the same orientation. They are in fact, rotated with respect to each other, for instance this yellow unit cell is rotated with respect to this yellow unit cell, which is further rotated with respect to this yellow unit cell. But, the whole composite 6-fold unit cell is a combination of these 3 unit cells, now if you try to calculate the number of lattice points per cell for this larger cell, we see that the lattice points are divertices of this unit of the cell.

Actually make a contribution of 1 3rd to this larger cell and therefore, net contribution from 6 of these is 2, the lattice point at the centre of the cell makes a complete contribution, which is a full contribution to the cell which happens to be 1. Therefore, the net contribution or the net content of this larger cell is 2 plus 1 which is 3, which is what we would expect, because the original cell was a primitive cell. And this is three times larger in area, therefore it is not surprising that this is a triply non-periodic hexagonal cell.

Now, this is not the conventional cell which we use, and the reason is very obvious the conventional cell is a parallelogram has to be a parallelogram in two dimension, and in three-dimensions as we shall see it is a parallelepiped. Since, this hexagonal cell is not a parallelogram, this is not the typical unit cell we would use, for the description of an hexagonal cell. But, the importance of this kind of a cell comes into picture, when we want to understand the hexagonal symmetry.

In other words, this cell has a hexagonal symmetry which the smaller primitive unit cell, which lacks this kind of an hexagonal symmetry. So, to summarise the important points of this slide, the hexagon shape cell often which is shown in many textbooks is chosen, to illustrate the 6-fold symmetry which this lattice has. And we have to be absolutely clear this is not a conventional or the typical choice of an unit cell, and this is triply non-periodic or triply non-primitive unit cell, term it is triply non-primitive unit cell. It is three times in area as compared to the blue unit cell and therefore, it is got a net area which is three times and it is content in terms of lattice point is also three times.

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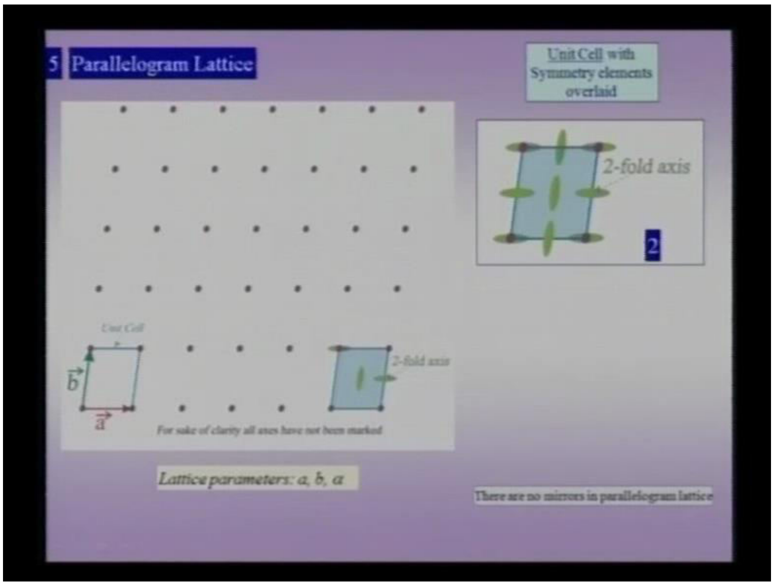


The fifth two-dimensional lattice which we shall consider now, is the general parallelogram lattice, in other words in this case there are no constraints, either on the lattice parameters a and b or on the included angle. The fifth the lattice we consider which is the final lattice in two dimension is the general parallelogram lattice. The general parallelogram lattice has no constraints, as far as any of the lattice parameter goes a, b and alpha, all can take any of the possible values. The unit cell is the general parallelogram and on the right hand side of the figure, the unit cell is shown in a larger figure.

Now, when I look at this lattice and when I overlay the symmetry operators as before, I can see that this lattice has only 2-fold rotational symmetry operators, this is the minimum a lattice in two dimensional has which is a 2-fold. Hence this brings about two important points, number 1 even though the parallelogram lattice is the most general or the lowest symmetry lattice, it still has 2-folds. And the other point being any two dimensional lattice would have 2-fold symmetries, so where are the 2-fold located within the unit cell, as shown in the right hand side, they are located at the vertices the edge centres and also the face centres or of the centre of the side.

All the symmetry operators within the unit cell are shown on the figure on the right hand side, and you can clearly see that there are symmetry operators are the vertices, the edge centres and also are the face centres. If I want to give a short hand notation for the symmetry of this kind of a lattice, all I can say about it is, it has got a 2-fold rotational symmetry. And clearly this lacks any higher order rotational symmetry operator, like a 3-fold, a 6-fold or a 4-fold and additionally, there are no mirror planes in a general parallelogram lattice.

So, to summarise what is a general parallelogram lattice, it is a lattice which can be obtained from for instance, a square lattice by sharing the lattice and also distorting it along x and y directions, in any pre-manner. And in the process we find that, the lattice parameters a, b and alpha all can take values which are not constraints in any way, alpha can take any value, a and b need not be equal to each other, or in any other way b constraint to have a certain value.

The unit cell is a general parallelogram, in other words a parallelogram means the opposite sides are parallel, but it is not a rhombus lattice as in the case of the previous example we considered, which was the 120 degree rhombus lattice. This kind of a lattice has only 2-fold of symmetries and the 2-fold symmetries have been overlaid on the unit cell, as shown in the figure on the right hand side.

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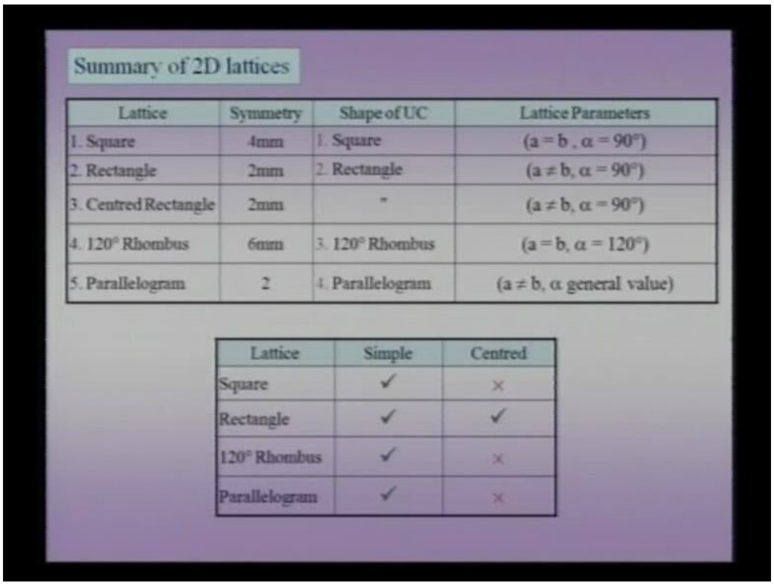

Summary of 2D lattices

| Lattice | Symmetry | Shape of UC | Lattice Parameters |
|---|---|---|---|
| 1. Square | 4mm | 1. Square | $(a = b, \alpha = 90°)$ |
| 2. Rectangle | 2mm | 2. Rectangle | $(a \neq b, \alpha = 90°)$ |
| 3. Centred Rectangle | 2mm | " | $(a \neq b, \alpha = 90°)$ |
| 4. 120° Rhombus | 6mm | 3. 120° Rhombus | $(a = b, \alpha = 120°)$ |
| 5. Parallelogram | 2 | 4. Parallelogram | $(a \neq b, \alpha$ general value$)$ |

| Lattice | Simple | Centred |
|---|---|---|
| Square | ✓ | × |
| Rectangle | ✓ | ✓ |
| 120° Rhombus | ✓ | × |
| Parallelogram | ✓ | × |

Now, let us summarise all the 2 D lattices and look at their symmetry, look at the shape of their unit cell and also consider their lattice parameters. One thing and additionally there is a table at the bottom, which tells us that which of these lattices have centring possibilities and which of them do not have a centring possibilities. So, the lattices in two dimensions are the square lattice, the rectangle lattice, the centered rectangle lattice, the 120 degree rhombus lattice, and the parallelogram lattice.

The symmetries possible, square lattice has a 4 mm symmetry, the rectangle lattice has a 2 mm symmetry, the centered rectangle lattice also has a 2 mm symmetry, the rhombus lattice has a 6 mm symmetry and the parallelogram lattice has the least possible symmetry, for any lattice which is a 2-fold symmetry. One point which we have been making so far, and we will repeat again and again, because this is a really a point of confusion is the fact that, often we use unit cells with describe lattices; but at no way these two should be confused.

The shape of the unit cells which we can choose, and the typical unit cells chosen are for the square lattice we choose a square unit cell, the rectangle lattice we choose a rectangle unit cell. And the rectangle unit cell is also chosen for the centered rectangle lattice, for the 120 degree rhombus lattice we choose a 120 degree rhombus unit cell and finally, for the parallelogram lattice which is the most general possible, we choose a parallelogram unit cell. The lattice parameters on the constraints which we had considered so far is also summarised in this table, and we can clearly see that for the square unit cell a is equal to b and alpha is not equal to 90 degrees.

For the rectangle unit cell a is not equal to b, but alpha is still constraint to be 90 degrees, so it is for the centered rectangle lattice, for the 120 degree rhombus lattice a is equal to b, but alpha is equal to 120 degrees, for the most general possible parallelogram lattice there is no constraints either a or b or an alpha. Just to summarise the data above in a different form of a table, we consider the table as shown in the figure below. And the table shows you that for the square for the 120 degree rhombus, and the parallelogram there are no centered variations of the lattice. The centered lattice is possible only in the case of the rectangle lattice before.

Student: Sir, how it is possible

So, in the audience here I have student, and Mr Patel has a question, why is that only for the rectangle lattice we have the centring possibility, we will consider this all the possibilities and why some of them are possible in the coming slides in lot of detail. Let me review these lattices we have done so far, before I progress into some further consideration of about these two dimensional lattices. So, let us start with a stage where we try to construct a two dimensional lattice, and we said there are five distinct two dimensional lattices, there are no more possibilities just five no less possibilities, so that has to be absolutely clear.

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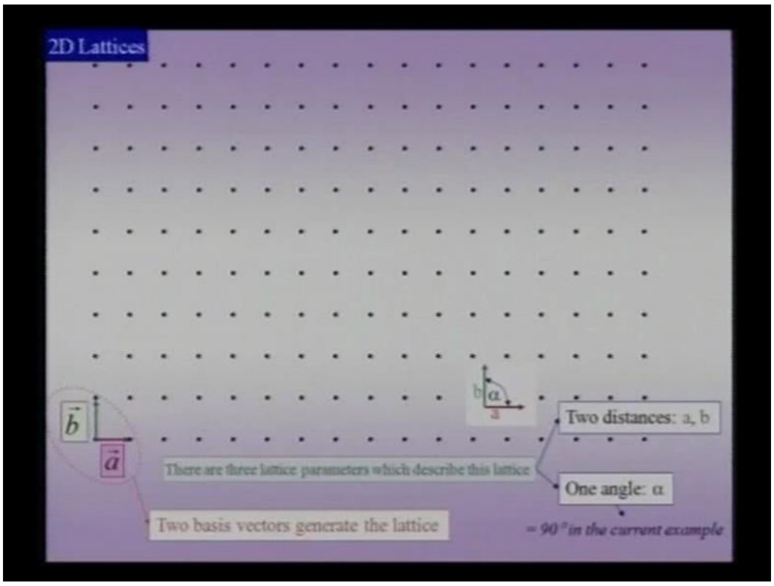


And these lattices are generated by two translation vectors, which are non co-linear, because if they are co-linear they cannot spread themselves out in two dimensions, there are three lattice parameters which go on to describe a lattice in two dimensions. These are two length scales a and b, and there is an included angle between the two, which is typically given a symbol alpha. In the example shown in the figure, this alpha has been constraint to be 90 degrees, but in no way we should mistake that lattices always need to have this included angle as 90 degrees, this is just a starting example.

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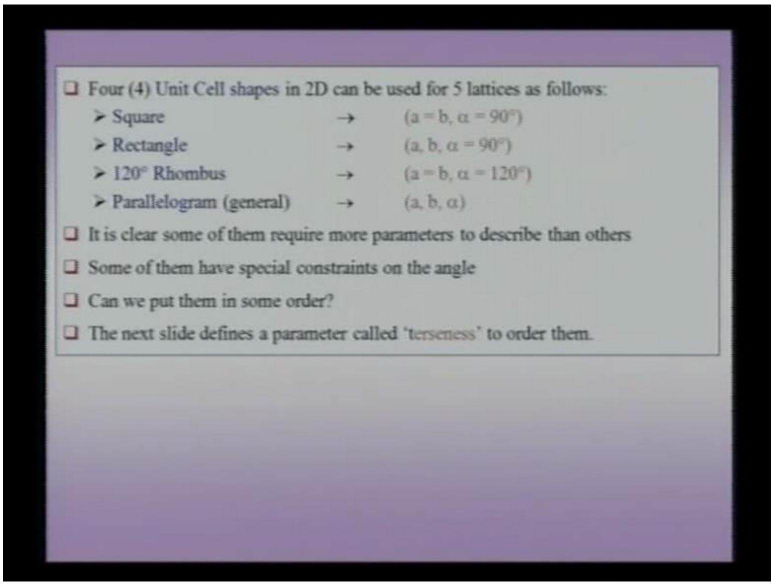


Another point will considered that there are four typical unit cells, which we can use to describe these five lattices possible in two dimensions, and these unit cells are the square, the rectangle, the rhombus and the general parallelogram.

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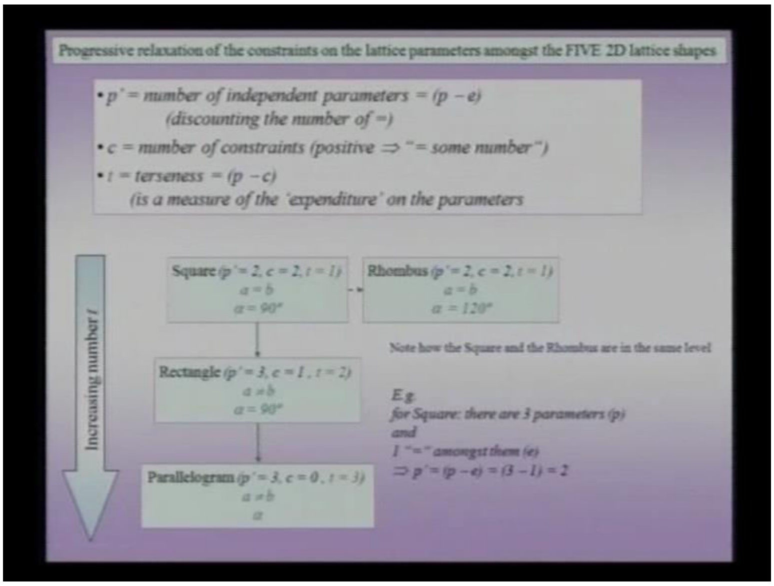


Further, what we did was that we tried to sort of put these five possible unit cells, in terms of the expenditure on the parameters, in other words if I make more expenditure on the parameter, it comes lower down in the list. And I assign it a value which is known as the terseness, if a unit cell is low on expenditure in terms of the lattice parameters, it is placed above in this list and it has a lower, it is in other words it is it has lower value for the terseness.

And in doing so we also understood for instance, in terms of expenditure, in terms of lattice parameters the square and the rhombus are of the same level which have a terseness value of 1. So, what is the terseness value, it is the difference between the number of parameters which want to describe a lattice, which is 3 in two dimensions minus the number of constraints which are imposed for a certain kind of lattice. For instance, in the square case there are two constraints and these constraints are a equal to b and alpha is constrained to be 90 degrees.

In the case of the rhombus lattice the constraints are a equal to b and alpha is equal to 120 degrees, and in both cases I get the value 3 minus 2 is equal to 1, 3 minus 2 is equal to 1. Therefore, the square on the rhombus are in the same level, as far as expenditure in parameters goes, this is merely a classification based on expenditure in lattice parameters. And has nothing to do with the symmetry concept which is an independent concept, we shall invoke later when we try to classify these lattices, in terms of the crystal structures.

The rectangle lattice there is only one constraint on the angle, the a and b can take independent parameter values, therefore the terseness value for this 3 minus 1 is equal to 2. The parallelogram lattice has no constraints and therefore, c value is equal to 0 and I can, therefore derive a terseness as 3 minus 0 is equal to 3 and therefore, automatically the parallelogram unit cell lies at the bottom of this kind of an hierarchy.

So, we also consider these lattices in detail and when we consider these lattices, we were focusing our attention on three aspects; and I will go to the board and write down these three aspects, so that they are for your reference.

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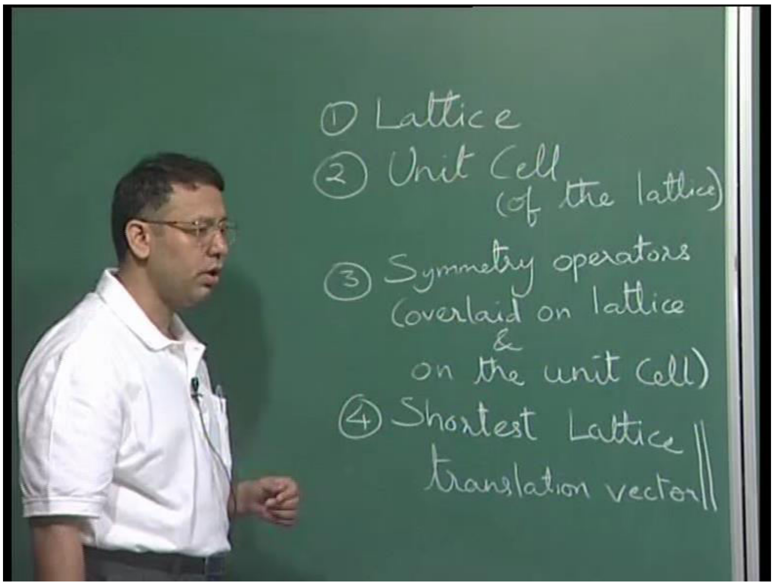


The first one being the lattice itself, the second one being the unit cell of the lattice, so we focus on these four things, then we were considering lattices. These are the lattice itself which is nothing but an ray of points the unit cell of the lattice and we had considered various shapes of unit cell of the lattices. We overlaid the symmetry operators on the lattice of course, not extensively to not in order not to confuse, the students and on other hand, also in the unit cell. And finally, we also considered the shortest lattice translation vector and we had pointed out, this has an important barring especially when we deal with this locations, so I shall returns to my slides now.

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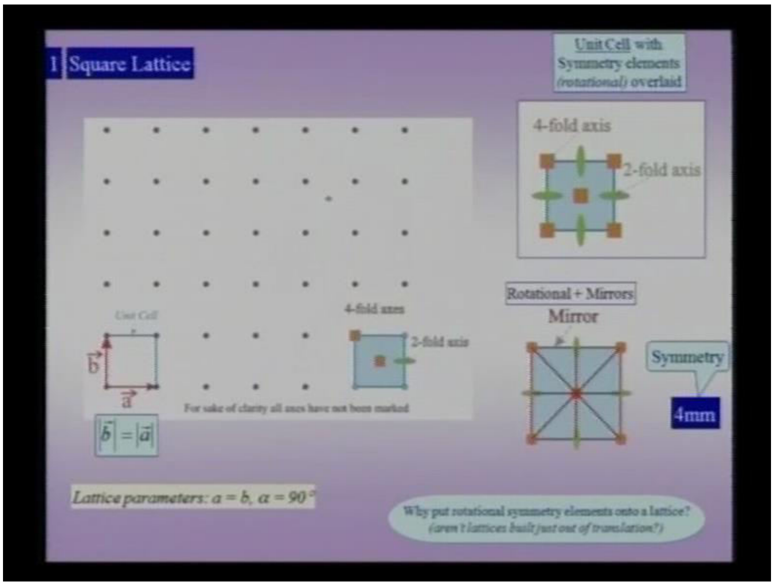

So, when I consider the square lattice, I can see that it has got a 4-fold symmetry in addition to 2-fold symmetries. It is got mirrors for instance, this unit cell shows a diagonal mirror and a vertical mirror, we said that when we write down the shorthand notation, I can call the diagonal mirror for this as m 2 and the vertical mirror as m 1. And therefore, I can write a short hand notation for this, in terms as a 4 mm symmetry or 4 m 1 m 2, I had also pointed out I need not write for instance the other diagonal or the horizontal mirror, because the vertical mirror is automatically taken to the horizontal mirror by the application of the 4-fold rotation axis.

Similarly, one diagonal is taken to the other diagonal by the rotation application of the 4-fold rotation axis, which is located at the centre of the unit cell. We also noted even though we always write a is equal to b, what we mean is that the modulus of the a vector is equal to modulus of the b vector, but a and b vectors are actually distinct. And the important just of the square lattice is the presence of the 4-fold symmetry, and if this 4-fold symmetry is absent, then we cannot call that kind of a lattice as a square lattice.

In other words, the definitive signature of a square lattice is the presence of this 4-fold rotational symmetry axis, this fourfold symmetry axis it is to be noted, is located not only at the lattice points. But, in addition at the point which is exactly between all the lattice points, in addition there are twofold rotation axis which we noted, which are located at edges of the cells or exactly in between two of the 4-fold axis. Now, when I have a unit cell overlaid with these symmetry operators, we noted that I do not have to do anything more.

Because, this unit cell will now be operated upon by the a and b translation vectors, to generate an infinite two dimensional lattice. And when it is does so to the lattice points, it does so in addition to the symmetry operators, in other words the infinite picture would have all the symmetry all the lattice points and midpoints like here, having the 4-fold rotational symmetry. Now, we asked ourselves a question that why do we put symmetry operators on top of a lattice, and the answer to that we saw that, we are future going to construct crystals. And the classification of crystals is based on symmetry and not merely the lattice translation vector which we had seen, which is the first requirement for something to be considered a lattice.

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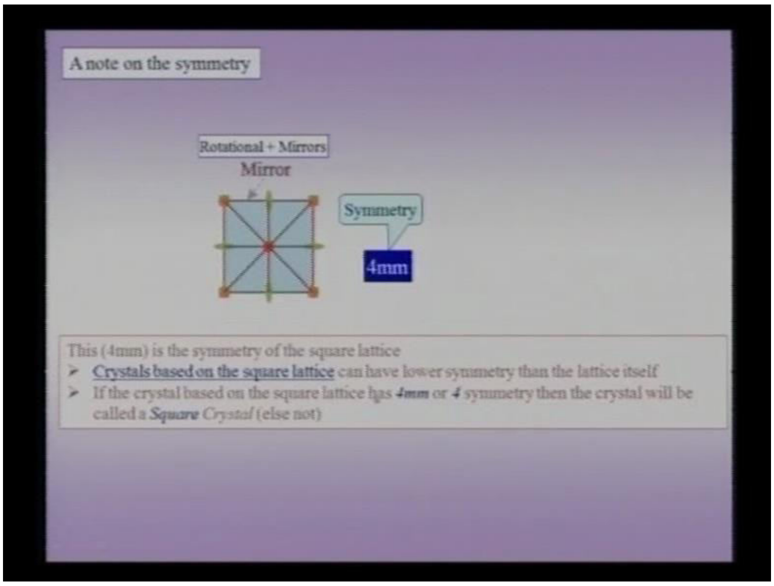


In addition we had made a point in the passing of course, we will return to it in considerable detail later, that if a crystal is based on the square lattice, it can have 4 mm symmetry which is identical to the symmetry of the lattice. Or it would have a certain lower symmetry like a four symmetry, in other words in such a crystal, the mirror planes have been lost. But, still if the four symmetry is present such a crystal would be called as a square crystal, otherwise it will not be called as square crystal.

Just to reiterate the point, in other words crystals which can be called square crystals should have at least the 4-fold symmetry to surviving, even after we have decorated the lattice with the motive to generate the crystal. So, even though we have lattices generate only out of translation vectors, we are very much interested in considering symmetries when we are dealing lattices.

Student: Sir, what is the main difference 4 mm and four symmetry

Mr. Patel has a very important question, that what is the difference between a 4 and a 4 mm symmetry, this aspect we have considered when we dealt with the topic on symmetry. And perhaps we will come to it back again when we actually try to generate crystals, at that point of time we will actually consider examples where in there will be crystals with four symmetry there will be crystals with 4 mm symmetry. And it will be clarified beyond doubt that what kind of a motive will give you a four symmetry and what kind of a motive will give you a 4 mm symmetry.

So, it is a very good question, but we will return it to in considerable detail at a stage when we actually make these kind of crystals. The second lattice which we considered was a rectangle lattice, and the rectangle lattice has nothing but 2-fold axis.

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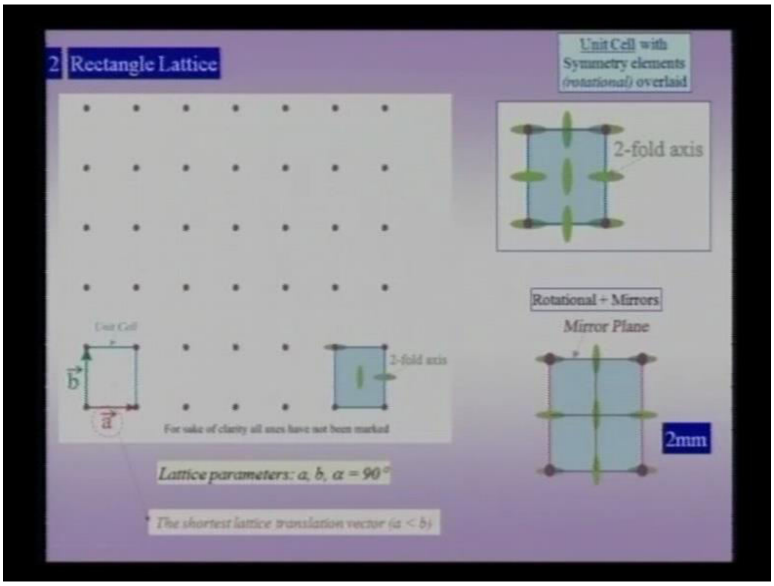


But, more importantly the constraint between row of in the vertical direction, the row in the horizontal direction which are nothing but the a and b directions is 90 degrees. So, this is later on we will see that the signature of parallelogram lattice is also a 2-fold, but this is a special kind of a parallelogram lattice, wherein the angle which is being constraint to be 90 degrees.

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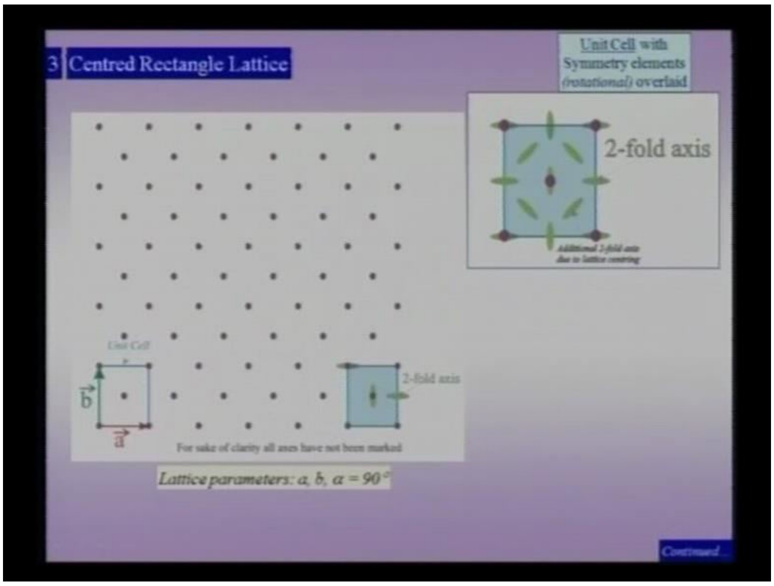


The centred rectangle lattice is a variant of the rectangle lattice, where in addition to the lattice points the corners there is a one at the centre.

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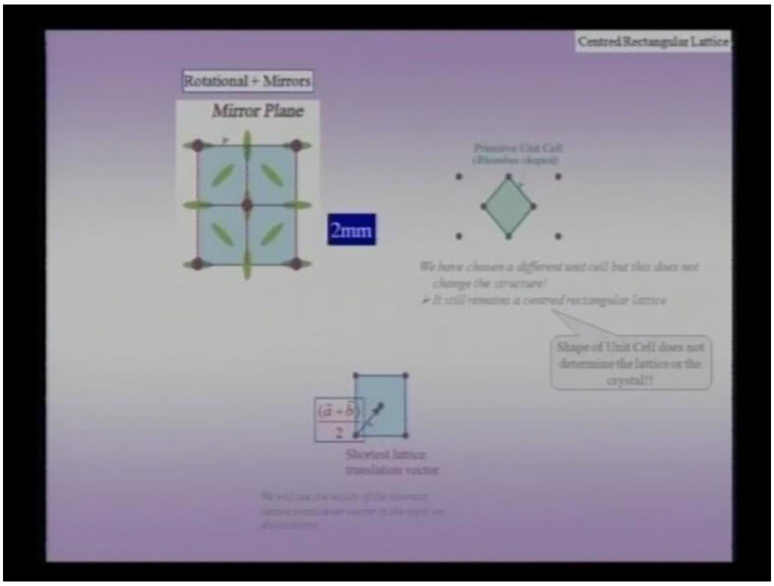


Now, if you look at it we could have describe this same lattice, by a primitive unit cell which is the shape of a rhombus, but typically we would prefer to use this kind of a rectangular unit cell. And we have to be clear here that, our choice of the unit cell is not going to affect the lattice in any way, the lattice exists, because of the surroundings that which is around each point. If you have a point here, you can clearly see in the case of the normal rectangle lattice, you had a point, you want to reach the next point for instance the diagonal direction, you have to go a and b direct vectors.

While in this case you can actually go a plus b by 2, which is the shortest lattice translation vector which we had considered. So, it is the surrounding of each point which determines the kind of lattice and it is not the kind of unit cell, we will choose to represent this lattice, as example if I had very clearly by this green unit cell. And when we had dealt with the concept of unit cell we had said that, we could actually choose doubly non-primitive, triply non-primitive and the choice is infinite.

But, those infinite choices in no way affect the kind of lattices which are possible in two dimensions, which is constrained to be just five. So, there are only five possible lattices in two dimensions and our choice unit cells is infinite even for a single given lattice, but the preferred lattice in this case of the centred rectangle is a rectangle unit cell which is been shown in blue.

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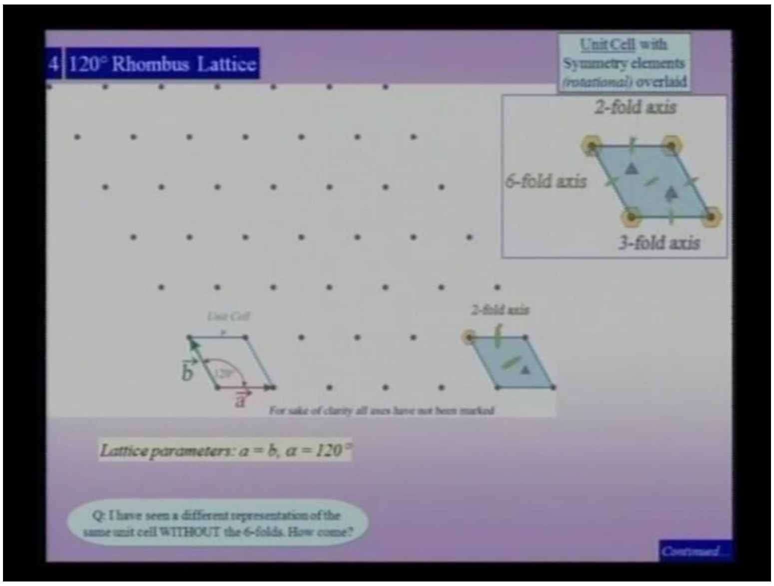


The 120 degree rhombus lattice, as the word rhombus implies as a lattice parameter a equal to b and alpha equal to 120 degrees, and the important signature of this kind of a lattice is the presence of the 6-fold symmetry. So, of course, such a lattice in addition to the 6-fold symmetry has 3-fold axis as well, which we had told is secondary important, because that is not the primary signature of this lattice. Nevertheless, if a lattice which is present which as shown in the unit cell on the right hand side, it will have 3-fold axis and also the 2-fold axis which is shown in green colour.

This position of 3-fold axis at the centroid of the triangle which is half the unit cell dimension. And the important question which we asked is that, there are sometimes representations of the rhombus lattice without the 6-folds how is this possible, so we will answer this question in some little detail, little later.

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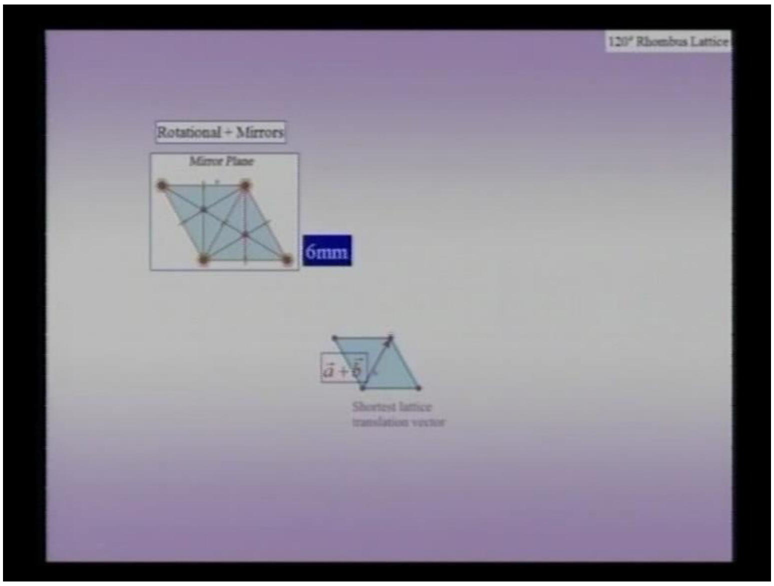


We yet pointed out the shortest lattice translation vector was along the diagonal of the rhombus unit cell, which has a length of a plus b vectorially added. In terms of the mod of the length of the vector you can clearly see, the modulus of this a plus b vector is nothing but equal to a or b, so in terms of length it is same as a or b, but in terms of vectorial addition it is a plus b.

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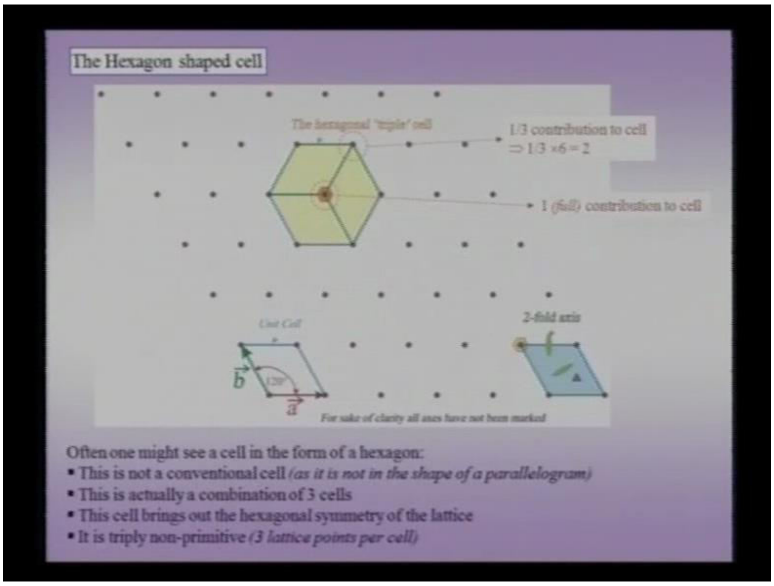


We would also consider the alternate possibility, in terms of a cell to describe these kind of hexagonal lattice, which was the hexagon shape cell. And we get pointed out it is three times in all area, three times in terms of content of lattice points and it is not the conventionally chosen unit cell to describe hexagonal lattice. But, there is a reason why we would sometime want to consider such a three triply non-periodic hexagonal cell and the reason is that, this brings out the definitive signature of the hexagonal lattice which is nothing but hexagonal symmetry.

And since unit cells are generated by merely translation vectors, I cannot go from this cell to this cell by merely translation, I need to rotate this implies clearly, this is the composite of 3 unit cells and not a single unit cell translated to a new position. So, in no way is my description of the lattice altered by this alternate choice of a cell, which is triply non-primitive, but we should keep in mind that this brings out the hexagonal symmetry.

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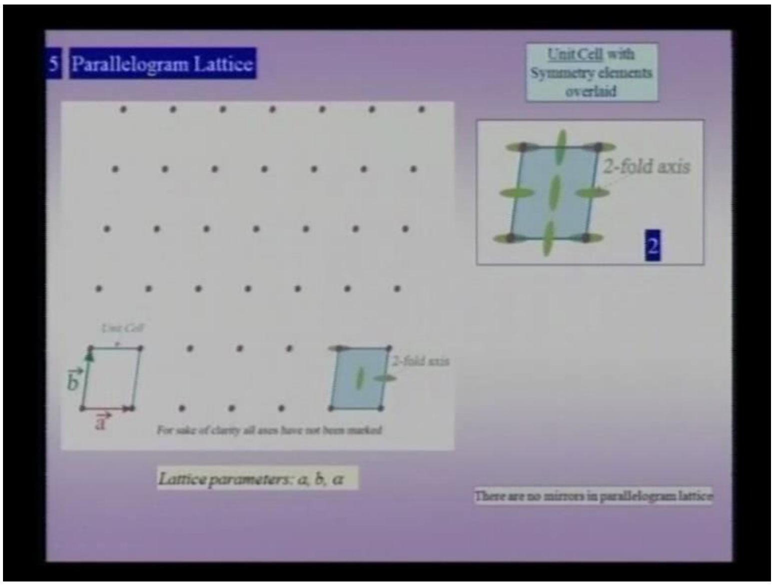


These symmetry a lattice, is the parallelogram lattice of course, as you can clearly see when I mean least symmetry, I am talking in double language, because in terms of the symmetry operators it is also called the exactly like the rectangle lattice. But, in terms of expenditure of parameters it is more, now we have no constraints either a or b or alpha, in the rectangle case there was a constraint on alpha to be 90 degrees. This kind of a lattice has no mirror planes, and the only possibility here is the 2-folds, we had noticed that the rectangle lattice still had mirror planes.

We also went on to summarize 2 D lattices and the important thing is said was that, the only kind of lattice which shows a centering possibility was the rectangle lattice. Mr. Patel has asked, why is that there is this possibility and we last we will answer, that concept also little more detail we answer.

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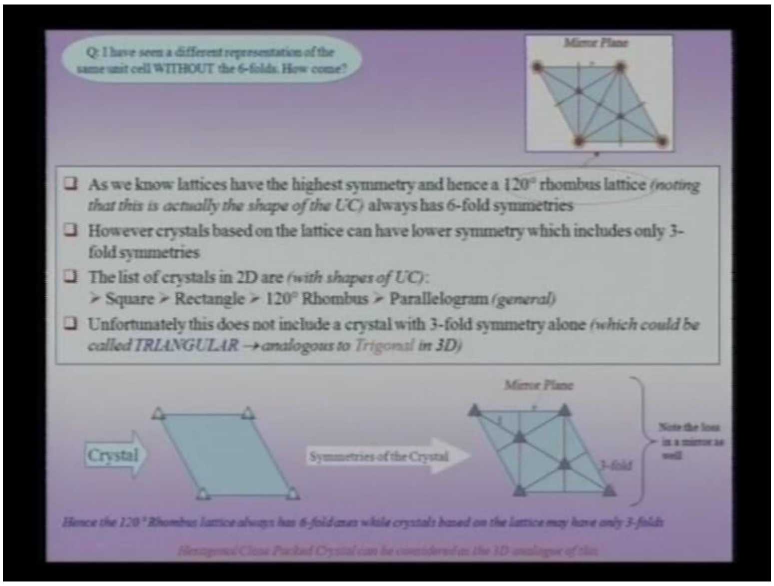


Now, first question we will answer now is that, why is that sometimes we see representation and we have to refer at the figure of the bottom a representation, wherein you have no 6folds. And still you see that the unit cell is the conventional 120 degree rhombus unit cell, so this was a symmetry, and as we had previously seen the one which is the complete overlaying of symmetry operators on unit cell, is the figure on the top which contains the 6-folds and in addition also the 3-folds.

If you look at the figure at the bottom and compare it with the figure at the top you would see that, 6-folds have been replaced by 3-folds, and in addition you will notice that the mirrors the cell is just mirrors. And in the case of the hexagonal lattice or also missing here, these original verticals which are present, there are these inclined mirrors with the two inclinations which is nothing but the 3-fold rotation acting any one of this mirror.

So, if we have mirror like this, if it is rotated by 120 degrees, which is the signature or which is the mode as operand is the 3-threefold axis. Then from this vertical mirror you can go to this mirror to this mirror by mere action 3-fold axis, but you would notice that those mirrors are present in the alternate diagram which you might have seen. But, these mirrors which were at the edges of the original unit cell are missing in the case of the alternate representation.

Now, the point we need to note and I will read this for you, we know that the lattices have higher symmetry and hence 120 degree rhombus lattice, always has 6-fold symmetries. We need to know at the crystals based on a lattice in have lower symmetry, when I say can have it means, it can have equal symmetry for also can have lower symmetry, which then there is a possibilities, since there can be a lower symmetry one, there are only one 3-fold symmetries present.

Now, when we are talking about the crystals, so when I have this kind of a lattice it has to have this kind of hexagonal symmetry, but when we try to make a crystal out of it, for instance there is a crystal which have constructed here, by taking the 120 degree rhombus lattice. But, by putting an additional triangle on top of every lattice point, so now my lattice is the usual 120 degree rhombus lattice with hexagonal symmetry. But, I am putting a motive to make a crystal, what is the motive, the motive is a triangle.

Now, if I try to look at the symmetry of this kind of a crystal, now I am looking at the symmetry of the crystal and not merely the symmetry of the lattice, then I clearly see the this crystal has low 6-folds rotational symmetry. It has only got 3-folds as shown in the figure on the right hand side, therefore the original 6-fold symmetry which were sitting at every lattice point is now been lower. And it has been lower to 3-fold and why it has been lowered, it has been lower, because now the motive has a lower symmetry.

We actually explore this concept of putting a motive and lowering of symmetry inconsiderable detail later. But, at this point all I want to point out is that, if you have a lattice and in this case specifically hexagonal lattice, or which is been this is called 120 degree rhombus lattice. Then such a lattice will definitely have 6-fold symmetry axis in addition to the 3-fold, but when we try to make a crystal of it, and in this specific example having on the slide here, is by putting triangle at each lattice point.

And one of the constraint we said, when we put motive point of lattice was that the motive should not be rotated or translated with respect to the lattice point, when we go from one lattice point to the other which is exactly we have done here. And when I do that I will make a crystal, the crystal itself has a lower symmetry as compare to the lattice from which I started. Now, if I want to overlay on the unit cell, the symmetry of this crystal now those are shown in figure at the right hand bottom and those symmetries have no 6-fold there, there only 3-fold, and which is what sometimes you would see textbooks. And what we need to understand when you look at the figure is that, that it is been additionally added in, so that we can understand crystals based on the 120 degree rhombus lattice, which can come under the class of the 120 degree rhombus crystal.

Now, there was a additional possibilities with crystallographers would have taken reports to what they could have done was that, they could have called generate an additional type of crystal, for instance they could have called something called a triangular crystal, which are crystals with 3-fold rotational symmetry. In other words, you would have had crystals with 6-folds rotational symmetry which you could have called hexagonal crystals in two dimensions of force.

And you could have used those crystals with only 3-folds rotational symmetry as triangular crystals, but unfortunately there are no separate terms for the triangular crystals, they have been merged with the hexagonal crystals. Since, there is one term for hexagonal crystals, those hexagonal crystals contains those which have 6-fold symmetry and those without 6-fold symmetry. And interesting point note is that, I have use the term hexagonal, for instance one example would be the hexagonal close pack crystal, can be considered as a three dimensional analog of this two-dimensional hexagonal crystal.

Actually to be more precise, I will have consider hexagon lattice which is nothing but an two dimensional hexagonal net, which has been grown in third dimension to make a prism. And that is the hexagonal lattice and a special case of that hexagonal lattice would be hexagonal close practice still, which are crystals three dimensions. So, even though we are considered this example here lot of points, and lot of questions might be coming in your mind and Mr. Patel has 1 question now.

Student: Symmetry of crystal based on motive for ((Refer Time: 38:48)) actually if we want to change some motive, then symmetry I think of crystal will be change.

Where is that sentence

Student: Here a triangle is there if I put some hexagonal, then ((Refer Time: 39:07))

So, the question you are asking in other words Mr. Patel's question is that, how will the symmetry of crystal change, if I instead of putting triangular motive, if I put a hexagonal motive, and in that case you would retain the very important question. So, let me repeat the question again, I had made a crystal from the hexagonal lattice by putting a triangle at each lattice point. Now, is to putting the triangle if I put hexagonal, for instance a green hexagonal at each lattice point, what would have happen to the crystal, which I would have be generating.

Then in that case the crystal I would be generating, would have a symmetry shown on the top right hand figure, which would actually have the 6-fold axis, in additionally 3-fold and 2-fold axis. In other words, I would have had all the symmetries possible for the lattice of course, I need to we are little careful, when I mentioned this and it will compare when we a consider more examples. But, I need to put this hexagon at each lattice point, in a very careful way, I cannot randomly put this hexagon, I will assume that I am putting the hexagon the way exactly this hexagonal sitting on the lattice.

In other words, orientation such that, the mirror planes and the rotations are coinciding with that of the lattice, I am not rotating it in a very arbitrary way, as compare to the hexagon shown. If I do it that way, then I would get a crystal which has 6 mm symmetry, which is identical to the symmetry of the lattice, in other words I would have not lowered the symmetry of the lattice, when I go from the lattice to the crystal, a very important question.

But, we will return to this question, and many more such questions which will be coming to your mind a when we consider more examples of how we are going to put motives on the lattices to generate crystals. At this point merely this was an example, so what when you are reading in textbooks and considering especially in this two dimensions lattices, you should not get confused buy a representation which has only 3-fold axis shown on the lattice.

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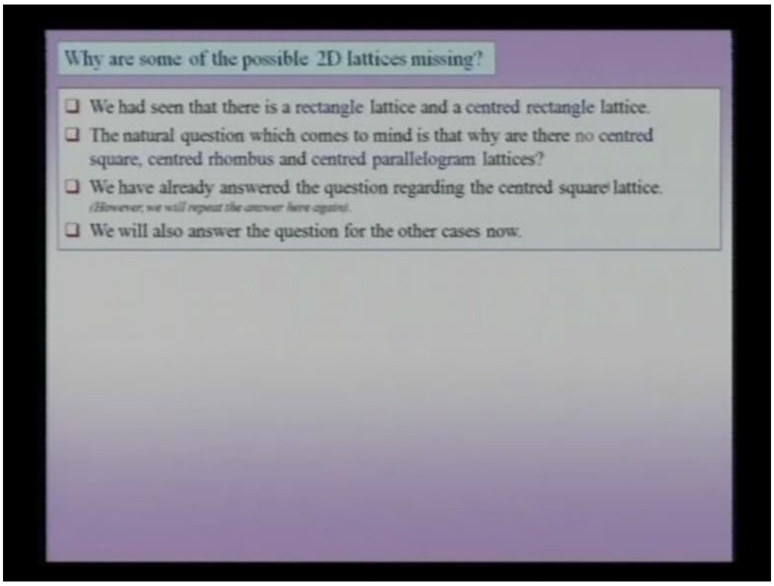


We would asked ourselves a question previously that, why are some of the two dimensional lattices missing like for instance, we could have had in other words, there is no centered square, there is no centered a rhombus, there is no centered parallelogram lattice. Why is that these lattices are missing, we had in implicitly answered this question with regard to this square lattice, but we will consider this example again and we will start from there.

And we will answer this question for the other cases as well, in doing so we will repeatedly invoked the concept of the unit cell, and try to understand the lattices based on the unit cell. But, again to re iterate lattices exist independent of the unit cell, and there are distinct classes of lattices which are based on translation vectors and identical surroundings to each lattice point, and that is the criteria when we try to understand the lattices. So, let us start with the case of the centered square lattice.

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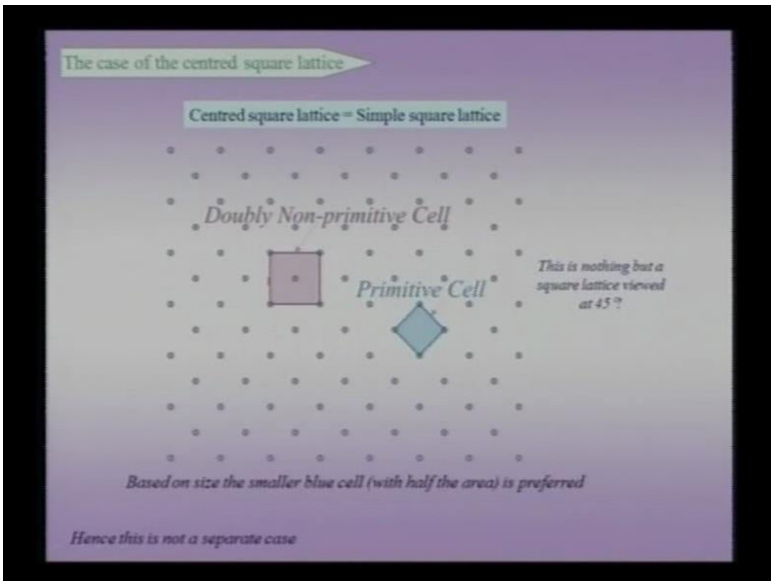


And we seen before that the centered square lattice is nothing but a simple square lattice, in other words if I were to draw this picture of this lattice rotation of 45 degrees, then I would viewing it the vertical direction would be this. And actually I would see it like a normal square lattice, I have two choices to make, either doubly non primitive unit cell or a singly non primitive cell. And when we dealt with the criteria of choice of unit cell we said that first is symmetry, second is the size and of course, the size and symmetry criteria fail we would go for some kind of a convention.

Since, the size the symmetry of these two unit cells is identical the size factor tells us actually choose the smaller primitive unit cell, the doubly non primitive unit cell, the double the area of the blue cell which is the primitive unit cell. And therefore, when I am trying to talk about the centered square lattice, I am actually non-generating a new type of lattice, all I am doing is viewing the old lattice at certain angle. So, in other words there is not a separate case a top, so in other words this centered square lattice is nothing but a simple square lattice, therefore I do not have to make a new case for this case of the square lattice.

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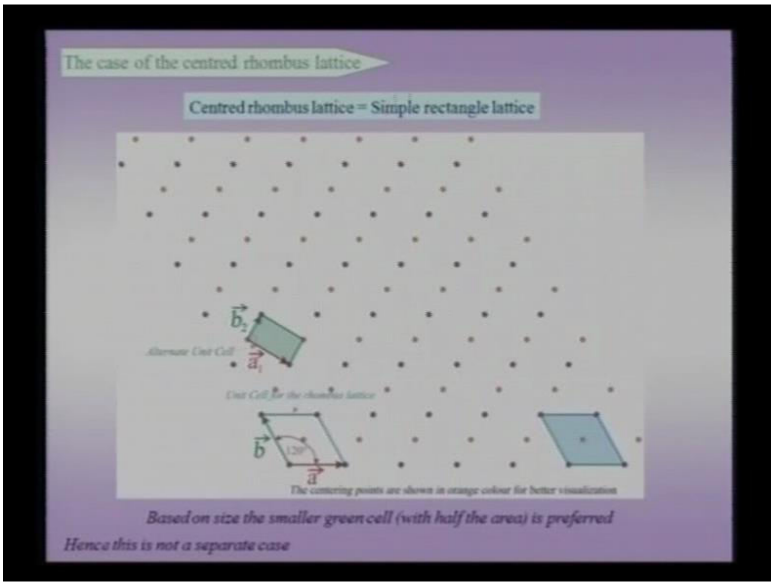


Now, let us consider the case of the rhombus lattice, because we do not consider the second type of lattice which is the rectangle lattice, wherein there are two distinct cases, normal rectangle lattice and the centered rectangle lattice. So, if the case of the centered rhombus lattice, why is that we do not have a separate entry something called is the centered rhombus lattice. So, let us look at the centred rhombus lattice, wherein just for sake of clarity, I have shown the centering points in orange color, we should note that actually lattice points zero dimensional.

Therefore, they cannot have color, taste or any those attributes which we associate with physical objects, they are mathematical entities with zero dimensions and therefore, they cannot have color as well. But, I have just shaded them, like I have used a finite kind of a circle, filled a circle to represent a lattice points I have use different color, but please note there all lattice points equal to each other, just for the sake of clarity the orange lattice point is the one which is being generated.

But, taking the normal rhombus unit cell, but by putting an additional lattice point which is now orange in color. Now, if I look at this rhombus lattice and now again look at this lattice viewing along the 60 degree diagonal, I can clearly see I can construct two basis vectors usually go on to generate this new centered rhombus lattice, these are the a 1 vector and b, I should have made maybe b 1, but does not matter I call this as a b 2 vector. So, I have an a 1 vector and b 2 vector which go on to generate this centered rhombus lattice, in other words what is this other lattice which we can now view along the 30 degrees of the 60 degree angle it is nothing but a simple rectangle lattice.

In other words, when I try to generate a centered rhombus lattice, I was not actually generating new type of a lattice, all I was doing was generating a simple rectangle lattice. Of course, I could have lived with still calling this is the centered rhombus lattice, but we can clearly see that this is neither preferred in terms of the new classification, which does not deserve. But, additionally if you want from the unit cell point of also this is a smaller unit cell, the green unit cell is smaller than the blue unit cell.

And therefore, I would stick to my older classification, wherein I will call this a simple rectangle lattice, which is now generated by these bases vector, which are inclined with respect to the original basis vectors. So, clearly this centered rhombus lattice has no independents existence it is nothing but a simple rectangle lattice view in a different way. So, based on the surrounding criteria we see that, this is a simple rectangle lattice, so now we can go on to the next kind of lattice which is missing in the list, and that kind of a lattice is the case of the centered parallelogram lattice.

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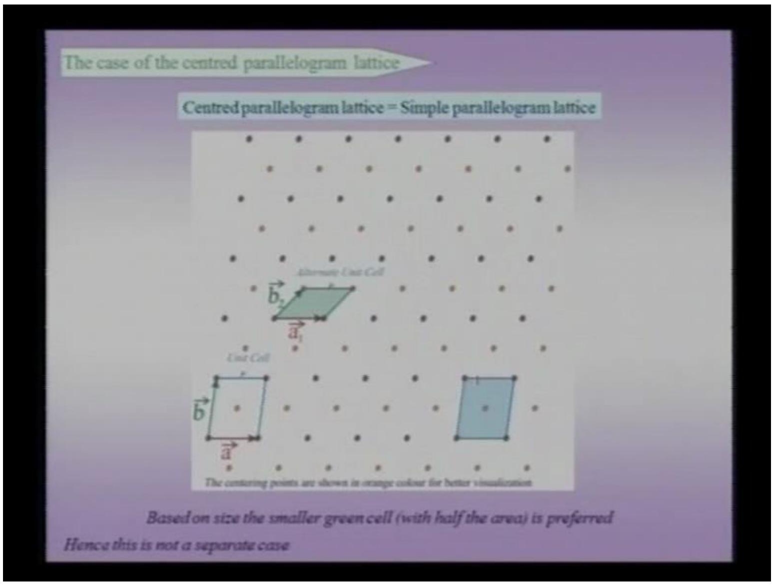


We noted before that the parallelogram lattice is the one, where there is no constraints either on a or b or the included angle and we also said it is the lattice of the lower symmetry. Now, as before what we will do is that we will color the centering point in a different way, and this case we are going to give it in orange color, while the original lattice points are in this maroon or brawn color. The original lattice was generated by these two basis vectors a and b, and now the centering point is this orange, and now I have this in finite lattice of course, the portion of it is shown here in this slide.

If you notice this a little more carefully, I can actually now define two alternate basis vector, this red vector a 1 and this alternate green vector b 2 which can now go on to generate this new kind of centered parallelogram lattice. Clearly this new choice tells us, this is not new case at all, because if you look at the relationship between a 1 and b 2 and included angle you will notice that, it is exactly same as that of purely parallelogram lattice. In other words, a 1 is not related to b 2 and in addition special angle which would describe these kind of a choice.

In other words, based on the fact of this unit cell is smaller than the blue unit cell, in other words this is primitive and this is doubly non-primitive, in other words the area of this would be half, the area of a doubly non primitive unit cell, it is clearly the choice of the green unit cell is better. And to state it differently when I am talking in the language of lattices, this is not a new lattice at all it is nothing but a parallelogram lattice where in every drawn the basis vectors.

So, the centered parallelogram lattice is nothing but a simple parallelogram lattice and when I want to make a choice of unit cell, I will go ahead and make a choice of green one in preference to the blue one, hence this is also not a new kind of lattice.

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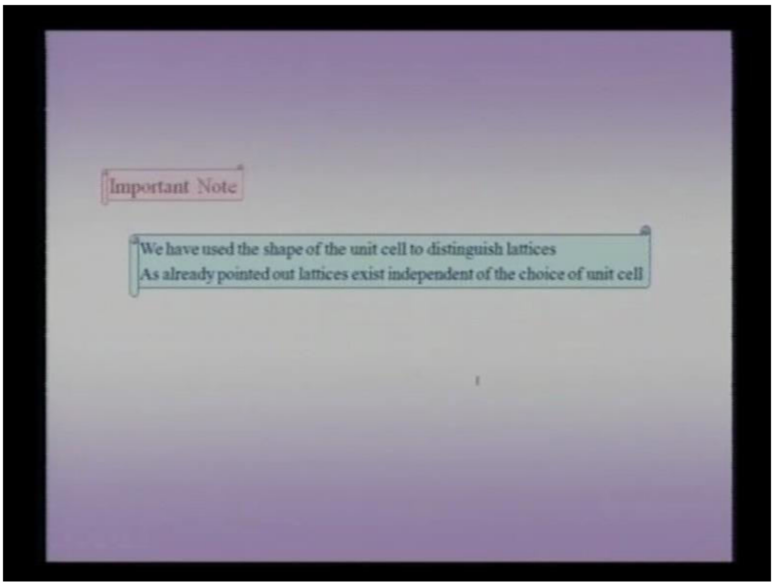

And therefore, our collection of lattices is not altered by this additional consideration of centring on the square, on the 120 degree rhombus or on the general parallelogram lattices. Again to reemphasise the important point, we have used is the shape of their unit cell distinguish lattices, and but lattices exists in totally independent of our choice of unit cell.