Structure of Materials
Prof. Anandh Subramaniam
Department of Material Science and Engineering
Indian Institute of Technology, Kanpur

Lecture - 12
Geometry of Crystals: Symmetry, Lattices (continued) and Miller Indices

Mr. Patel has the question let me address that question first.

Student: I want to know how we can define residual stress within the materials

Very good question, I have been using the term residual stress, but I have not defined, and we have also looked at the causes for instance, the residual stress can come from intrinsic defects, like vacancies and dislocations.

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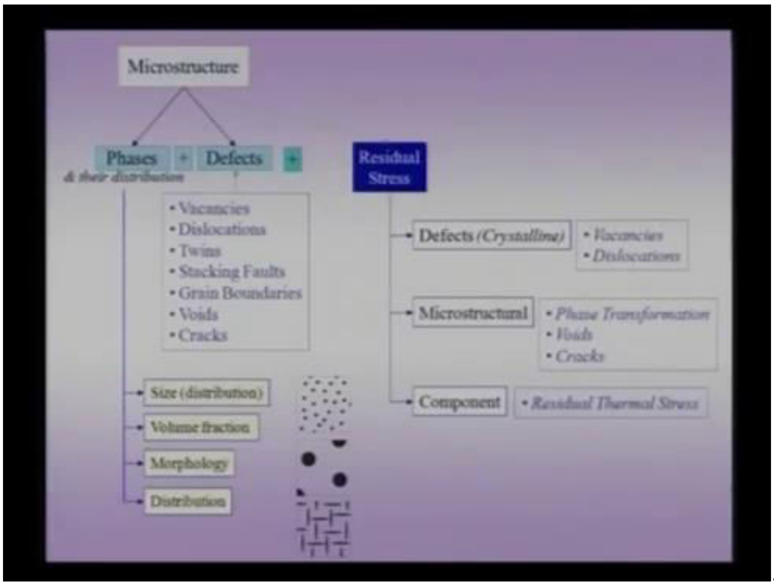

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They can come from process like phase transformations, they can be of voids and cracks of course, voids and cracks themselves are not sources of residual stress. But, if you have a far field stress, they can have a amplifies voids and cracks and if you have making a compound for instance, even the component could be the simple component like even slab of material. You could have residual thermal stresses, and I have also pointed out that, this residual thermal stress could even be intentionally introduced for beneficial purposes.

Typically what you want might want to do, is we want to introduce residual compressive stress on the surface, which has very good beneficial effect. So, how do we define these residual stress, residual stress is the stress present in the material in the absence of any external loading, so let me write that down.

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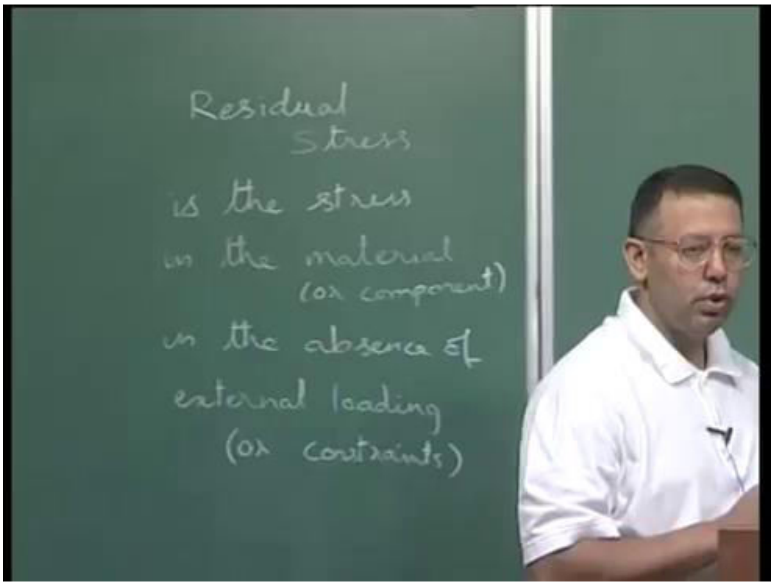

Now, it is important to note of course, if you do have external loading, then this residual stress would be modified, but the very definition of the residual stress is that, if I do not apply any external stress, still I should be able to have some stress in the material. Now, one important point we noted is that, a body in equilibrium that is the body free standing body, cannot be fully totally compressive or not totally tensile. In other words, a residual stresses a one sign, will have to balance of other residual stresses sign, so that the net body, which is now a free standing body is an equilibrium. So, let me for instance give a trivial kind of an example, wherein suppose I have a cross section of material, for instance a slab of material.

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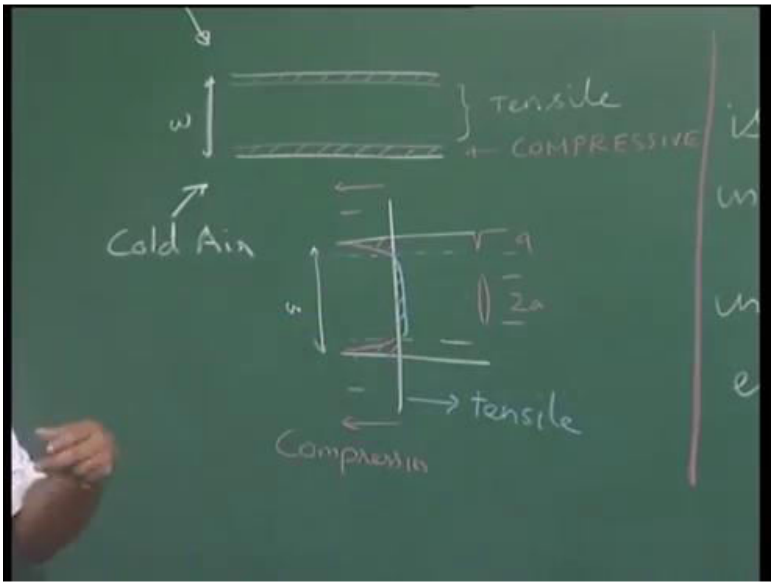

Now, for instance suppose this is a slab of glass, which I want to make and I want to toughen the glass, so what will I do, I while solidify the glass will blow cold air from the top on the both sides. So, what will happen the surface will solidify first, so I am drawing small layer on the surface which has solidified. And later on the inner layer will solidify and the inner region, and when the inner region is solidifies the glass will contract.

And in the process it will actually pull the material, which is now the surface inward, and it will introduce the compressive stresses on the surface. But, if the body has to be an equilibrium, then the surface compressive stress as to be balanced by tensile stresses within the body, so I will assume for this sort of the approximation, that this is my compressive region, which I marked in red.

Of course, as you know this is now as variation in stress which I can plot, but so I will just marking the compressive region in red now. And now the inner region has to be tensile to balance the compressive stresses, now suppose I look at this body in cross section. ((Refer Time: 04:39)) This is my cross section, this is my cross section again and this is my dimension here is for instance w, and this is my layer. Then if I plot my stresses, so what will happen is that you will have large compressive stresses, which of course, is negative value, suppose I compression side is negative.

Then I have large compression stress and then, it will go to 0 here, this will be like this across something like that, because I am not drawing the exact mathematical curve, but some kind of appropriation, and the inner region will be tensile. But, it will be in such a way seen that this length of course, I have not drawn the scale, typically this would be the small layer, ((Refer Time: 05:22)) this will be larger layer assuming that, my overall stress have to be balance out.

In other word, this area which is the negative side has to balance out the area in the positive side, in other words this is now my tensile stress. Now, because this value of the tensile stress spread over a large area or a large volume, the magnitude is small and that is why it will not play much detrimental effect. And overall effect of this kind of stress is my body is beneficial of course, one thing why we are suffering to class is that, we know that we want to avoid crack propagation glass, glasses are very brutal.

And what kind of the glass we are talking about, we are talking about the typical plate glass which are nothing but, silicate glasses, now in such kind of glasses, since they are very brutal, the crack tip is very sharp and therefore, cracks can propagate. If I am talking about ductile material some of the analyses may have to be slightly modify, but in the brutal material, cracks propagation is perhaps very easy mode of fracture. Now, in the absence of any kind of the compress stresses and in a general body like this, we note that cracks of length a on the surface are equivalent in effect to cracks of length, 2 a of the interior.

So, suppose have a crack a on the surface, this is my crack of the length a, this is equivalent to a crack of length 2 a inside, in other words cracks on the surface are twice as detrimental as cracks on the interior of the material. And therefore, it is for me very imperative that I protect my surface, which also the agent were all the other element sat like, ((Refer Time: 07:01)) agent etcetera etcetera. Therefore, I need to protect my surface and that is where I introduce the compress which is large in magnitude, but over a small region.

And the tensile stresses are small in magnitude over a larger region, and the overall effect is beneficial, because this small magnitude is tensile stress is acting in a region, were cracks are half is detrimental as on the surface. So, this overall process introducing compressive stress on the surface, gives me the beneficial effect as far of the fracture of glass techniques.

So, typically suppose I am doing the experiment in which I am trying to bend this glass, so bending kind of experiment, it will break surface compressor stress helps me in improving the fracture properties of this class glass. So, residual stress are stress, which are present in the absence in external agent, and they can be detrimental in many cases like for instant, they can lead to a wrapping of the component. They themselves lead to cracking of the body, if suppose the body happens to very brutal, but if engineer properly, we can use residual stresses to our beneficial effect.

And that was the last point we saw that, for a free standing body you cannot ((Refer Time: 08:08)) free standing body in pure compression or few tension, overall we have to balance one with other. So that, the overall body is net equilibrium, please proceed with the question.

Student: ((Refer Time: 08:21))

The way we are doing the whole process is that what we do, we are having a molten pool of glass for instance, I am blowing some air on the surface and I just blow air for sufficient region to form a thin layer of solidify glass. I make sure that layer is very thin and obviously the heat transfer, because of the air when only take place by convection only on the surface. A below that it has to express to conduction through the glass, which would be the less sufficient agent.

After solidly thing I will stop this air blowing, and now the remaining glass to solidify perhaps in the normal way, suppose I continue with the blowing of air also, but for now simplicity I will stop blowing the air, the remaining bulk of the glass solidifies, it pulls in the remaining. So, I already solidly define one words, that is why the residual compressor is coming, which is larger in the magnitude over the small region, the topic we are considering next, is the important topic of the miller indices.

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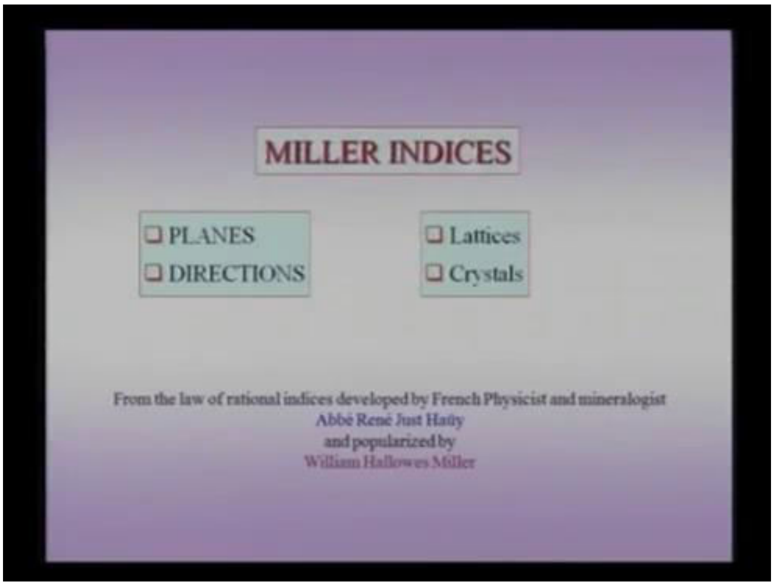

This is an important topic, because in crystallography plains and directions are represented by miller indices. And in fact, we have already used informally, and perhaps little measure these miller indices, when we were trying to describe various aspects of the crystals. The concept of miller indices is irrevocable and inextricably connected with the concept of the crystals, and the concept of the lattice, so we need to understand the miller indices in detail and thoroughly.

The important point note to write it outside is that, miller indices are can be for plains and they also can be for directions. Miller indices for plains are straightly more difficult to understand compared to miller indices for direction, miller indices for direction is as simple for understanding normal vectoral directions. And other second point to be noted is that, the miller indices can be used equally well to describe lattices, or to describe crystals, and we have already seen that lattices are not the same as crystals.

And especially this concept becomes more powerful when we try to describe, for instants the crystals of lower symmetry, in other words we are dealing with a family of direction in a crystal. So, we have to note that we have miller indices for directions, which are pretty simple and straight forward to understand, and the miller indices for planes, wherein there is a certain prescribe procedure to arrive at them, and we will see that procedure.

And also we have to note that miller indices can be for lattice or for a crystals, and the important difference becomes all the more obvious, when we are talking about the concept known as the family of direction or family of plains. So, let us take up this important concept, and try to understand the miller indices.

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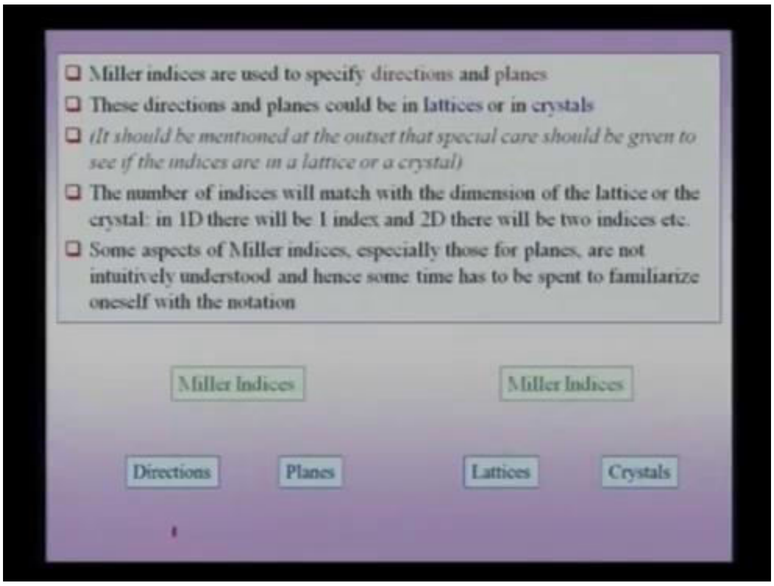

So, as I will slide here, let me explain that miller indices are use to specify directions and planes, these directions and planes could be in lattice or in crystals, it should be mention write at the outside, the special care should be taken to see if the miller indices are for plane or lattice or for the crystals. So, please remember that distinction have to be absolutely clear right from the beginning. The number of the indices will match with the dimension of the lattice or of the crystal, this is an important point to note, in one dimension they will be just one index to describe the miller indices.

In two there will be two indices, in three dimension which will be the typical dimension we will be dealing with in normal crystals, there will be three indices to describe a crystal or a lattice. As I pointed some aspect of the miller indices, especially those for planes are not intuitively understood that means, there is a certain procedure involved which is not. what you might call intuitive. And therefore, some time as to be spent over this miller indices concept of planes to understand them. So, once again we have miller indices planes and direction and we will could have miller indices for lattice and crystals.

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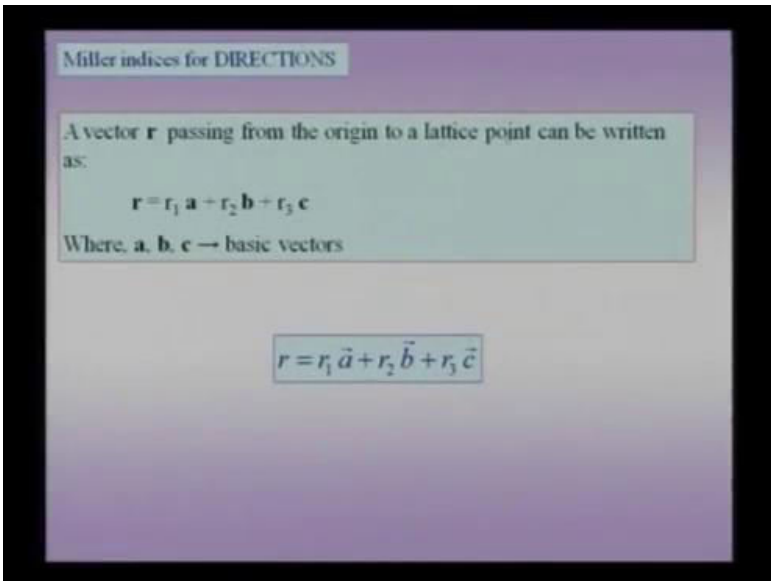

So, how do we understand the miller indices for direction, we know from our coordinate geometry, that a vector r passing from the any lattice point can be written as r 1 into a, r 1 into b and r 3 into c, were a, b, c are the basic vector along the three coordinate axis.

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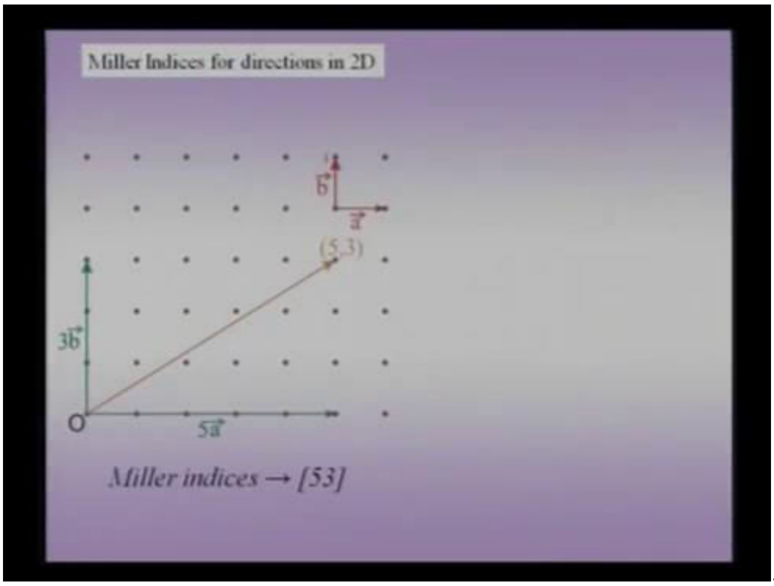

Now, in miller indices for instance let me start with the simple example, for this I have a vector which is shown in this orange color, which starts from the origin and which ends in the point (5, 3). And my coordinate axis now, this happens to a simple kind of crystal, we are coordinate axis are orthogonal to each other, and are describe with the basis vector a and b. Now, this is as simplest as coordinate geometry, so I get the miller indices by taking this vector and which projection, for in this vector is 5 unit along a direction, 3 units along b direction.

And therefore, I write down those coefficient the 5 and 3, and I will get the miller indices for this direction as (5, 3), so this is the very very simple kind of the procedure, and very simple to understand. So, let me go through the example once again for sake of clarity, so I have vector which goes from O to say a point p, and the coordinate system in this case happens simple it could in effect you will see later. That it could be any general coordinate system were in the lance of a and b, it could be any and include that angle also would be any, any obituary value, so you got this place a and b.

And this vector connects the origin which is (0, 0) to the point (5, 3), so the projections of this vector on the x axis the a direction, b direction are 5 and 3. So, I write that now and the important point note is the bracket, so I enclose them in this kind of square brackets. So, miller indices for direction are always enclose in this kind of square brackets, and my miller indices for direction I have shown is (5, 3), since I am working in two dimension I have pointed out number of indices will match with the number of dimensions of space which is now 2; so my miller index for this would be (5, 3).

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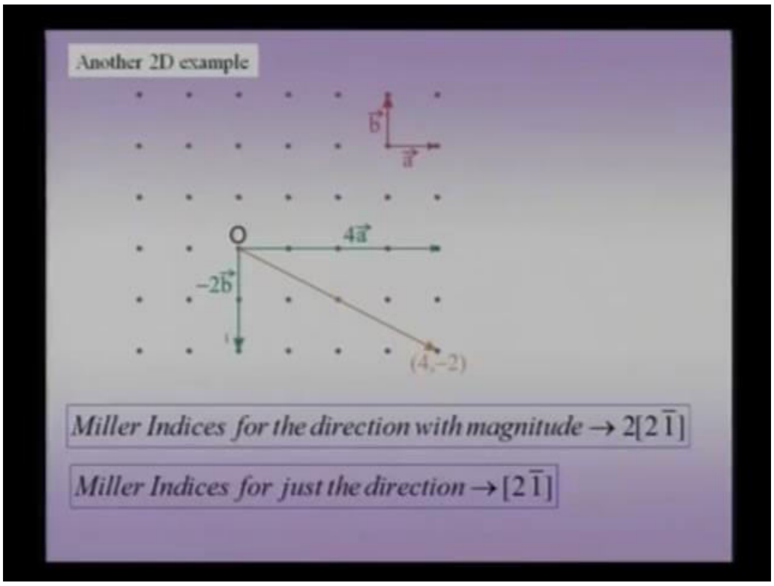

Let us consider some more example to understand the concept of miller indices, so again I am referring myself to the same two dimensional lattice, so the miller indices (5, 3) was for the lattice. So, I was (5, 3) which I do last time was for a lattice, and I consider the same lattice again, but now consider at different kind of a vector, so now my origin is here and the vector says point p connects the point 4 minus 2. So, this is 4 units along the a directions and minus 2 units along the b directions, so in the factorial I could write it as 4 a minus 2 b, the way I have normally would represent the vector in this factorial notation.

So, I will write this is in 4 a plus 2 b, this as you can see in two dimension, since we are in the current example we are dealing with only by two dimension, so only values for r 1 and r 2 and my basis vector r a and b here. So, my projections and 4 and minus 2, therefore I can write this miller indices as 4 minus 2, but then I could factor out my factors and write it as two types two minus 1. And the miller indices for the direction excluding the magnitude would be 2 minus 1, so let me go through steps on the board, so that the simple concept is sort of free emphasis.

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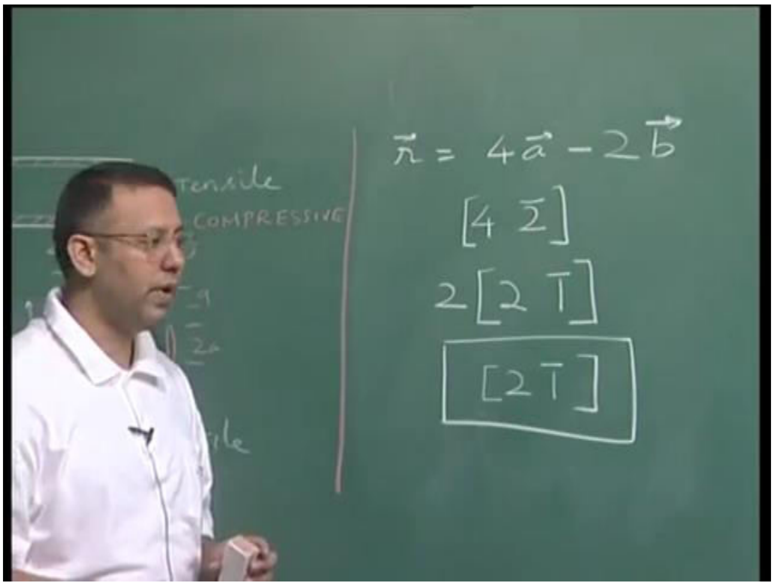

So, I have 4 units along a minus 2 units along b, and if I want to write the miller indices for the whole thing, I can write is as 4 and I will put the minus on top as 2 bar, I will not write the minus in front I will write on the minus on the top, so this is the convention. I can pull out the common factor and write it as 2 2 times here and there is 1 bar, now this kind of representation of the miller indices for this direction includes the magnitude. But, typically miller indices are we exclude the magnitude, we want to refer to a only direction and typically direction in the lattice.

In that case I will drop that 2 and I will write it has 2 minus 1, so this will be my direction for the miller indices for the particular vector, which I do on the computer. ((Refer Time: 17:11)) So, this is the direction 2 1 bar, so a few important point note here, the number 1 I mention the square brackets, number 2 the negative as written in the bars above the numbers. Number 3 that I typically drop the magnitude out from the common factor and take it to the least, I use only the least common factor in side.

And an important point, now the miller indices just not represent as single directions, but an entire host of the directions in this crystal, which are all parallel. And which have this magnitude of 2 1 bar, which is said other word, suppose I take another vector starting from here this point and I go parallel to this vector, in other words this vector they can be 1 unit along this 2 unit, 1 2 and finish up here. So, this is my starting point, so this is not my origin now, which is somewhere else, so I call this point x and I will call this point y.

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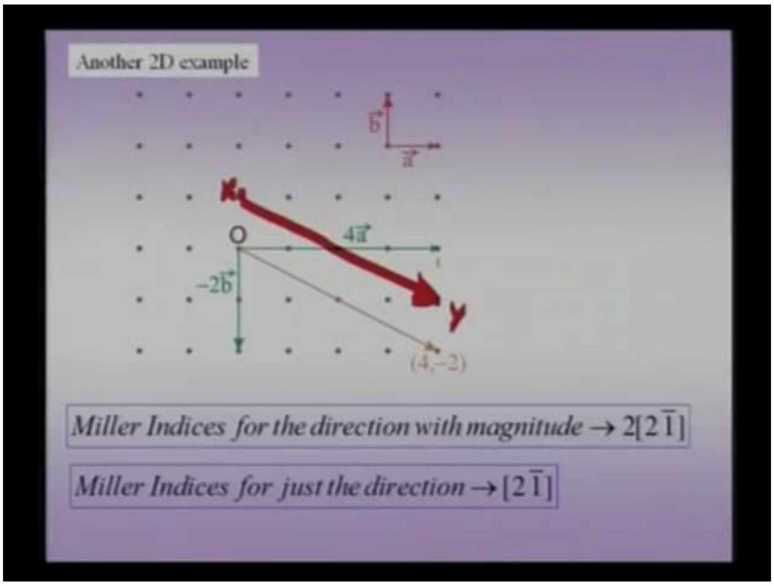

And now my vector would connect these point x and y, let me try to join the straight line so that, which passes through this lattice point. Now, I want to locate the miller indices for this direction, it would be exactly identical I will not have to introduce new miller indices, it will also represent by the miller indices to 1 bar, so this is noted here. So, whenever I am writing a miller indices, it is not a direction, but a set of all directions which are going parallel to each other.

In other word in any point of the lattice I could make that as an origin, and I draw vector which is of course, if I want to take account the magnitude, like for instance the magnitude of this thing including magnitude this would be my representation of this. And dropping the magnitude this is my representation, I need write this two I can start any point of the lattice and draw a vector, and all those vectors of these would be clarified or would be used by the same miller indices, Sowmya has a doubt.

Student: Giving the miller indices is it necessary that from the starting point of the direction which is shift the origin there, like if we have to write the miller indices of x, y, do we need to shift the origin at x and then, write is it necessary.

I am not sure I am understanding your question fully, but let me answer whatever it is possible, the origin can be anywhere in the lattice the new origin, because every lattice I fix origin for some purpose. Normal lattice does not have any origin, the origin is the concept of our mind, therefore the normal lattice again I can choose my origin anywhere, but suppose I have already chosen my origin O, and I constructed this vector. And I represent that miller indices that vector as 2 1, actually the 2 1 will be half the length I want to call this point p, this point ((Refer Time: 20:02)) here as p.

And this point as mid between somewhere, so I call this as a m point midpoint, so actually this vector having this magnitude 2 times in square brackets 2 1 bar is this O p vector. The vector 2 1 bar is actually half the length and connect O 2 M right it is half the length, now all this from O M kind of vectors I can translate this any were lattice use my origin anywhere, would be presented by the same miller indices. So, there is no origins irrelevant as for as the miller indices scope, but there will be special circumstances where we will worry about origins also.

Suppose, I have a single crystal and I am worrying about, what you called certain process occurring then, I may have to specify the disc, actually preferring to this particular miller indices or this particular directions and not anything also that might be done, keeping in the contact extent view. But, in general as when we write down member like this as 2 1 bar in square the brackets, there is no specific association to any lattice point, at this point of time. But, I may want to do later, but that would be under though circumstances, so we will seen that it is pretty straight forward constructed miller indices for any direction in a crystal.

And we just go to through procedure for finding the miller indices for many orbitary directions, now for instance in the couple of example we consider before, we always started as another points starting point is origin. Suppose, we do not start with the origin, then how do I go about constructing the miller indexes for any directions, any orbitary direction we consider here. Of course, I do so within example for a direction like a, b.

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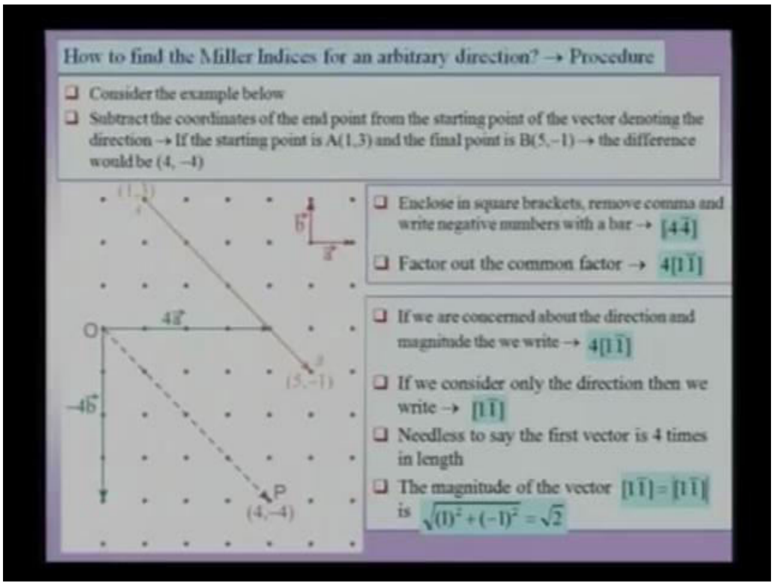

So, this is my orange direction a b and with respect to the origin for insane is the a b direction, as a tail of arrow at 1 3 and the head at 5 minus 1 this is respect to this coordinate system in green, which is O center at the origin O. So, this is my old orthogonal system where in my a and b vector are as follows, and therefore the negative b is downward directions. So, what we do, we subtract the coordinates of the n point from the starting point of the vector denoting the directions.

So, we subtract say this coordinates and this coordinates from 5 minus 1, so the difference would be 5 minus 1 would be 4, which is shown here minus 1, minus 3 would be minus 4, so it will be 4 minus 4. So, this is the subtraction position of the b minus a coordinates, I would first enclose this in square brackets and remove the comma, see for instance I am dimensioning coordinates or differences in coordinates I would use a comma between the two numbers. Here, I remove those that comma and I would write the negatives numbers of bar, so this directions would be in square bracket 4 4 bar, as you can see here.

Then I would as before factor of the common factor 4, so you will have a 1 1 bar, suppose there only constitute the magnitude I would erase this 4, and leave it as a 1 1 bar. So, I am not worried about the magnitude of the direction, but just the direction, then I will write as 1 1 bar this is written here, so becoming the direction, then I write 1 1 bar and if I constant with the magnitude n direction, then I will write it as a 4 times 1 1 bar. So, it needs to say the first vector which is the 4 1 1 bar is 4 times in length, as compare to a vector which is 1 1 bar, the magnitude of the vector which can be written as a mode of the miller indexes is nothing but, square root of 1 square plus 1 minus 1 square which is root 2.

So, the length of this vector a to say the some point x, here in point would be root 2 and there four root 2 here in the total vector which joins a to b, so it is very very simple which like the normal coordinate geometry. And of course, I say pointed out I could always translate this vectors a b to the origin, I can put this vector on the origin and that means, I take a to O automatically b will go to p. And therefore, a equivalent direction as far as latitude goes, and fact this all this vector a b and O P all belongs to the same set, in other word they all describe the same millenary indices.

And I could describe this O P also by this millenary indices 4 1 1 bar in square brackets or I am just concern about the direction, then I could call it a 1 1 bar directions. So, we have seen that for the procedure for getting millinery indices orbitary directions, is to subtract the coordinates of the n point all this starting point from n point, then en close in it square brackets. And put the negatives top as bars factor of common factor, and we got the millinery indexes for the direction, so reasonably straight forward procedure.

Now, as I told you, that if I want to write general direction in two dimensions, for a directions I will call as h k and in three dimensions, we told we need three indices to describe a direction. And therefore, I will write a general directions as h k l and the length of vector represented by this millineries h k l, is square root of h k l plus k square plus l square. So, and suppose corresponding suppose I go to four dimensions, then I will the 4 h k l m one of those kind of representation, and they will have four indices and enclose square brackets.

And the magnitude of that vector will be square root of h square plus k square plus m square, so this is for millinery indices in three dimensions. So, to understand this millineries indices better, let us focus on the cubic system and we already seen cubic lattices. We have seen cubic units cells, we also seen cubic crystals at is some simple example of cubic crystals, more complicated examples we will consider later. And therefore, I would like to know all the important direction and planes in a cubic system, so for instance this is a cubic in front of me and equivalently have a cube here, which you can see ((Refer Time: 26:28)).

And now I want to understand and important directions as obvious, the important directions since this cube or obviously, the edge directions, it is represent by h like this, so this is my direction. Or one of the other h is for instance, this could be another h for instance or this could be another h, so this one is h it could be face diagonal, like this is a face diagonal as you can see here ((Refer Time: 26:51)). So, let me come and show you, so this is my face diagonal which is a important direction, it could be one of the other face diagonals like this, or this face diagonal it could be a body diagonal.

So, it start from the origin goes to another body diagonals, it could be another body diagonal for instance, it could be like this, so this are for instance the important directions. There are more what called higher order directions which be connecting for instance a corner, to half the face diagonal for instance like this, to this is half the edge and the vector goes like this, call it me equivalently place it here. So, it connects the vertex to half the face diagonal, and the procedure for generating all this direction exactly same as we saw in two dimensions, but we will consider them one dimensions.

So, let us go back to a slides, ((Refer Time: 27:53)) and we see that for instance, now assuming this is my x directions, the miller indices for this directions would be 1 0 0. Of course, I can easily obtain this, because mow and I am not taking the account magnitude, if I am talking the account magnitude, then the vector would start from this point and would at this enter this point which I can call point a. So, this is b by vector, if I worried about the magnitude for the general direction 1 0 0 not taking at account magnitude would be this directions 1 0 0.

The 0 1 0 on the other hand, suppose you mean this is my x axis with this is my x axis which is directions, this my y axis and this is my z axis. So, the vector along the y direction be 1 0 vector, and how do obtain it this is my 0, 1, 0 and I subtract it from a 0, 0, 0 and I get 0 1 0. Similarly, the third vector is 0 0 1, which is as you can see for cube are equivalent, face diagonal is represented by vector like this, for this diagonal vector a invalid notation would be 1 1 0.

And exactly I would obtain the same way I did my previous operation, this is for instance the coordinate of this point here, would be 1, 1, 0 and the origin 0 0 0, so my subtraction mean to 1, 1, 0 and I make my bear indices, my enclosing square brackets numbers 1 1 0. Therefore, I got my millinery indices this 1 1 0 equivalent face diagonals are for instance, this one which is 0 1 1, this one which is 1 0 1 and I could have opposite of this directions. For instance, if this directions is 0 1 1 my opposite directions which would be from this point ((Refer Time: 29:50)) to this point.

So, let me draw the directions would be 0 1 bar 1 bar, so it is opposite directions, another word if have any direction h k l, so this is my general directions which is given h k l, then my opposite direction to that, this is my h k l opposite direction would be h bar k bar l bar.

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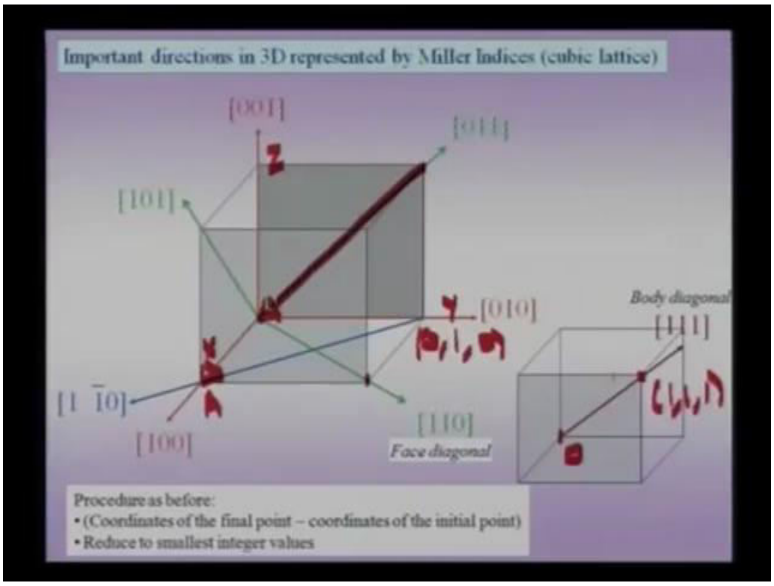

The other important directions, when I am talking about millinery indices is the as I told you the body diagonal, which is now the given by index 1 1 1, because this point here for instance has coordinates of 1, 1, 1 and this is my origin O, which is one it is 0 0 0. And therefore, by subtraction I obtain millinery indices as 1, 1, 1, so for a cube we are seen perhaps these kind of indices which has very small indices as their values for h k l are called low index plane directions.

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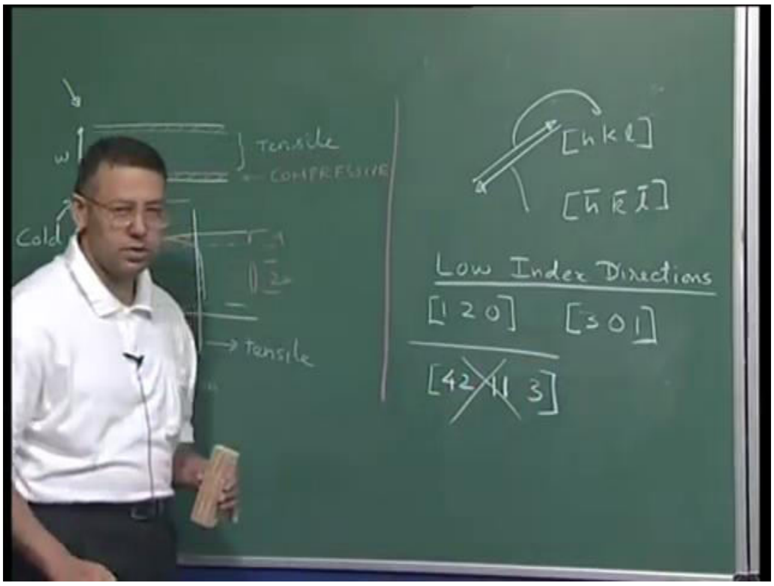

So, we also introduce that concept call the low index directions, so let me write down some examples of low index directions like 1 2 0, apart from the example we already for instance, 1 1 1 would be a low index direction. We could have some other directions for instance, we could have is for instance 3 0 1 for instance, and also will later see that the same rules apply it we are talking about low index directions or low index plane. Wherein, this integers are small numbers, for instance an example of an index which would not be low index direction, would be say for instance, let me chose and extreme kind of the example say 42 11 3.

So, this would not be called, this will not come under category of low index directions, now the importance of the low index directions will become later clear, when we talk about slip, and other kind of things. And we have to remember that often when you doing any experiment, you would be worried about these kind of directions, which are the low index directions, which would have more physical means. The concept of family of directions is an important one, and this is specially, so when you talking about properties.

We already seen that according to the nominal principle, the symmetry of any property at least has to be equal to this symmetry of the crystal. For instance suppose have a this were cubic crystals, I could actually take a for instance even a body center cubic central for instance, and already mentions suppose I have a property, for instance it could be any one of the properties which I need to consider. If it is along this direction it has a certain value major value, then it would be exactly equal along this directions and so along the other directions.

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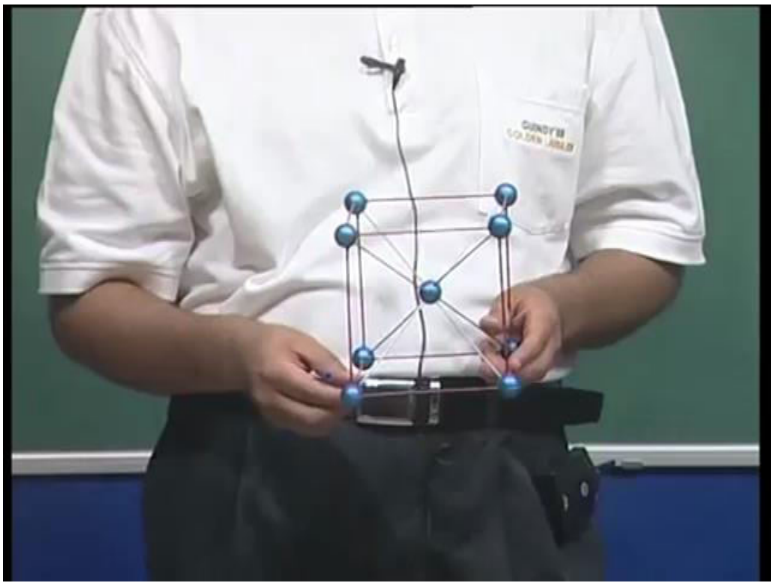

So, all these three directions, which are related now by three fold which is going along the body diagonal, would have exactly the same major of the property. And suppose I major value of property like this, then the opposite directions will also have the same property need us to say, so this is coming from the cubic symmetry, symmetry of the this crystals triangle. Similarly, suppose I have a property certain value of along this directions, then for instance the conductivity of crystal or it could be other kind of vectoral property.

Then the conductivity we exactly identical along this also and so for this face diagonal or this face diagonal, and it has the certain value of the body diagonal, then it will have the same value along all the body diagonal of this crystal. So, in another words when I am doing experiment, I do not have to major this property along all these directions, all I need to do is pick one of these directions, for instance the x axis directions. And major value of my property, then I know from the symmetry of the crystal that the property will be exactly identical along the other directions of the cube, this steam 1 0 0 kind of directions.

Therefore, I have a concept now, something know as the family of directions, one concept we already seen is the parallel set of directions, wherein I could just translate the vector somewhere. And move it from one lattice, the origin from the lattice point the another any other lattice point and my millinery indices could be no different for that directions. And that is the set of direction, but this I am describing is the word use to family and this is a family of directions.

So, is an important a concept in millinery indices, and as far as the properties of the crystals go and I have to remember, then when I am describing the family of directions what other rules which apply. So, the important definition of the family of directions is a set of direction related by symmetry operation of the lattice or the crystal, is called a family of directions. So, in other words let me repeat the sentence I again a asset of directions related by symmetry operation of the lattice, or the crystal is called a family of directions.

In other words, it symmetrically related set of directions, similarly later on see we can describe family of planes. So, now we have introduce the family concept of the family of directions, and a family of directions is represented by the miller index notation, as the same original h k l, actually the preferred direction notation would u v w, I do not know I am using at scale before.

So, let me replace it by u v w, typically the preferred notation for directions is u v w and for planes is h k l, so let me replace that, so I will call as this as u v w. So, let me write this in bracket as u v w, so this my preferred notation and I will reserve the h k l notation for planes, but important thing to note here is the presents of the kind of brackets.

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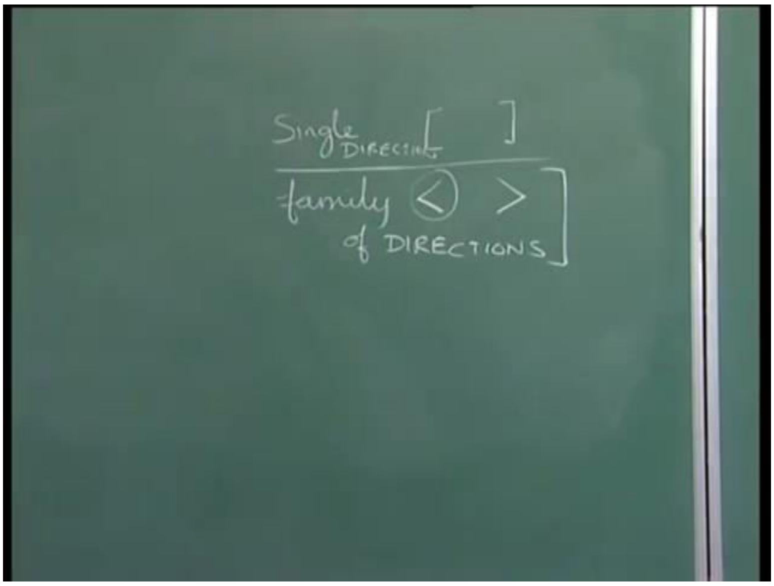

So, know as single directions is represented by, so I can represent a family of directions as by different bracket, and they are written as u v w those brackets. Now, so sometimes this kind of sign, this kind of bracket is also use the represent an averaging process, plus please remember this have to be, this is dusting from an averaging process, wherein which context it will be clear, that is a not a family of directions. But, before I go down to designing the list of members in a family, just what I mean the family of directions as I told you in the cubic crystals.

For instance this one ((Refer Time: 37:42)) the 1 0 0, 0 1 0, 0 0 1 all belong to the same family, I have to ask two questions. So, the two questions are is one considering the lattice or the crystals this has to be absolutely clear, and what crystal is system is one considering when we are describing the family, this is the very important question. Without considering the crystals system hike and more importantly the point group, I will not able to identify the members of the family.

We will take up examples to clarify this point in detail, but these two question have to be asked or write at the outside, now when we were talking about the millinery indices for a directions in a lattice, verses in a crystal for a distance. So, I am considering direction in a lattice verses distance in a crystal, we have to note that we already seen in this detail. In fact, that the crystal can have symmetry equal to that or lower than that of the lattice. So, that means that if the symmetry of the crystal is lower than that of the lattice, then two members belonging to the same family in the lattice, mid not belong to the same in family in a crystal, and we will take up to example to explain this.

So, let me repeat this important point, that there could two members which belong to the same family in the lattice, but when we talking about the crystal made out of the lattice, then they may or may not belong to the same family. So, that depends on the symmetry of the crystal you consider, again suppose I am talking about the cubic crystal, then I mention this point group issue, then suppose there is a member, all the members I want identify for instance of the 1 1 0 direction.

Now, suppose the crystal has a point group 4 by M 3 bar 2 by M, then the numbers of the member of the family would be different compare to for instance, the crystal had a point group symmetry 2 3. Therefore, I need whatever the point group of the crystals, before I write down the members of the family, and this is because crystals can have lower symmetry than a lattice. So, this has to be absolutely understood and the we already seen in other examples to actually, I make this point a clear beyond doubt. And as far as the millinery indices and there family goes, we will take up example to explain this concept.

So, once again to summarize this slide, a symmetry lattice set of directions, and later on symmetry ((Refer Time: 40:12)) planes is called the family of directions or a family of planes. And the symmetry word is the key word, when you talking about the family, and this will automatically be clear considering the point groups symmetry of the lattice or the crystal.

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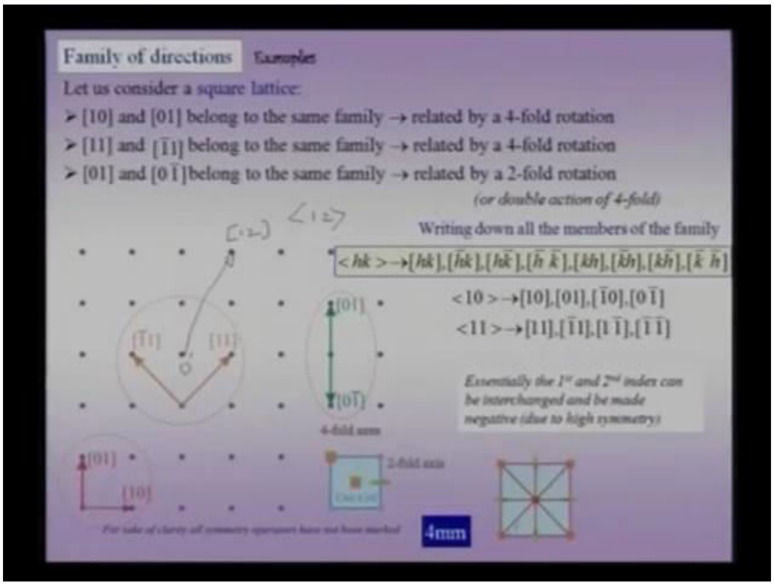

Now, let me write down take some examples to clarify this pint of the family of direction, and in this case i have for instance a square lattice. So, this is not crystal, this is lattice and I want identify a family of directions, now for instance I can start with a directions like 1 0 that means, the directions here, clear 1 0. And the write down all the members of this family, then I would write down as 1 0, 0 1, 1 bar 0 and 0 1 bar, the 1 bar 0 being opposite direction into the 1 0, the 0 1 bar being opposite directions to 0 1, so I have four members belong into the family 0 1.

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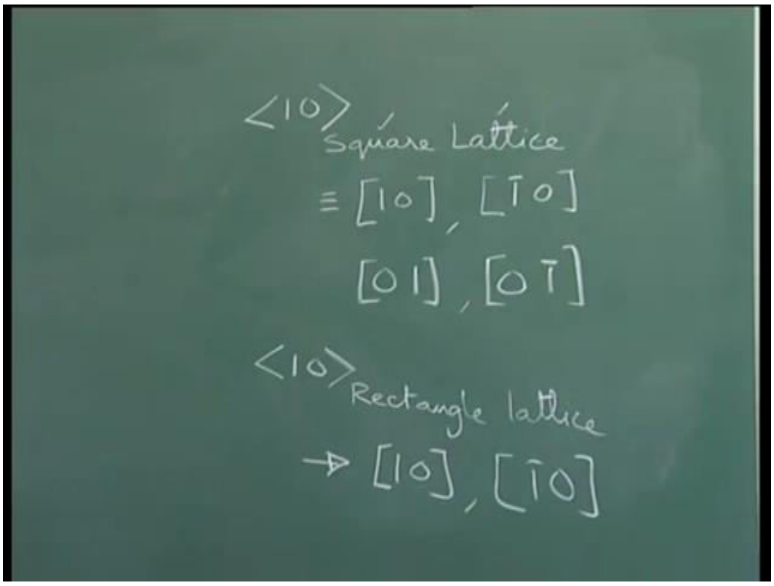

So, let me write them down on the board for further clarity, so this words square take of the symmetry and the word lattice tell me I am talking about the lattice, I am writing for subscript for clarity. So, the members of this family are, so there brackets on the left hand side it is this kind of bracket, the right hand side bracket is the square bracket, and I think this bracket are called characterized. So, this kind of bracket on the right hand side and therefore, members belongs into this family, and each one of one members is equivalent to all the members based on the symmetry of the square lattice.

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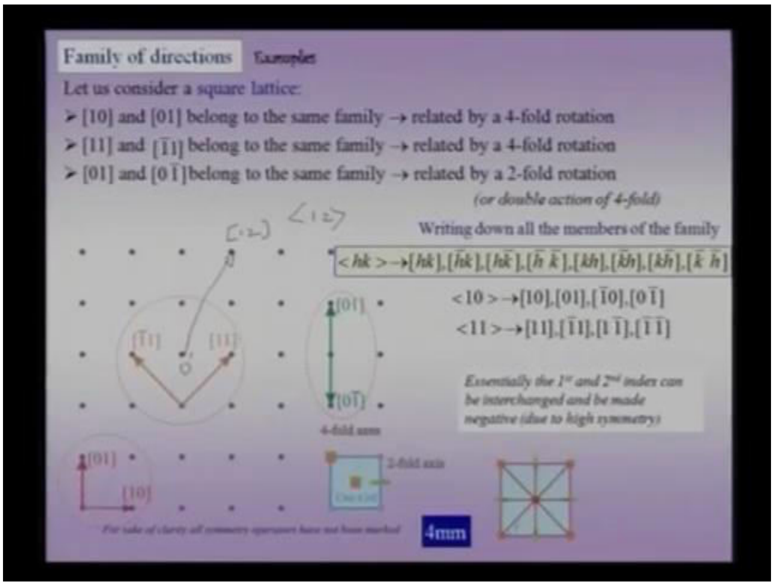

So, that point has to be kept in mind, we already seen the symmetry of the square lattice is nothing but, 4 m m symmetry, so I got 4 m m symmetry for the square lattices. Now, how do I understand this relation, in terms of the language of symmetry, once I am talking about 1 0 1 from this my 1 0 direction and this my 0 1 directions, they are related by a fourfold symmetry. Of course, use other symmetry operate also to relate them member of instance, but I using the fourfold percent simplicity.

So, fourfold rotation to take my 1 0 direction to 0 1 directions and therefore, we are equivalent as far as the, there role in the lattice and therefore, they belong to the same family, the 1 1 and 1 bar 1 belong to the same family. So, let me write down the family of directions, for instance for this 1 1, what are the member of this family, they are 1 1, 1 bar 1, 1 1 bar and 1 bar 1 bar. The 1 1 is opposite in directions in to 1 bar 1 bar the 1 bar

1 is suppose direction in 1 1 bar, so four member to this family, and all of them have a identical role as for as the lattice goes.

So, how do relate my 1 1 direction to the 1 bar 1 in the, in terms of the symmetry operator due, they are related by fourfold rotation again, 90 degree rotation would take my 1 1 to 1 1 bar I could alternately use the vertical mirror also, which is present for this mirror, to take this two that direction by just use a fourfold. So, any of the symmetry operator can be use to relate this two directions, and since they are related by symmetry operator of the lattice. And now this are symmetry operators of the lattice and therefore, they are equivalent and they belong to the same family.

Now, you talk about 0 1 and 0 1 bar, they can be related by 2 4 or a double action of the fourfold and therefore, the belong to the same family. So, similarly I can start identifying various directions in this crystals, I could take a certain lower index directions, once let me chose 1, I could chose index starting say from this point, go the all way here and this would be my directions and I can find out all the members of this family. So, this suppose this my new origin O prime, then this vector from this along the x directions it is 1, along the y direction it is 2, so I can represent it as a 1 2 kind of a directions.

And I can write down all the members of this family, which would now the generic name for the family would be 1 2 and I can write down all the members. Now, if I want to write down for any general direction h k, I can write down the rule of all the members of the family would be, they will be h k, h bar k, h k bar, h bar k bar, k bar h, k h bar, k bar h bar. And some other this are just opposite directions to the h bar k bar, in other words the generic rule for this type system of very high symmetric, happens to be I can exchange in second index.

And any of the indices can be made negative, and this is because high symmetry of this square lattice, this point absolutely kept in mind, I cannot do this exchange of indices, and permutation of indices and changing of them into negative, without understanding the symmetry of the lattice. So, let me now jump a little lower, and choose a rectangle lattice, and see that how certain numbers belong to a same family in square lattices. But, now I have the lattice of lower symmetry and therefore, those members no belong to same family.

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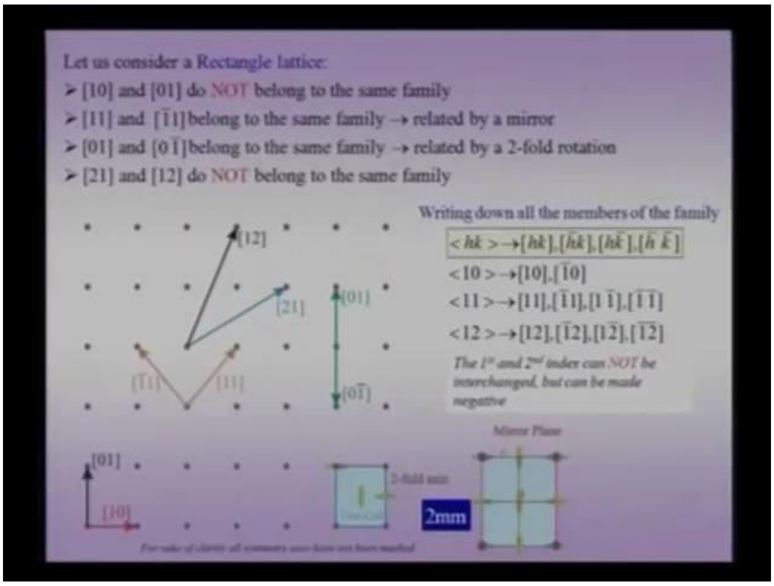

So, let me take very first example for instance for 0 1, it is not belong to the same family as 1 0, so for instant previously the 1 0 and the 0 1 where belonging to the same family, and that were by fourfold relation. But, there is no fourfold in rectangular lattice, you know the symmetry of the rectangle lattice 2 m m, we have consider this before, and all the symmetry of the operator of lattice shown here within the unit cell, in the right hand side below. Therefore, then I want to write down the family of the plane 1 0, I would not include the 0 1 has the member of the family.

So, movement I go into the rectangle lattice whichever square lattice, and I want to write down the members of this ((Refer Time: 46:25)) family, now 1 0 and now I will have mention. So, rectangle lattice therefore, I will see that I cannot include 0 1 as the member of the same family, so I will write down the members of the same family as 1 0 and 1 bar 0. So, two other member which were originally part of the same family do not any longer belong to, the same family when you are talking about the rectangle lattice, and this is because of lower symmetry of the lattice.

So, 1 0 will have it is family with members of 1 0 and 1 bar 0, the 0 1 family will have it is own members as 0 1 and 0 1 bar, so this the originally the four members are split into two family, two nuclear family, because of the lower symmetry of the lattice. However, if two directions related by some symmetry operator, then you can continue to relate them by belong to the same family, see 1 1 and 1 bar 1, which was an example we saw before. 1 1 and 1 bar 1, we have originally belong to the same family in the square lattice, they continue to belong to the same family in the rectangle lattice. So, this is my 1 1 direction and this is my 1 bar 1 direction, and they are related by a mirror plane, this vertical mirror.

So, this is mirror plane exist therefore, my 1 1 and 1 bar 1 belong to the same family, and there is also an horizontal mirror that means, the 0 1, 0 1 bar belong to the same family which is what had return for the 1 0, which is exactly equal to the 0 1, also because of the presence of vertical and horizontal mirrors. On the other hand, the direction like 1 2, which is drawn here, does not belong to the 2 1 and different color for these direction implied that it will not belong to the same family.

So, 1 2 belong to the different family, the 2 1 belongs to the different family, because there is no symmetry operator taking place one to the other. In the case of a cubic lattice which we consider below before they would have belong to the same family, and intact you would have the mirror classing, diagonal mirror would take the 1 2 to the 2 1 in the case of the square lattice, and in the case of rectangle lattice, they would not belong to the same family. So, I can write down the general formula writing down all the members of the family, for instant I can write the formula for h k has h k, h bar k, h k bar, h bar k bar.

So, if have any general h k, the members of the family would be 1, 2, 3, 4 members and they would be h k, h bar k, h k bar, h bar k bar and with clearly see that the first second index cannot be inter change, but they can be made negative. So, this is ma rule for the rectangle lattices, and this is something which is born in my, now I will just summarize this important example. So, what I am contrasting here, is the square lattice with example the rectangle lattice, the rectangle lattice with 2 m m symmetry, the square lattice are 4 mm symmetry.

And we have to remember, one of the mirror in the square lattice is a diagonal mirror and therefore, when we will come to a rectangle lattice certain members, which originally belong to the same family the square lattice, no longer belong to the same family in the rectangle lattice. And the example which belong to the same family 1 1 and 1 bar belong to the same family, which are related by the mirror 0 1 and 0 1 bar belong to the same family related by the twofold direction, by twofold rotation.

1 0 and 0 1 do not belong to the same family, because there is no symmetry operator taking one to the other, 2 1 and 1 2 do not belong to the same family, because there is again no symmetry operator taking one to other. On the hand for instance, square lattice to 2 1 and 1 2 belong to the same family and they related by the diagonal mirror. So, this is clear from even these two example, that I cannot blindly write down the members of a family. I cannot go for the instants and somebody tells me that, I have a lattice, please write down the members of the family, I cannot do so without asking the important question, what is the symmetry of the system.

If you do not answer the question, I cannot write down the members, and only the symmetric related set would form the members of the family. Now, as before we will try to make some crystal out of this square lattice, and try to write down the members of the family. And again we will note the same thing like, I cannot just because some crystal is based on square lattice, I cannot use equivalent based on the lattices, so I have to consider equivalents in the crystal.

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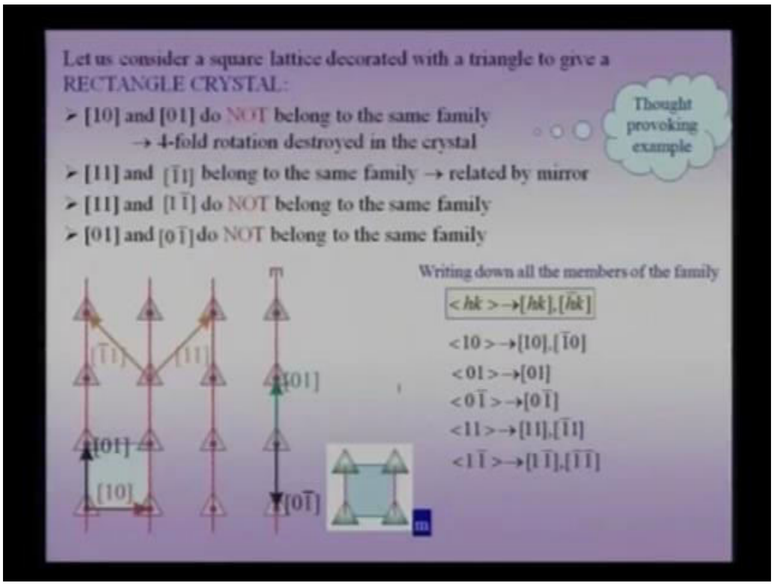

And we already seen especially, for instance this example we have seen, wherein we took a square lattice and decorate with the triangle, that for instant water flow from the top direction, or water flow from the bottom direction they are not equivalent. And this are not property of non equivalent should also come out, in our definition of the family out of the direction which are related by the symmetry, so this is absolutely clear.

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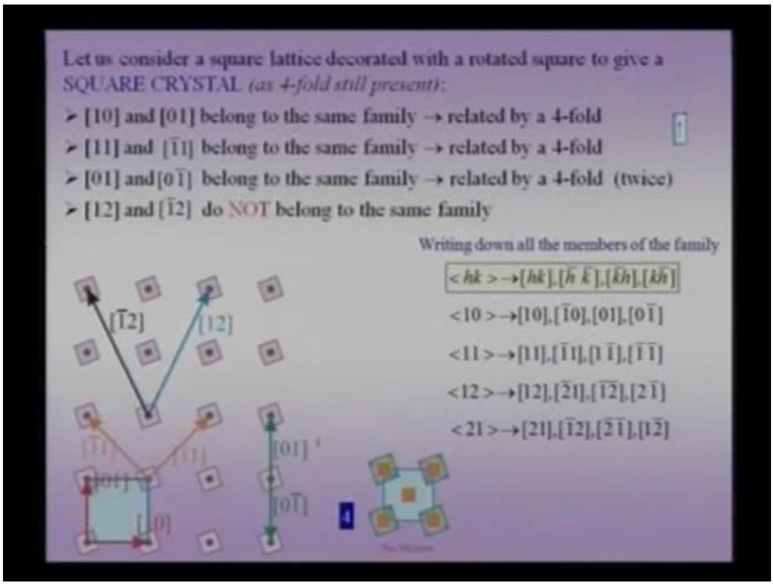

Now, let us consider a square crystals, now this is not the square crystals like the one we could construct starting with the square lattice and putting a circular motive, which is would give me a 4 m m 1 0 and 0 1 belong to the same family which are related to the fourfold. They were not related in the rectangle crystal, but here now they are related and they belong to the same family, 1 1 and 1 bar 1 belong to the same family as they again related by the fourfold. So, this is my 1 1, 1 bar 1, they are related and for that why they have given orange color, both of them.

The two red vector also belong to the same family, 0 1 and 0 bar belong to the same family related by the fourfold twice, so my 0 1 and 0 1 bar now are belong to the same, because I can fourfold twice. And they give me the same kind of a, and they fore they belong to the same family, 1 2 and 1 bar 2 do not belong to the same family, in the square lattice, they would belong to the same family. In a crystal with 4 m m symmetry that is I take square lattice and put a circle as a motive, in other word I would create the crystal 4 m m symmetry 1 2 and 1 bar 2 would be the same family.

Now, this is a square crystals, because it is still got the fourfold axis present, so it comes under the class of square crystals, but it is not highest symmetry of the square crystals would have 4 m m symmetry, this square has only 4 symmetry. Therefore, in spite of being the square crystal 1 2 and 1 bar 2 do not belong to the same family, so this point has to be absolute clear. Not only the information that belong to the square crystals need to be note, I need to not the particular symmetry of those square crystal, so the information nearly the square crystal is not stuffiest.

For me to generate the family of direction or later on as you see the family of planes, I have to the particular symmetry, and in this case because, only a fourfold present by 1 2 present cannot be related to 1 bar 2 by any kind of a fourfold operation, therefore they do not belong to the same family. So, let me write down the general for all the member h k belonging to the four crystal, crystals having four symmetry, they will be h k, h bar k bar, k bar h and k h bar.

So, you can see that the h and k can be exchange in position, but with one of them going negative, so h k becomes k, then k bar h becomes one of the must be negative or you can make k h bar and h k can go to h bar k bar. So, for instant suppose I choose a direction 2 1, what are the members of this family, they will be 2 1, 1 bar 2, 2 bar 1 bar which is equivalent to the to this kind of thing and 1 2 bar. But, for instant suppose I take 2 1, then 1 2 is not the member of the family of...

Suppose, I am taking the 2 1, 1 bar 2 bar is not the member of the 2 1 family, it has it is own family, which is of the 1 2 kind of a family, so that point as to be absolute clear. So, the two mirrors the vertical and the diagonal mirror are the vertical, and the horizontal diagonal mirrors are missing from this crystals. And therefore, we note that for instants 1 0 and 0 1 belong to the same family, which are related by a fourfold, they were not related in the rectangle crystal, but here now they are related and belong to the same family.

1 1 and 1 bar 1 belong to the same family has they are again related by a fourfold, so this is my 1 1 and 1 bar 1 they are related, and that is why they given the orange color both of them. The two red vector also belonging to the same family, 0 1 and 0 1 bar belong to the same family related to the fourfold twice, so my 0 1 and 0 1 bar, now are belonging to the same, because I can operate my fourfold twice, and they give me the same kind of a... And therefore, they belong to the same family, 1 2 and 1 bar 2 do not belong to the same family, in the square lattice they would belong to same family.

In a crystal with the 4 m m symmetry that is I take my square lattice and put a circle as the motive, in other words I would create the crystal with 4 m m symmetry, 1 2 and 1 bar 2 could be the belong to the same family. Now, this is a square crystal, because it is still got the four fold axis present, so it comes under the class of square crystal, but it is not the highest symmetry square crystal, would have 4 m m symmetry, this square crystal have only four symmetry.

Therefore, in spite of being square crystal 1 2 and 1 bar 2 do not belong to the same family, so this point has to be absolute clear, not only is the information that it belong to the square crystals a need to be known. I need to know the particular symmetry of that square crystals, so the information nearly to the square crystals is not sufficient for me, for me to generate the family of directions, or later on you will see the family on planes, I have to know the particular symmetry.

And in this case, because the fourfold present my 1 2 direction cannot be related to the 1 bar 2 by a any kind of the fourfold operation, therefore they do not belong to the same family. So, let me write down a general procedure for all the member h k belonging to the four crystal, crystal having four symmetry they will be h k, h bar k bar, k bar h and k h bar. So, you can see that the h and k can be exchange in position, but with one of them going negative, so if h k becomes k h, then k bar h becomes one of the must be negative, or you can make a h bar and h k can go to the h bar h bar full negative.

So, for instant suppose I choose a direction 2 1, what are the direction of this family, they will be 2 1, 1 bar 2, 2 bar 1 bar which is equivalent to the this kind of thing, and 1 2 bar. But, for instant suppose I take 2 1, then 1 2 it is not a member of the family, suppose I am taking the family 2 1, 1 bar 2 bar is not the member of the 2 1 family have, it is own family which is 1 2 kind of the family, so that point have to be absolutely clear.

So, this slight more than the amply explains the fact that, it is not enough if I know the square lattice, it is not enough if I know it is a square crystal, I need to know the symmetry of the square crystal which I am considering, before I write down the members of the family. And therefore, if I already know it is a square crystal of four symmetry and I make the property along the 2 1 direction, then I do not have to make the measurement along the 1 bar 2, 2 bar 1 bar I would know the property would have the same family along the those directions.

On the other hand, I will have to make only the additional measurement among the 1 2 direction, because 1 2 direction is not related 2 1. If I have talking about the crystals like this, were in I have a square lattice with a circle motive with the 4 m m symmetry, then I do not have to make even that, because the symmetry of the crystal is very high. So, this is the clear cut example to explain, how do I generate the my family of direction for a crystal, let me consider one more example to sort of clarifying the matter beyond doubt.

So, let me take the rectangle crystals and in the rectangle crystal, we would notice that for instance 1 0 and 0 1 had originally belong in the same member, for instant the square crystal at four symmetry. And if I originally consider the rectangle crystal, we would notice that 1 1 and 1 bar 1 had belong to the same family, and 1 1 and 1 1 bar, let me see go back here ((Refer Time: 59:52)), so we had 1 1 and 1 bar 1 had belong to an same family. Here let me summarize what is the belonging to the same family and what not, 1 1 and 1 bar 1 belong to the same family related by the mirror, so this is my crystal which I have construct before.

So, let me summarize how the crystal was construct, I take my square lattice and put triangle and each lattice point to create a crystals, the only serving symmetry common between the motive and the lattice is n. And therefore, the crystal has m symmetry, since it has only m symmetry are call it a rectangle crystal, in this rectangle crystal 1 0 and 0 1, do not belong to the same family. Because, the crystal has fourfold axis, 1 1 and 1 bar 1 continuous belong to the same family, because they are related by the vertical mirror you can see this vertical mirror, which is shown in orange color.

1 1 and 1 1 bar do not belong to the same family, 0 1 and 0 1 bar do not belong to the same family, so here now you are seeing that, originally large family are getting spilt into the related into smaller and smaller pieces. And there are families like 0 1, which are only one member 0 1, even 0 1 bar is not belonging to the same family. So, my 0 1 is a vertical direction 0 1 bar is the downward direction, and they do not belong to the same family. If I made a property measurement, which has the same symmetry of the crystal not an higher symmetry, equal to that kind of the crystal property consider, then we already seen one property respect to water flow.

Now, the upward direction, if I make the measurement of the property, I will have to make the additional measurement along the downward direction, and these two values would be different. Now, of course how difference depends on the motive, and in the physical property, but they will be different. So, again 1 0 and 0 1 are different members, different family, so I see the lot of family emerging 0 1 has only one member, 0 1 bar just one member, 1 1 has two members, 1 1 bars has two members like 1 1 bar and 1 bar 1 bar.

So, you can see that the originally, so what you might call a joint families are now broken down to nuclear families, when you consider the lower symmetry crystal, Kavitha ask the question.

Student: Sir, if the mirror indies necessary be an integers

Very good question, in fact let me go down to the board to explain, Kavitha's question is that do miller indices have always to be integers, so let me start with the lattice point and go to some point, which another lattice point.

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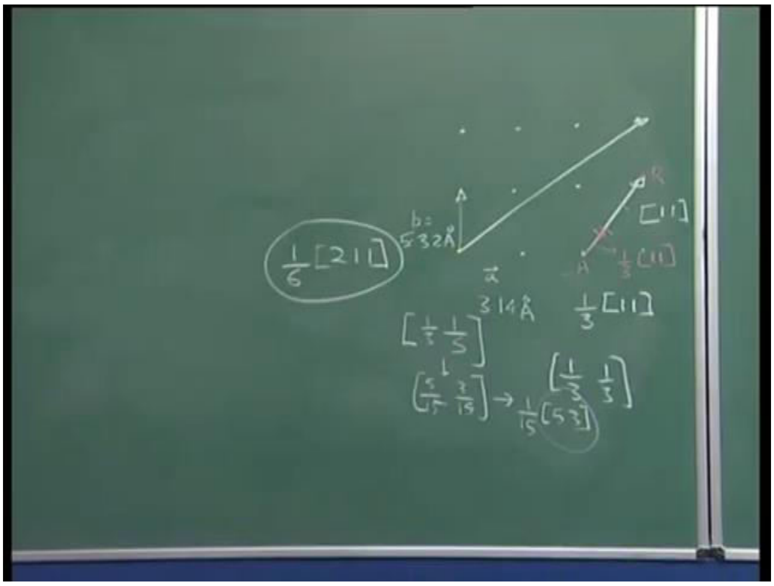

Of course, I am drawing the square kind of lattice, but it could be in general many lattice, so as long I start from the lattice point, and land up with any other lattice point. Now, what is in what term I may consider are this number in some units no, they are in the multiple of the lattice parameter. So, in other word just a along this direction, could actually be 3.14 ion, storm and this another be could be some other number, which could be say 5.32 ion storms.

But, I never put in this number, so the point number 1, why they start to be look at the look very nice and beautiful, which is the reason why we want to use the miller indices, we do not want always dealing with these kind of numbers. We want to be dealing with the integers, and since I am considering the perfect lattice transmission which course from one lattice point other, therefore these will be in the integral multiple of the lattice translation vector. And therefore, you will always land up in the integers.

Now, for instance my direction, let me take the simple direction for like this for mirror indices would be 1 1, what if I write 1 3rd 1 1, you can do this operation, all I am saying here is that, in other word suppose I want to expand it, I will write it has 1 3rd, 1 3rd. But, has you saw previously when I do miller indices, I always factor out common factor, and I would write my indices 1 3rd, 1 1 which nothing but, be the vector which is 1 3rd... So, this is my a b and I split my a b into three part and this would be my 1 3rd 1 1, if I were to start putting more difficult kind of indices, because I put, ((Refer Time: 1:04:25)) I put say for instants 1 5th. So, what I would do, I would my normal rationalization process, I will write down this has 5 by 15 and 3 by 15, and I will pull out my 0 from here to here, then I will pull out my 1 by 15 outside and write down this has 5 3.

So, this is the 5 3 vector, which is 1 15th in length, normal translation vector would always be the integers, because which is starting from one lattice point and landing up at other lattice point. And you are ignoring the details, that this actually in terms of physical dimension, there could be some orbitary number. Since, we are talking about the perfect lattice translation vector, they would always have the integer values. But, we could in principle have and we will see example of this especially, when we consider dislocations, and we will talking about partial dislocations, we will see those partial dislocation do not have has a Burgers vector complete integers.

And you have to represent them with these kind of having partial, so let me speak I had given the example for instance, you could have the partial dislocations, which has the index like this ((Refer Time: 1:05:42)). So, this kind of number come and this obviously, tells you, it is not the perfect lattice translation vector, it does not join one lattice point and another lattice point. If it did so, then the automatically the numbers will be integers, and even if I had numbers like this, what I would do I would write whatever is coming within the bracket has integers and pull out all the, what do you call fractional part outside, this I would do.

So, that I still have an lattice translation vector inside and the magnitude given outside this is how I will deal with this kind of vector. So, very important question and in fact, as I said we will have to deal with these kind of fractional indices later on having a full understand there are normal lattice translation vector always has to be having integral value because they collect one lattice point out other. And we have already factor out the other dimensional values.

So, just summarizing this example ones more that you have seen that a family of a direction cannot be written down mechanically we need to know the symmetric later it had said; that means, the symmetry operation of the crystal. And in this for instance this rectangle crystal had m symmetry we have seen that the rectangle lattice would have two m m symmetry, but this is just m symmetry.

And therefore, the originally members originally belong to the same family would not belong to the same family in the lower symmetry crystal which I have constructed from this square or the rectangle lattice, so this point has to be completely clear. And the generally for this rule would be h k and h bar k and this inversion of h index is possible because of the mirror plane which is vertical mirror plane. So, it is a important note that all direction relate by symmetry only if they related by the symmetry form of family and no other case I can relate them and call them members of the family.