

Steel Making
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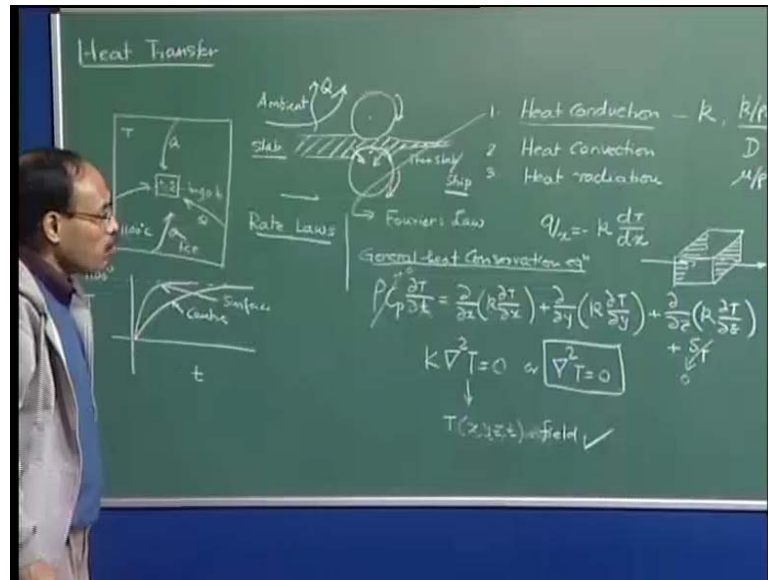
Module No. # 01

Lecture No. # 06

Now having said and talked about thermodynamics as well as fluid flow or fluid dynamics. We will now move on further and talk about the next segment of the science based steel making which is get transfer. Now, thermodynamics as we have seen will allow us to predict the feasibility of a process. We can perform many equilibrium calculations and get good insight of what is happening within a steel making system.

And fluid flow as we have seen, particularly all the liquid state processing operation involves some kinds of a fluid motion. So, knowledge of fluid flow is also important as it will be more and more evident in order to calculate the rate of the processes which is of our prime interest, but by now, I think you have been able to appreciate that to handle with thermodynamics, to handle with good dynamics, one has to have very good analytical capabilities; the good background in mathematics is a prerequisites. So, never ever think that steel making is a highly qualitative subject. It evolved as an art, but today, it is highly scientific and we have to have good analytical and mathematical abilities in order to really move further.

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Now, moving on to the next segment, which is heat transfer. I must first explain to you that what is the relevance of the subject, and as you all know that large tonnage of a ferrolyse and deoxidizer elements as well as steel scraps are routinely added in molten steel and these solids which are invariably projected into steel at room temperature. This could give a thermal interaction between the resided solid as well as the melt, because they are at the similar temperatures.

And as a result of which, the allowing editions or various deoxidizer editions will proceed to melt and then dissolve in steel, and the overall weight of efficiency of these edition techniques depends on the rate of heat transfer as well as the rate of mass transfer. We will discuss these in succession, but apart from that also, when you are talking of particularly solid state processing operation, for example, solid state processing operation implies beyond continues casting which involves slab reheating, then rolling, then you have galvanizing and all these operations are carried out at relatively high temperature, where the prevalent temperature plays a very important role in terms of the efficiency of the processes.

For example, if you have a furnace, and in this furnace, if I have kept, you know, if I have kept an ingot, for example, this is an ingot and this is the furnace of a temperature of T and then the heat. So, Q is flowing from the furnace, and this is the furnace; I write it as fce, and so, Q heat is transferred into the ingot personally mechanism which we are

going to discuss in a minute, and then, as a result of this transport of heat, the object temperature continuously increases.

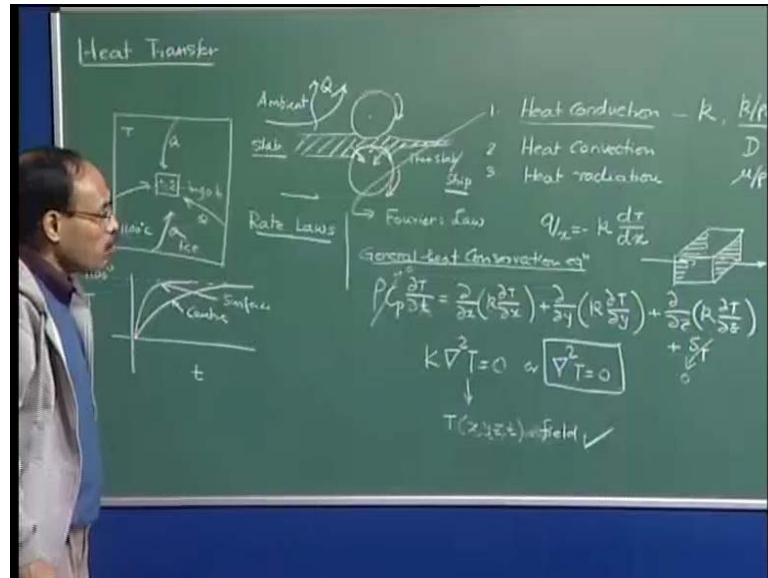
Now, the rate at which the surface temperature increases, in the rate at which the inside temperature increases are dramatically different because of the resistance of solidified or solid material to heat flow. Now, if you want to predict the homogenization time, suppose I have an ingot and I place it within the furnace and I want to now find out that at what time, the ingot has become completely homogeneous. And this is only then, I am going to take out the ingot from the furnace and subject it to either soaking pits or the same temperature is maintained or straight forward taking it to the rolling mill.

So, prediction of complete homogenization time or an uniform rolling temperature in their heat furnace is of key interest, and unless we know the fundamentals of heat flow within the furnace itself, from the furnace environment to the solid surface, and then, within the solid surface, you will not be able to calculate the optimum reheat in time which essentially gives a constant reheating, a constant ingot temperature at the end of the cycle.

So, innumerable descriptions can be given where heat transfer can be shown to play decisive role. So, there is no doubt that we will have to have good background of heat transfer in order to tackle efficiency of many metallurgical processes. Roll and the slab is moving between the rolls. There is going to be a contact resistance; there is going to be heat flow from, in that the roll is going to be cool, and then, the slab is moving between the rolls and the rolls are going to take away.

So, you have the direction of movement is like this, and then, this the role product comes out like this and we have enormous amount of heat flow between the roll surface and the certified material, and also, because of mechanical working, you know some amount of heat is going to be evolve, just like the way heat evolves during solidification process.

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So, we have heat transfer from this; heat is being lost by various mechanisms. Then, there is heat evaluation within the solid because of mechanical working conversion of mechanical working to heat energy, where there is a efficiency factor is involved here, and then, because of the contact between the roll and this roll is always water cool to maintain high, relatively low temperature to maintain its low deformability in our deformation and to maintain or to have more heat transfer rates. So, we have heat flow from the solid surface into the roll. So, all this things, the heat loss to the environment, the heat evaluation due to mechanical working, heat flow to the roll and all these things are going to determine the overall temperature at which the rolling is going to be carried out.

So, if you want to predict the evaluation of temperature within this that is the very complex task. You have deformation taking place; you have heat transfer taking place of course. There is no fluid flow, but there is solid flow here. The volume remains constant, but the dimensionality of the object changes; it is the whole industry complex task, and you have to predict that for temperature, the slab is going to come out. You have to have good background of heat transfer and mechanical working process or the deformation theory. So, I can go on and on, and before I really end this part, I would now like to show that well, we are talking of heat generation; we are talking of heat flow; we are talking of heat flow between the roll and the solid.

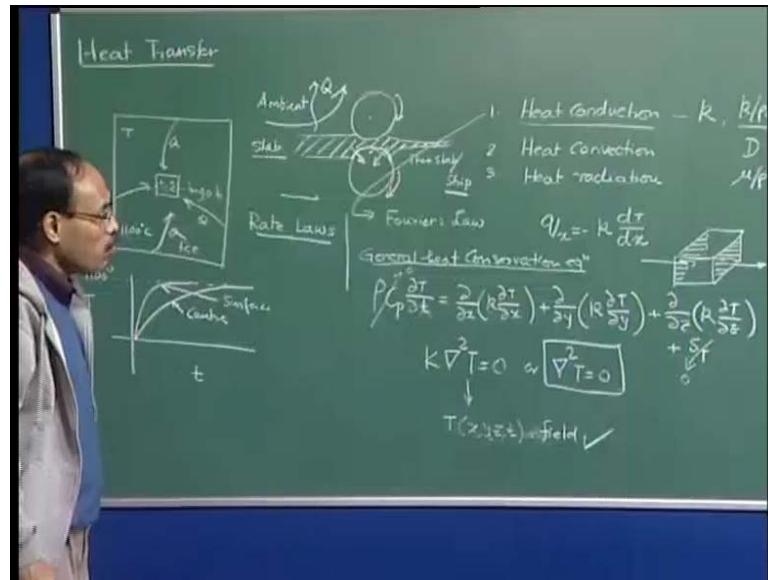
And this, there are various mechanisms by which the heat flows actually and we have to discuss first the mechanisms of heat flow, and then, possibly we will be able to tackle this sort of a problem or this sort of a problem. Now, there are three fundamental mechanisms by which heat flows common objects, and within a solid object, the primary mode of heat flow is called heat conduction.

As I said the primary mode, it is the only mode of heat flow within the solid. In the solid, no other mode of heat flow can be there. Now, when heat flows from the solid to the embedded atmosphere, it can flow via three different mechanisms - it can flow via conduction and there is second which we call as a heat convection or convective heat transfer, and then, finally, we have heat radiation or radiative heat transfer.

Heat conduction which takes place within the solid is because of a material property, which is known as the thermal conductivity of the material. If there is no thermal conductivity and there is no heat flow within. So, heat conduction basically is a diffusion process, just like the way mass flows from one region to another region, because of the property mass diffusion, heat conduction is also termed as diffusion of heat.

You must remember that there is a diffusion of momentum and the diffusion coefficient of momentum is viscosity. The diffusion of mass for the diffusion coefficient is the mass diffusivity and diffusion of heat for which the diffusion coefficient is the thermal conductivity or you can interpret it as thermal diffusivity, which is k divided by ρc . So, this is the actually thermal diffusivity and we have mass diffusivity and μ by ρ and this is the kinematic viscosity or momentum diffusivity, and the properties which are responsible of diffusion are basically thermal conductivity, mass diffusion and viscosity. Convection on the other hand, as the name suggest request some flow. So, if there is no flow, there cannot be any convection.

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So, as a result of fluid convection or movement of fluid, we can have heat flow in this system for one point to another point, for example, I have solid here, solid is moving here. If there is some motion here within the liquid gas phase, so, this is an ambient; this we call as an ambient or the atmosphere which is above the solidifying. So, this is the slab which is being rolled, slab is made into and they derive either a thin slab which is coming out or a strip which is coming out.

So, heat will be lost from here to here and we can combined, and if you have a fluid motion here, if the, you know, flow is going like this, so, this flow here. If the ambient contains here, can take you some amount of heat and the flow of heat, because of this fluid motion start this convection. While conduction and convection requires a materially medium, radiation does not require a radiation material medium. For example, the heat that we get from sun is through radiation; it does not require material medium and there are certain laws which you must first understand in order to move on further.

So, the rate laws of, which you also call constitutive equations. So, this rate laws are an important that if heat conduction is taking place within the solid from this point to this point, therefore, flow of heat from this point to this point is going to be purely by conduction. Now, the heat is flowing by conduction. I must be able to know that at what rate heat is flowing. That is the most important, and then, only possibly I can talk of that what is a homogenization time of the income.

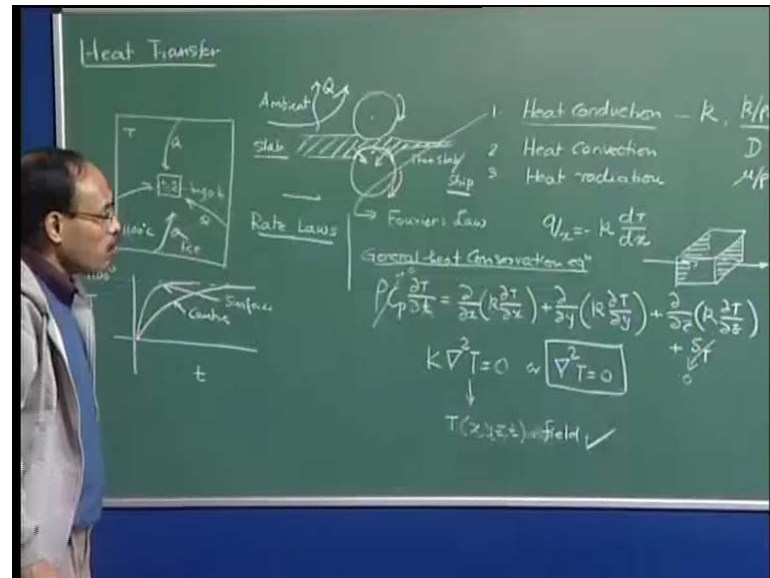
So, heat conduction we have number 1; so, I am not going to write it again. So, this is Fourier's law. As we all know, the Fourier's law is nothing but the heat flux q along the x direction is equal to k temperature gradient. This is a vector quantity, which is a heat flux. This is a thermal conductivity and this is... We can similarly write the heat flux along y direction. The definition of flux all of you must be knowing it is the amount of energy; heat flux is amount of energy per unit area per unit time and that say area is normal to the direction of heat flow.

So, heat flows from a region of higher temperature to a region to a, to a, lower region of lower temperature and that is why it is preceded by convention a sign of negative. Now, this thermal conductivity a material, for example, I can have a q_y , I can have a q_z , and in that case, I may not have k . In all three directions, k may be a direction sensitive property also or structure sensitive property, and in that case, if the structure in this direction and structure of the material in this direction are not identical. We can have, we can anticipate that thermal conductivity in different directions and be safer.

If thermal conductivity are same in all directions, then we say that the material or the thermal conductivity is isotropic. On the other hand, if thermal conduct is different in different direction x direction, we have k_x ; y direction we have k_y , and z direction, we have k_z . Then, we say it is a non-isotropic material.

Now, based on this weight equation that the generalized heat conservation equation is derived, the generalized heat conduction. So, in order to calculate the flux, I will require that flux is as I said at what rate temperature is flowing from this point to this point. So, therefore, in order to know that, I have to know this distance and I have to know the temperature difference T_1 minus T_2 , and then only I will be able to calculate flux and then only I will be able to calculate that in 10 minutes of time, how much of heat really has moved from this point to this point.

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So, therefore, the first and the foremost thing is while the object dimension is known, I have to map the temperature field. I have to know the T field, and it is only by knowing the T field, I will be able to temperature field; I will be able to calculate the rate of heat flow from one point to another point and there only I will be able to tell that well certain set is the homogenization time for the important, so on.

On what basis the temperature is calculated? I have formally obtained the temperature. The temperature is known from by writing a thermal energy balance equation. So, basically these also you have done. I am going to just mention if you take a control volume, and then, we say that well, in that particular control volume, we have heat flow along three direction, heat is flowing, but due to conduction in x direction, it is living on this particular phase.

For example, so, it is entering on this phase, this is the phase that I am talking about and this is the phase I am talking about, so, into this imperial decimal is small control volume, heat is entering as well as heat is leaving. If we fluid in three different directions, how much of heat is coming in and the net difference of this, that is, between heat flowing in and between heat flowing out and heat flowing in will determine whether the temperature has increased or decreased.

Also within the control volume, we may have a thermal source because of may be a phase transformation reaction, because of may be a mechanical working process. So, there may be some heat generation also. So, therefore, we can say that whether the net rate of change of temperature of the body is going to be determine by the heat flow, heat coming in and heat going out in three different space directions as well as the rate of heat generation, and therefore, equation if we values this equation, write this equation in terms of what that this means, that you have heat coming in minus heat going out in all three direction plus or minus heat generated in the system is equal to net rate of heat accumulation.

This is the statement energy conservation statement in what, and if I use Fourier's law to convert this statement into mathematical term, this is what we have seen. Also you get a partial differential equation which looks in this familiar form. Most of you must be knowing what is this equation called, this equation in its, so thus that is the generation term that I am talking about thermal heat generation. If it is negative, then it is a heat consumption in the process.

And this represents the net heat flux of heat due to conduction along x direction, along y direction, along z direction respectively, and this is the heat accumulation tab. ρ is the density of the solid under conservation C_p is the and this equation is basically written for constant density and constant heat capacity although thermal conductivity is can be assumed to be a function of temperature.

As you see, I have written it for isotropic thermal conductivity, because along x direction also I have written k , along y direction is also k , along z direction is also k . Therefore, I am essentially talking about an isothermal isotropic material, but nevertheless, here the thermal conductivity can be taken to be a function of temperature, because I have maintained it within the bracket, or alternatively, if we assume that it is a steady state process, so, this term will become equal to 0. If there is no heat source term which is also equal to 0, and if k is equal to constant, in that case, we can say that well it is or which is a very popular equation, the name of phase is the Laplace equation.

So, therefore, given the boundary conditions, now as you see, we have this is a second order term. So, two constants of integration comes from here, two from here, two from here. So, given the source term, six boundary condition to each for these terms, and one initial condition, I should be able to solve this equation.

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t, y, z
 T
 $k \frac{\partial T}{\partial x}$
 \rightarrow tangent
 x
 $T(x, y, z, t)$
 $\boxed{\rho c_p T \Delta x} \rightarrow J/s$
 $\frac{kg}{m^3} \cdot \frac{m}{s} \cdot m^2 \rightarrow m(kg/s)$
 $\rho c_p T$
 $\frac{kg}{s} \times \frac{J}{kg \cdot ^\circ C} \times ^\circ C \rightarrow J/s$

$$\rho c_p \frac{\partial T}{\partial t} + \frac{\partial}{\partial x}(\rho u c_p T) + \frac{\partial}{\partial y}(\rho v c_p T) + \frac{\partial}{\partial z}(\rho w c_p T)$$

$$= \frac{\partial}{\partial x}(k \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k \frac{\partial T}{\partial z}) + S_T$$

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Heat Transfer

Ambient Q
 Slab
 Surface
 Rate Laws
 Fourier's Law
 General Heat Conservation eq.
 $\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}(k \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k \frac{\partial T}{\partial z}) + S_T$
 $k \nabla^2 T = 0 \approx \boxed{\nabla^2 T = 0}$
 $T(x, y, z, t) = \text{field} \checkmark$

1. Heat conduction - $k, k/l$
 2. Heat convection - D
 3. Heat radiation - μ/k
 $q_x = -k \frac{dT}{dx}$
 $\nabla^2 T = 0$

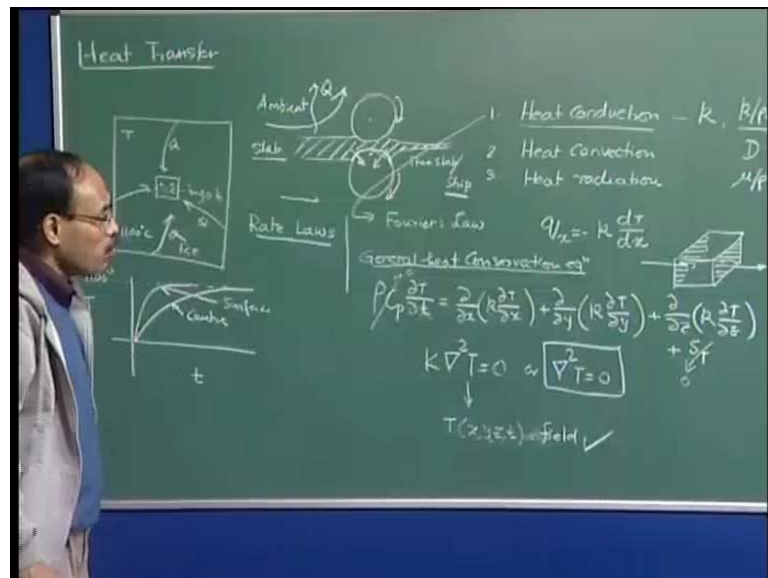
If I solve this equation, then I should be able to find out that what is the temperature distribution, and if I plot the temperature distribution as a function of distance, so, I can know that well at a value of at certain time, at certain distance y, at certain distance z, so,

I can plot T temperature as a function of x , because temperature is a function of x y z and T .

Now, if I fixed time, if I fix z , if I fix y which I have done as indicated, then I can see that well, I can make a plot of t versus x , and suppose, I get a plot like this and how do I know this plot, because I have solved this equation. So, by solving this equation, I have been able to map the T x y z and t . **This is now (())**. So, I have governing equation is there; boundary conditions are there. I have solved this. Again, I would repeat say mention here that we cannot solve it analytically, we have to solve it through numerical techniques.

So, assuming that, we can solve it one equation, one unknown. Given the thermal conductivity, given the source term, given the properties, it is one equation, one unknown. The unknown being T which is called the dependent variable and x y z t are the independent variables, and if you solve this equation, we can get the temperature field.

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And at a fixed value of time, fixed value of y , fixed value of z , I can plot the T versus x distance, and the slope of this line which I can very easily determine is nothing but and this itself is a measure, multiply this wave thermal conductivity becomes the flux, and

then, I will know that, yes, I have from this point to this point at what rate, it is really moving.

So, prediction of temperature, therefore, I can say the how can I find out the optimum reheat time. I will know for example, my knowledge of heat transfer tells me that well, the rate of heating at the center is going to be list, because it is the heat has to reach from the furnace to the surface, and then, it has to go inside at the solid and this is the conduction process which is a molecular process, and of the three mechanisms of heat flow which I have indicated here, heat transfer way conduction that is for this process.

So, therefore, I can say that this center point, if my target temperature that I want this is my optimum reheat temperature of 1100 degree centigrade, then I can say that the surface is going to achieve this reheat temperature like this and the center is going to receive the over the reheat temperature like this.

So, this is the center point of the ingot and this is the surface of the ingot. The surface is going to be heated much more rapidly; so, it is going to heated up rapidly. As you see, the slope is more steeper, and therefore, it will approach the desired reheating temperature which is the equilibrium temperature, which is the furnace temperature actually.

So, the surface is going to attain that uniform temperature quite rapidly. On the other hand, the center is going to attain the reheat temperature or the optimum furnace, the equilibrium furnace temperature at a much later stage. So, by monitoring the temperature of the surface, so, I can now having solve this equation, I will plot temperature at the center as a function of time from my numerical results, and then, tell, well it is after this time for certain such furnace condition, I am going to get an optimum uniform temperature also my solid and the time required to attain that is going to be the desired or the optimum reheat time.

So, conduction is very important for us and conduction, for example, whenever you will have solids, conduction is the dominant or the only mechanism of heat flow. Therefore, you will have to know the physics of heat flow. At the molecular level, the governing equations, the boundary conditions, etcetera, you know, quite exhaustively in order to

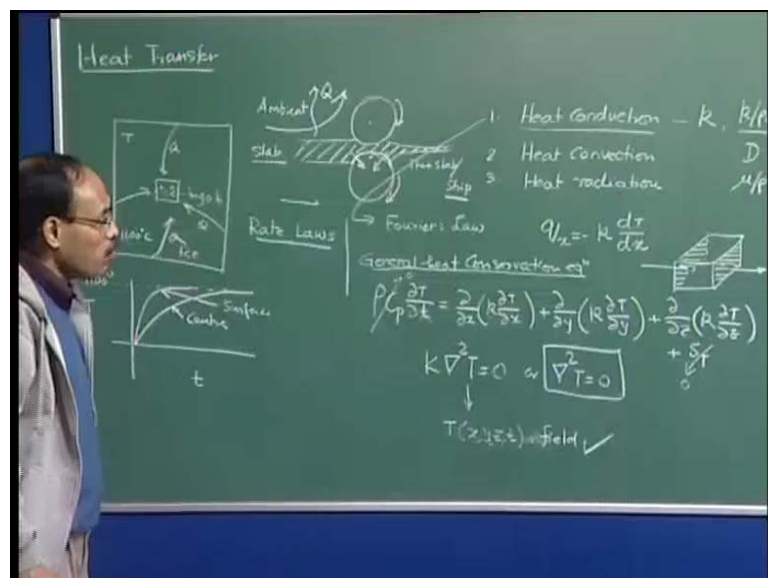
tackle the problem and make certain predictions. We should be using which will be of relevance to top floor engineers and the designers as well.

So, next we go on to convection. Now, convection, due to convection heat flows over a large distance, due to conduction, heat flows only over a small distance. You see, if you take a rod and then you put the rod, suppose the rod is 2 feet long, and then, one end of the rod, you put it inside the furnace.

Now, you are holding the other hand and the heat is going to take really much time to heat at this particular end, because heat has to flow within the solid rod molecule after molecule, one molecule will pass on it heats to the next molecule, next molecule, and that is the way it is going to reach and this is a very slow process.

So, you will see may be after half an hour or 1 hour, you are feeling some hotness at the other end which you are holding. On the other hand, imagine in a winter night, you are put in a convection heater in your room to one ((C)). So, the moment you put the switch and the fan as well as the heater in the blow are goes on, you will feel that the heat is immediately coming to you.

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And therefore, what you conclude that well, you know, it is the flow of heat assisted by fluid motion which is the fan, which is flowing the heat into the room is much more rapid than the conduction heat or the radiation heat. Also you can imagine will you have

a rod for example, usual old fashion type room heater. So, those rods basically heat flows from the surface of the coils to ambient; it is because of not conduction not convection but because of radiations which is a third mechanism, which I am going to discuss.

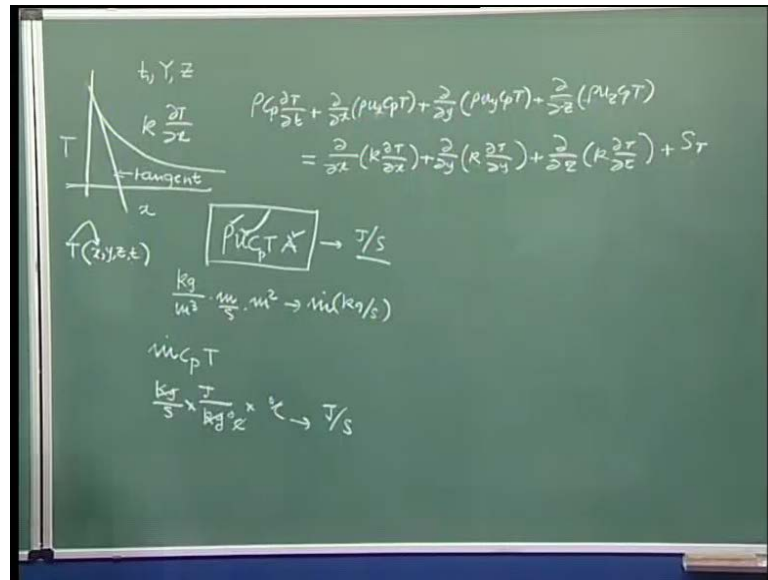
So, for example, air conditioner, convection blowers which you used in the winter nights, these are typical example and they demonstrate how heat can be blew, you know, from one place to another place or transferred over a large distance assisted by fluid motion.

So, now, if you set the fan speed at a lower level, you are, suppose you are sitting in a distance and the blower is suppose a few feet away from you. If you set the fan speed at low level, then you do not feel even though your desired temperature, the thermo state of the device may be at the unit temperature, but your controlling you are only changing the fan speed. So, the fan speed is low. You feel that the convection heater, the heat is coming at a very slow rate.

Now, if increase the fan speed, you will find that heat is coming to you at a much faster rate, you are feeling warm very quickly and this tells us that the rate of fluid motion or the speed of which the fluid is flowing, that is going to play a very important role. Determine the rate at which a heat is going to go or you transferred from one point to another point.

So, therefore, it is understood at this particular stage, that before we can talk of heat convection or conductive heat transfer theory, we must understand that we have to have some knowledge of fluid flow. If you do not know, at what rate the fluid is flowing? In that case, you will be not able to predict the rate of the convective heat transfer.

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So, we can say that if we take look at this particular term multiplied by area, so, what is rho? rho is kg per meter cube; u is meter per second; area is meter square. So, this term rho, u and A, this basically is nothing but meter cube meter cube cancel, so, this is actually the mass flow rate which is kg per second.

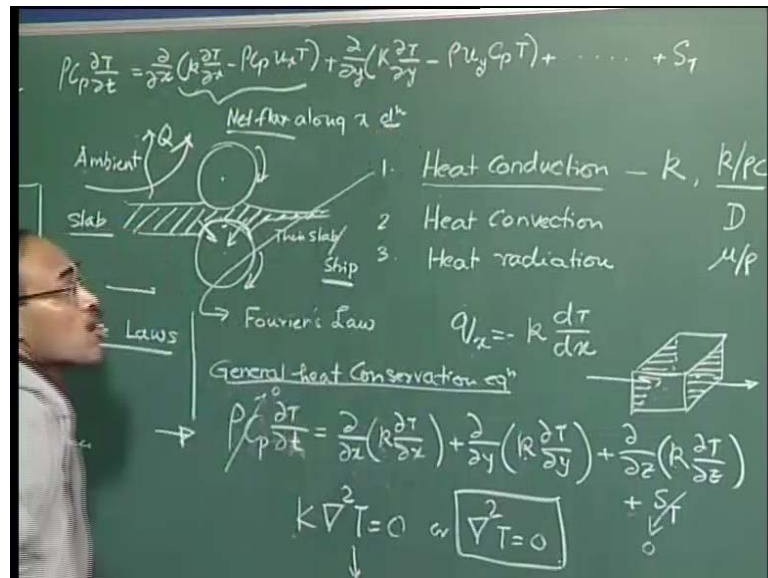
Then, we have mass flow rate into C p; this is C is C p into T. So, this C p is what we have kg per second, and then, we have C p which is joules per unit mass per degree centigrade and this is per degree centigrade. Therefore, this is kg and this is this and this term basically tells us this is the rate of, this is joules per degrees joules per second. So, the rate of energy transfer that is what is this particular term.

So, this is the rate of energy transfer because of what? Because of fluid motion. I have not taken the differential temperature. I have taken the temperature of the fluid and this is now, you know, there is a fluid motion, and therefore, this term, the rate of heat flow joules per second which is the dimension of this particular term and this joules per second, and at the rate of energy transfer from one point to another point is so late because of the fluid motion.

And therefore, it is understood while I may, you know, have knowledge of rho, I may have knowledge of C p; I may have knowledge of temperature also by putting in a device, but unless and until I have some knowledge of the fluid flow, I will not be able to

quantify whether it is 5 joules per second or 10 joules per second. Therefore, knowledge of fluid flow is a prerequisite to the calculation of convective heat transfer rates.

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While in conduction as you have seen, there is no fluid flow involve. So, the rate law is a Fourier's equation and we can therefore solve the governing energy equation, and without in the knowledge of fluid flow, because there is no fluid flow within the solid, we should be able to predict, but before we can address convection. Therefore, we have to know about the fluid flow.

And how do you know fluid flow? We know fluid flow either through experimental techniques to insert a flow and find out the velocity or as I have indicated in the, in the previous lecture on fluid flow, that we have governing equations and we solved those governing equations which are nothing but Navier-Stokes equations.

And once we solve the Navier-Stokes equation, we have a very good idea about the fluid motion in that system itself, and having obtained the fluid flow, having obtain information about flow in the system, we should be able to calculate convective heat transport provided. We apply a similar energy conservation equation.

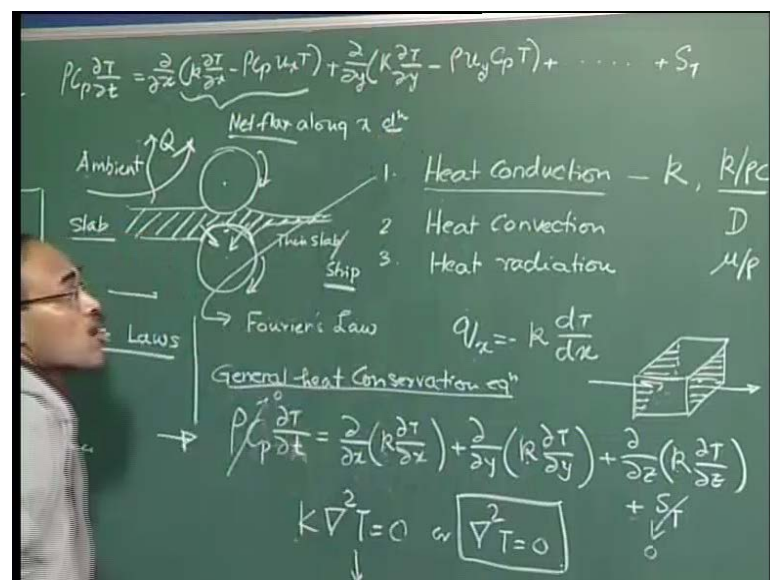
Now, the equivalent equation and you most appreciate, then the subject is getting highly quantitative now and all this would be useful, you know. So, we would like to predict the melting rates of deoxidizers, melting rates of allowing editions, heat flow through the

refractory's, heat loss to the finite small. All these theories are going to be useful, and we see, we have, we talk of, you know, we were assigned here talking of energy balance mechanistic approaches, we are talking of differential equations; we are talking of boundary conditions and subject is getting highly quantitative and complex as well.

So, if I now do a similar energy balance, so, what we have seen? We have weight of net rate of change of temperature of the control volume is because of heat flow, allow, you know, in net heat flux of heat in three different directions, but within a liquid, when heat flows from one point from another point, we have both heat conduction and heat convection both are operational.

Of course, if the transfer is occurring over a very large distance, this convection that determines so far diffusion or conduction, but nevertheless it is not correct to neglect conduction. Particularly as you will see that, suppose we are talking of flow of fluid near a wall. Now near a wall, what happens? The flow of the fluid slows down, because the wall is stationary, and as I have indicated when I was talking about fluid flow that along the solid wall, all the components of the velocities are 0.

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So, if the velocities are 0, so, near the wall, it is predominantly conduction. That is going to be important, and the other hand, if you move from the wall to the bulk of a system, the walls are here on both sides and we are talking of on the centre medians of wall here

is very little, and here, we will see one point to another point, heat flow will be by on that predominantly by convection.

So, therefore, when you talk of, if this element is a fluid element, so, and here, we trying to make an energy balance expression. So, within this control volume from one point to another point, heat will be flowing both because of conduction as well as both because of convection.

So, therefore, we can understand that if I have to write a balance equation, energy balance equation for a fluid element where heat is flowing because of both conduction and convection, I will not have fourth terms on the right hand side, but I am going to have seven terms x direction, one conduction term, and on top of their conduction in x direction, heat will be flowing by convection also. Y direction, heat is flowing, for example, here only due to conduction, but now, I am talking of a fluid element. So, in addition to that, I am going to have convection and flow is in the z direction.

So, while the governing equation for a reasonably descriptive or reasonably realistic governing equation for a control volume in conduction for heat flow due to conduction may contain only one, two, three, four, five term. You can see that if I am talking of conduction plus convection as the two modes of heat flow, in that case, I may have five plus three more terms, all together eight terms.

And now, we can once I write the equation, you will be able to appreciate at the similarities between conduction and convection. So, let me just write down. So, I have the same term ρC_p . This is the accumulation term that I have basically we write it like this. So, we have this now represents the net heat flux; this is the conduction heat flux; this is the convection heat flux and this represents the net heat flow due to conduction along the x direction.

And similarly, I can write the y direction equation. This is the conduction heat transfer in the y direction and this is now the velocity along y direction. I should write C_p also, and then, T and then I have desired directional flux and then I have the volumetric heat source term.

So, this equation typically is also written in this particular form. This is one way of writing where this is the net flux, and net flux, as I have mentioned, net flux along x direction, because flux unless is specify that reaction flux has really no meaning net flux along x direction.

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T, Y, Z
 T
 $k \frac{\partial T}{\partial z}$
 \rightarrow tangent
 z
 $T(x, y, z, t)$
 $\rho C_p \frac{\partial T}{\partial t} + \frac{\partial}{\partial x}(\rho u C_p T) + \frac{\partial}{\partial y}(\rho v C_p T) + \frac{\partial}{\partial z}(\rho w C_p T)$
 $= \frac{\partial}{\partial x}(k \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k \frac{\partial T}{\partial z}) + S_r$
 $\boxed{\rho u C_p T} \rightarrow J/s$
 $\frac{kg}{m^3} \cdot \frac{m}{s} \cdot \frac{m^2}{s} \cdot m^2 \rightarrow m^2 kg / s^2 \rightarrow W/m^2$
 $\rho C_p T$
 $\frac{kg}{s} \cdot \frac{J}{kg K} \cdot K \rightarrow J/s$

So, finally, I can write this equation in this particular form that well $\rho C_p \frac{\partial T}{\partial t}$. I bring in this negative term to the left hand side, and then, I say it is like this. I have the y direction term; I have the z direction term. Then, I put equality term and I write down the same C terms which I am having here as a conduction term. So, maybe I should write the third term so that you are going to get confused here. So, this is u_z , - the z component of the flow - but the difference between this is I have rearranged, I have brought this negative term on the left hand side.

So, this is the rate equation. If I have to now, so, this is again you can visualize it to be one equation and one unknown previous conditions stay well ρC_p and k are known, thermal heat source is known. So, what is the unknown here? The unknown is T only one unknown; one equation I am talking about only one unknown divided $u_x u_y$ and u_z are known.

So, given the velocity field, remember this, given the velocity field, given the thermo physical properties of the material. Under consideration, given the heat source term this corresponds to one equation and one unknown. So, therefore, if you have the right number of boundary conditions, incidentally this equation and this equation requires the same number of boundary condition, because the highest order of derivative is through there also an x direction, here also along x direction.

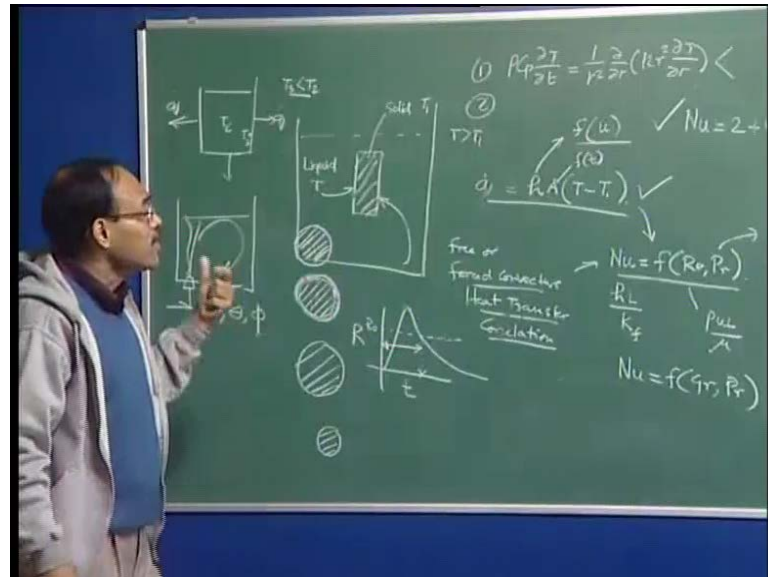
So, I would say that given the right amount of boundary conditions, given the thermo physical properties, this is one equation and one unknown, and then, I should be able to calculate or solve this equation and obtain the temperature fields, and once obtained, I obtained the temperature field. What I can do? I can again make a plot like this draw the slope and find out that what is the net flux because of conduction and convection together. So, looking at this particular equation, I would like to meet the one important point for you to note that along x direction, we have convection; along x direction, we have conduction.

Now, this is a second order derivative; this is a first order derivative. This essential implies that if I take a solid rod, in which, conduction is only mechanism. Physically this means, if I take a rod, and in this rod, as that conduction is the only mechanism. If I keep the center part with a matchstick, the heat will flow along the rod in this direction also; heat will flow in this direction also. So, that is the meaning of you can go in this direction also; you can go in this direction also, but now, imagine heat is flow in this direction.

So, if the fluid is flowing in this particular direction, so, for heat to flow, the large distance opposite to the direction of the flow is going to be very difficult. So, this is the one way behavior of the term. This is the first order derivative. So, the condition at a point, you will determine that what is happening on this side of fluid. It is not depend on this side. So, there is the fluid is flowing on this particular direction. The heats at this particular point will dependent heat. What is here? How much heat is here? That will determine how much heat is flowing here and not what is there, but on the other hand, if I am standing here, if I heat this point and this point, my temperature may dependent on at what rate, I am heating here; at what rate, I am heating here.

So, this is basically the second order derivative, and what I have said about the nature of the flow in one single direction that you know the downstream point has no role and that is refracted in the first for the first order derivative. Now, solving this equation, therefore, we will require I have to solve for the u_x , u_y and u_z field. Solve the velocity field first and use that velocity field to solve this particular equation.

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So, it is combustion task. Now, let us imagine I have a solid, what is my objective? I have a solid which I have the box in liquid. So, this is a liquid box for example, and I want find out that at what rate, it is melting; so, that means, from the liquid, the heat is going to the solid surface, and as I have mentioned that heat will be flowing to the solid surface by a combined mechanism of conduction convection.

So, I have to solve this equation in order to find out this particular slope at the temperature profile in the surface, and then, draw a tangent to find out the slope. So, it is going to be a really combustion task in order to find out the rate of. So, I have liquid here which is at a temperature of T and this is a solid which will undergo melting and this is at a temperature of T_1 ; so, T is greater than T_1 .

So, the heat is continuously flowing to the surface, and in order to find out the rate of melting, I have to know the rate at which the heat is going from the liquid to the solid, and therefore, that rate can only be determine provided and map the temperature

everywhere within the system, and at the surface, I draw that kind of a graph which I have shown here. Draw the tangent to the temperature versus distance profile and there by determine the net flux. So, it is going to be very complex task.

On the other hand, we can there are empirical ways to find out that what is, how much of heat is really flowing to the surface, and there we say that the rate at which heat is flowing to the surface, it depends on it is a heat transfer coefficient multiplied by area and then I would say T minus T_1 .

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t, y, z
 T
 $k \frac{\partial T}{\partial x}$
 tangent
 x
 $T(x, y, z, t)$
 $\vec{q} = -k \nabla T$
 $\frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{m}}{\text{s}} \cdot \text{m}^2 \rightarrow \text{W} (\text{kg/s})$
 $\dot{m} c_p T$
 $\frac{\text{kg}}{\text{s}} \times \frac{\text{J}}{\text{kg}^\circ \text{C}} \times ^\circ \text{C} \rightarrow \text{J/s}$

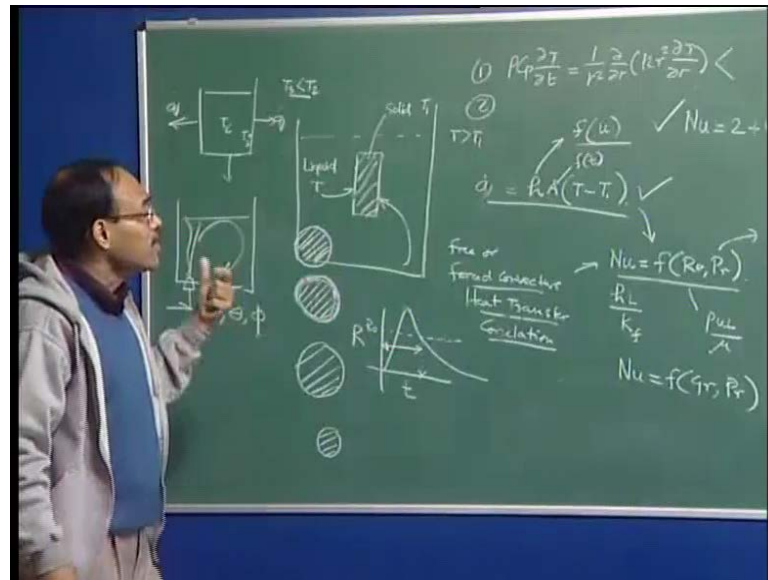
$$\rho c_p \frac{\partial T}{\partial t} + \frac{\partial}{\partial x} (\rho u c_p T) + \frac{\partial}{\partial y} (\rho v c_p T) + \frac{\partial}{\partial z} (\rho w c_p T)$$

$$= \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + S_T$$

$$\boxed{\rho \vec{u} \cdot \nabla T} \rightarrow \text{J/s}$$

So, now, this h which is called a heat transfer coefficient. This is actually related to the fluid flow itself. Now, for very simple situation, people have found out that a similar very simple equation of this can be derive for a simple situation. Simple situation means I am talking about, say for example, I have a flat vertical plate, and then, the fluid is flowing passed it very nicely in a laminar flow mode everything is under steady state.

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And this particular condition, one can solve this equation by analytical means; that means I do not have to use any computer; I do not use any numerical methods, but I can solve it just like, just following the procedure of solving of a ordinary differential equations, and then, I can find out the rate and it is found out that the rates of heat flux under those conditions that from a hot plate, at hot rate, heat is flowing to a liquid which is flowing fast from vertical plate.

We can solve the convection diffusion equation which is a convection diffusion equation or convection diffusion of thermal energy. So, you can solve a simplified version of this energy in order to get the heat flux and it is found out that such a heat flux solution can be cast into this particular form itself, in which, this parameter which is called as the heat transfer coefficient is a function of the velocity of the fluid.

So, this result is obtained for some classical bench mark solutions by solving of this kind of a governing equation. So, this tells us that no matter what is the problem, you know, even, if you have turbulent procedure, turbulent flows, even if you have multi phase flows, even if you have unsteady state behaviors in the system, it is possible that we need not have to solve those in a complex situations, but to express the heat flow, we have, we will have to heat flow to the solid surface, or from the solid surface, we can express use this particular rate equation.

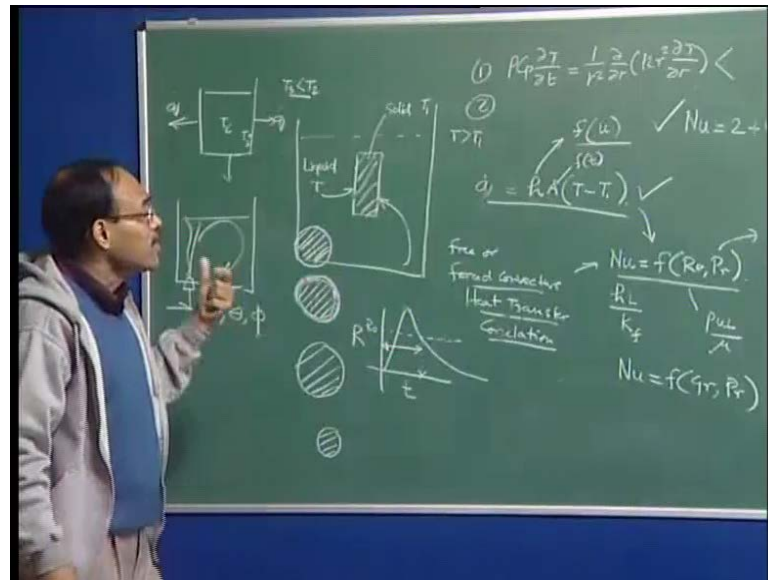
But here, the task is now that how do you obtain the heat transfer coefficient which is a function of fluid flow. So, influence of fluid flow is coming into the picture and it is through this heat h term which is small h . Now, basically the results are expressed in terms of per from the h is obtained. The results are obtained, for example, we say that in terms of dimensionless groups; so, nusselt number is a function of Reynolds number and Prandtl number.

You see your heat transfer, we are getting in and in into the heat transfer. These are, these are dimensionless number and this dimensionless number is nothing but heat transfer coefficient characteristic length scale and divided by thermal conductivity and you must remember that this thermal conductivity is the thermal conductivity of the fluid phase.

On the other hand, if you are talking about bio number which looks similar to Nusselt number. They are the thermal conductivity is the thermal conductivity of the solid. Reynolds number as we all know it is, so, Reynolds number can only be calculated provided we have some idea of the flow, and prandtl number is again the thermo physical property; so, it is α . So, it is actually I have written it in the first way. So, this is kinematic viscosity by thermal diffusivity which I have just now introduced.

This will be known to us, because this is the properties of the fluid. These terms - characteristic length and thermal conductivity of the fluid is also known to us. So, only uncertain parameter here is u , which is not known to us. So, somehow, if you can acquire the value of velocity in the system, we can apply a correlation like this and this type of a correlation is called forced convective heat transfer correlation. So, if you know the velocity, then use in a forced convective heat transfer correlation and find out the value of h ; h is unknown, h we have kept on the left hand side.

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So, this is treated as a dependent variable and these are independent variable. So, substitute the value of u into the right force convective heat transfer correlation. Obtain the value of h , substitute the value of h here and these temperatures which are known the melting temperature and the liquid temperature interfacial area is also known. So, we should be able to calculate the rate of heat flow to the surface.

Now, well, we have force convective heat transfer and we have free convection heat transfer also. There is, there is distinction forced convection means, for example, if you are the convection heater air conditioner, these are case of forced convection. So, if you have a fan moving on in a room and that causing heat to be distributed all over. So, we are using forced convection. So, therefore, in the context of steelmaking, I would say if I have a ladle which contains, you know, and I have a porous flag here and I start the contents of the ladle and there is a motion and this motion is going to add the distribution of heat in the system.

So, this is heat will flow from one point to another point due to convection of course and that convection is going to be forced convection and forced convection the origin of force convection here is gas injection. On the other hand, I can have a ladle with no gas injection, but that heat is going to be lossed from all surfaces. This may going to give raise to some differential in temperature T here and T_1 or T_2 here and T_3 here and we can except the T_3 is going to the lower than T_2 , because T_3 is closer to wall.

So, now, we can understand that temperature and density are related. So, higher is the temperature, then we can say lower is the density; lower is the temperature, higher is the density. So, differential temperature will cause differential density and heavier density material will sink down; lighter density material will raise up and this will create a convection current. The origin of which is the differential temperature of fluid and this sort of a convection which precipitates in the system or generates in the system due to temperature difference is called free convection heat transfer. So, we have not forced, but then, we have a free convection heat transfer.

So, we either have a free or forced convective heat transfer correlation and this sort of a correlation is for forced heat transfer correlation, force convection, and Nusselt's number as a function of Grashoff's number and Schmidt Prandtl number is a free convection heat transfer correlation. So, depending on the scenario, so, your job, why do you want to calculate the net rate of heat flow? What is your job? It boils down to not to solve this equation, not to solve this equation, but to first, identify the applicable heat transfer correlation to your case.

Try to understand whether is a free convection or a forced convection. Having understood that it is a forced convection, you understand the geometry of the scenario. Whether it is flow over a spherical object, whether if there is strong one directional flow or multi directional flow, whether it is laminar flow or a turbulent flow, you told to understand the process or the flow and the heat transfer well.

And then, you try to look at the literature and find out that whether you have a correlation available to the situation which you are interested in. If you are, if you denote, find such a correlation available, you have no option, but to carry out experiments and find out that what is the applicable correlation itself. For example, we have seen this correlation may be when you have done a heat transfer course. Flow past is fear. I think it is $0.6 \text{ Reynolds raise to the power half and Prandtl raise to the power } 1 \text{ by } 3$ which is called the popular range marshal correlation.

And this two shows that it is applicable to a spherical geometry. All heat transfer and mass transfer correlation as we will also see, you know, for spherical geometry, the distinguishable feature of the correlation is that it has 2 plus something and this 2

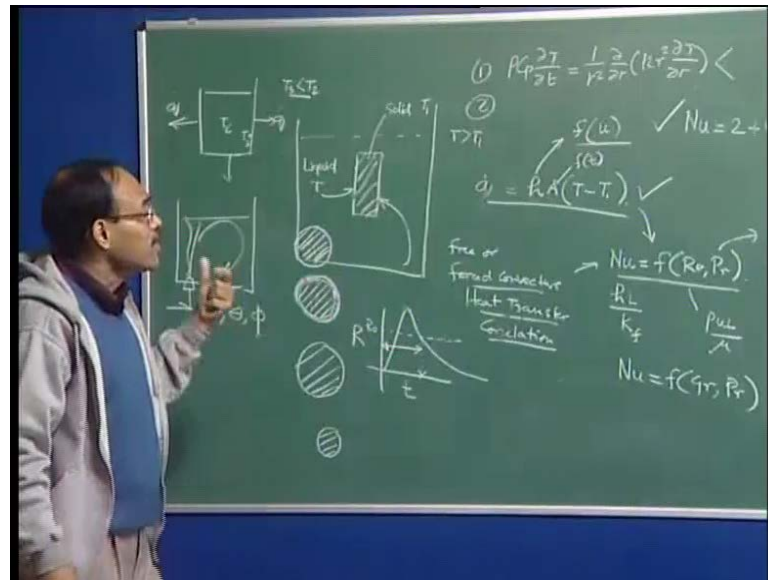
corresponds to pure conduction or pure diffusion situation. This component comes because of the presence of Reynolds number which is the forced convection scenario.

So, therefore, we can say that if you have the correlation already available flow passed a spherical object, it is to be understood that for all practical situation that we are interested in under steel making condition, a correlation may not be available in that way really forced problems to us. So, therefore, assuming that a forced convection correlation is available. Next would be to find out that what is a meaningful velocity scale that we have to use in that particular correlation.

So, in this correlation, the moment I say that well the velocity in the Reynolds scale is known to be it becomes a trivial exercise, because on the left hand side, now there is only one unknown and that unknown is the heat transfer coefficient. So, having obtained some information on the flow, I can calculate Reynolds number. Prandtl number is already known and fixed, and therefore, I can calculate Nusselt number from which I can calculate the heat transfer coefficient. Substitute the heat transfer coefficient in the rate equation, and then, I should be able to calculate the convective net heat flow to the solid object or the net interaction thermal interaction between the solid.

Now, during melting, we must understand that solidification as well as melting is going to take place particularly when you project a solid into liquid. Now, for example, if we, if we project this solid into a liquid and this solid has a room temperature is a deoxidizer element or an alloying element. So, typically you form a solid crust. Now, this crust when the material is under liquid submersed in the liquid, this crust melts back.

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So, initially, there was no crust. Some solidification of crust takes place. Then, the crust melts back, and once the crust melts back, then the original solid is going to be exposed to the liquid, so, it goes like this. So, the solid is coming. I can say this is a solid. The next stage is you are formed the crust and the third stage is may be again the solid is exposed. So, this is initial radius; the radius has increased; the radius has decreased. The original radius, so, here and here, the radius as that same and this duration from here to here essentially tells us about the shell growth and shell melt back period, and then, finally, the size decreases and eventually the object melts and vanishes into the system.

So, the R versus t curve radius curve, in this case is going to be something like if this is the original radius of 0. So, for some distance, the radius goes like this increases and then the radius decreases. So, therefore, this is the duration. You see, so the radius, as a function of time, the radius going to increases radius attains a maximum value, and at that particular point, the formation of shell is complete. The shell now starts to melt back and radius decreases, and at this particular time, the original sphere is released. So, this is the duration which corresponds to shell formation and shell melt back, and this duration from this point onwards to this, represents now the melting time of the allowing condition itself.

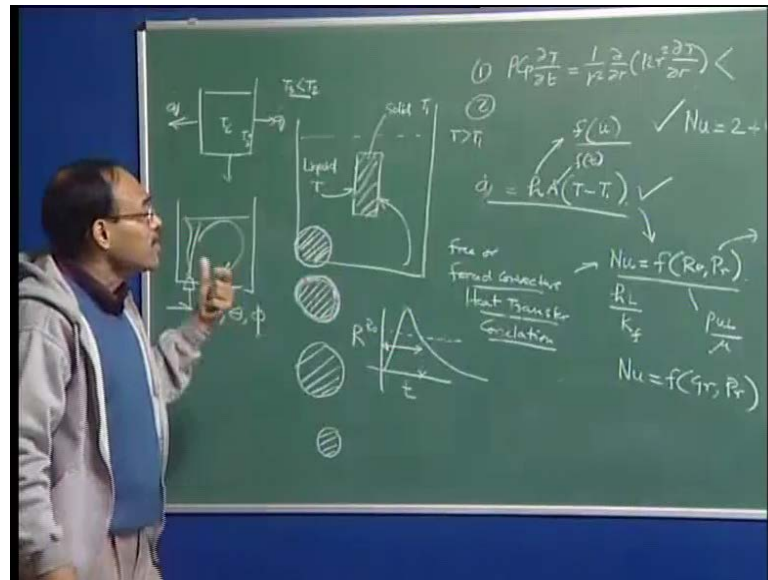
Now, therefore, in order to calculate this, we have to take into account the latent heat of solidification as well as of latent heat of formation and also we must solidification and melting. One is positive; another is negative, and we must also first understand that the geometry of the object is not constant all the time.

So, this area that I have to use in the rate expression will not remain constant during the melting scenario, this is going to continuously change as I have indicated. So, if you take πR^2 which is the our $4\pi R^2$ square, which represents the surface area of a sphere, it is understood that $4\pi R^2$ is going to continuously change during the process. So, taking an initial area is going to be highly misleading in this particular case; so, this A is going to be a function of t .

Now, therefore, the task is really not as simple as obtaining the value of heat transfer coefficient, and then substituting it in this equation and this and this is called a typically moving boundary problem. When you have a solid, so, size is changing because of the processors taking places in the system, and then, the boundary or the domain in which you are interested, because you want to find out the complete melting time of this particular edition. So, you want to find out the rate at which it is shrinking, and therefore, the entire time duration has to be taken into account in order to predict that time realistically.

So, we have to now take into account that for example, if I say that I want to write down the equation, heat flow equation within the solid, so same conduction equation that I have written will come here ρC_p , and if I say that, well, it is only one dimensional heat conduction within the solid, it is there is a radial symmetry. So, heat is flowing along the r direction; θ and ϕ are not important. Spherical coordinate system is what? Spherical coordinate system is r θ and ϕ .

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So, if here is theta and phi symmetry, I can say heat is flowing in the radial direction and this is actually $1/r^2$, this is the governing equation k/r^2 . Now, as far as this equation is concerned, I require I have to do some condition that at r is equal to I require that, because this is a derivative in terms r ; this is a second order derivative with respect to r . Therefore, two conditions on r . I need a condition at r is equal to 0, which is very known; this is a symmetry condition, and I need a condition at r is equal to some point, but that the radius itself is changing. As a function of time, I will not be able to apply boundary condition to this equation in an exact manner.

So, from somewhere, the value of r is to be calculated and that is an energy conservation, statement of energy conservation at the surface of the solid that the net rate at which heat is flowing into the solid plus heat which is evolved or consumed here because of the melting and solidification phenomenon plus the heat which is flowing because of conduction. All three things have to be taken into account in order to predict the temperature profile within the solid or melting time completely.

Note that in the first equation, no information is there on the fluid flow itself. How does the fluid flow part get into the equation or the process mathematically? It is because the flow of heat at the surface, I require boundary conditions. So, through the boundary conditions to that equation at, for example, if I say that I have r is equal to r_1 which is at this at a particular instant, suppose this is the radius of the vessel, I can say the rate at

which heat is flowing, I can be given in terms of this heat transfer coefficient which is the conductive heat transfer coefficient.

So, through the boundary condition, I will be able to incorporate the actual fluid flow. Therefore, this equation alone I cannot solve in melting problems. I have to solve this with an expression for the change in size, and therefore, additional energy conservation equation will be necessary; so, two equations and two unknowns. What are two unknowns? The one unknown is the temperature and the other unknown is the changing geometry of the object itself.

So, therefore, I will have a specific energy conservation equation as of the heat conduction equation. These two equations I will couple together, and then, I have two equations and two unknowns, and as a result of which, I should be able predict temperature as well as radius as they are changing as a function of time and based on such, I will be able to calculate that what is the complete melting time.

So, the subject is getting extremely complex, and if you can remember, these are single objects immersed in a fluid, if you have multiple objects, multiple features turbulent flows and all these kinds of things, the task is really complex and you have to have very good background in heat transfer. In numerical techniques, in fluid flow theory, in order to calculate a convective heat transposition fairly we will just take it.