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## Indian Institute of Technology, Kanpur Module No. # 01 Lecture No. # 40 Modeling and Measurements

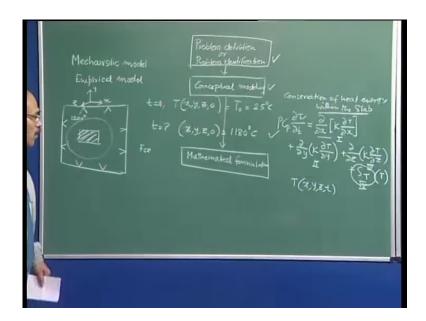
So, as I have indicated that mathematical modeling find wide spread applications in process analysis, in design, also in automation or control, finds quite specification in artificial intelligence. And finally, if you are thinking of optimizing a process in terms of some operating variables, you got to have a mathematical framework based on which we can carry out the automation task.

Now, automation or process control as I have indicated, is done on a dynamic fashion or on online fashion; that means, with the process, the mathematical model interacts continuously, and the mathematical model provides useful feedback for controlling of the process, as it could be in the case of melt level control or the role pass gap adjustment and so on.

On the other hand, process analysis, process design and process optimization are done on an offline fashion; that means, the steel making process is going on in the industry, while you can sit down in your laboratory with the aid of a computer, you can carry out some process analysis, some design and make certain recommendations to the shop floor personnel for improving the performance of the process.

About the artificial intelligence, the mathematical model can provide useful input to the artificial intelligence structure, or it can also obtain useful input from the artificial intelligence structure and embodied in the mathematical model.

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Now, how many types of mathematical models generally we encounter in steel making? Broadly speaking, we will say that we have a mechanistic model and empirical model, there are many subdivisions, which may be possible. Because, empirical models, for example, I can classify empirical models based on the stand point from which an empirical model can derive. For example, I can derive an empirical model purely based on regressional analysis; I hope you have heard this term.

On the other hand, I can carry out, I can derive an empirical model based on some dimensional reasoning or dimensional analysis, as well as some experimental data. Finally, I can derive the empirical model also from the view point of an artificial intelligence procedure or technique, are such as we can apply genetic algorithms, we can apply artificial neural nets and derive; so many further subdivisions are possible.

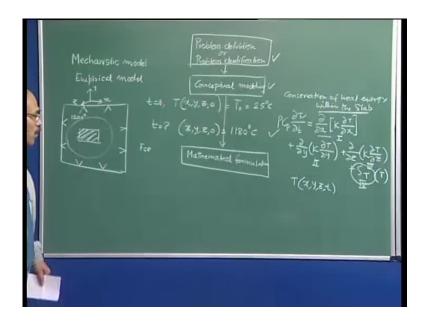
Mechanistic models, for example, we can drive it from various stand points. It can have... These are basically based on, as the name suggest, these are based on certain conservation principle. And people may subdivide mechanistic models also, say that which has a different kind of a connotation in the context of population balance, but nevertheless we would be talking about some conservation principle, some balance principle. On the other hand, the empirical model is going to be derived purely on experimental stand point, embodied in experimental data.

Because mechanistic model...So, the purpose of divisions I am not going to discuss here, that is not our immediate interest, we will consider models under only these two categories that either it is a mechanistic model or it is an empirical model as far as we have concerned. Now, mechanistic models, for example, they are based on the conservation principles, conservation of what? Conservation of mass, conservation of momentum, conservation of energy, conservation of population balance and all these kinds of concepts can be used to formulate the model. Because, mechanistic models are based on solid scientific principles, as the name itself indicates as the fundamental laws of physics these models are based on. So, therefore, a mechanistic model, a formulated in India, will equally work, provided certain, you know we have been able to derive the model realistically in other countries as well.

But for example, in empirical model, which we may develop, and which may be specific to a particular plan, may not work for all different plans, because empirical models contain experimental data points, which are specific to a given plan or given set of condition, so there lack universality. On other hand, mechanistic models are sufficiently general, they are not governed with geographical boundaries, they are sufficiently general and they work across the geographical boundaries, provided we can formulate the mechanistic models, correct.

For example, I can say that we can carry out a heat balance, and write a heat conservation equation, which can be very complex equation in terms of the geometry or the process itself, not a simple one-dimensional equation. So, I can write the heat conservation equation and say that (()) this represents the mechanistic model for a given process. Empirical models on the another hand, basically they are, look at them like a algebraic equations; on the other hand, in mechanistic models, most of the time these will appear as the partial differential equations to us, as we discuss this further, it will be a clear view.

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Now, I have also indicated that when we try to develop a mathematical model, we have to first define the problem for what we want to develop. So, nearly saying, I want to develop a model for a ladle, I want to develop a model for continues casting is not enough. So, the most important task is the problem identification, or problem definition, this is the most important; problem identification or problem definition, this is the first step in model, doing mathematical model... doing...

Let me illustrates with an example. Suppose I have a furnace, this is a furnace chamber, suppose it is a gas fired furnace, so you have burners from all sides. So, this is a top view, I am looking from the top, so that is the cross section of the furnace, and we have seen that there are burners there everywhere, from all sides, these are gas burners.

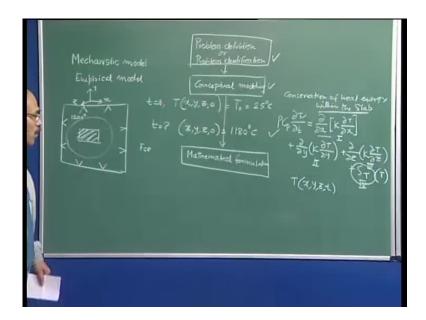
And my objective could be I have a slab which is sitting here, which is continuously cast slab, I have put that slab. And the interest here is may be to find out that after first time, following its insertion, the slab will attain certain temperature, because slabs basically... Once you have continuously cast slabs form, you take them, after sometime gap , into to heat furnace, heat them up, bring them up to the conduction temperature and soak them in a furnace, so that they are homogenous. And then you transport them for further processing in rolling mill, etcetera. So, therefore, I may have a furnace, which may be running at a temperature of 1200 degree centigrade. I may have inserted the slab at a time t is equal to 0, when the slab temperature, may be everywhere, all points, because this slab is a three-dimensional geometry, so temperature at x for all value of coordinates, if I defined the system by x y and z coordinates system, in that case, it represents that at t is equal to 0, so that is what it is, x y z and t, the temperature everywhere is T0, which may be say 25 degree centigrade

That means the slab under question, I have kept it all in continuous casting in the bay area, and it has pull down to room temperature, and now I am putting it back into a reheat furnace. And I want to find out that, give me the time, at what time? I do not know, for all point, the temperature becomes say 1180 degree centigrade. Just putting a reasonable value, so I want that the slab to be (()) in terms of its temperature, that everywhere, at some point of time, the temperature is going to be 1180 degree centigrade, I want to find them.

One more would be to direct the radiation pyrometer through some windows on to the surface, but then the radiation pyrometer will give you the surface temperature, you will not be able to find out that what is the exact temperature with that the slab is all through homogenous or not. So, the problem definition is clear here, I want to find out the homogenization time for a given slab, inserted with in initial temperature filed, to obtain a final temperature, all through of about 1180 degree centigrade; so this is the problem.

Now, once I have identified the problem, next comes the stage of what is called as a conceptual model. So definition...And this is the most important part, actually is a problem definition, identification, how clearly can you defined the problem? Mostly, with more certainty you will be able to define the problem, more easy it will be to formulate the mathematical equation.

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That will conceptual modeling... Conceptual modeling is a state, which is before the formulation, so after this, we have formulation - mathematical formulation. Before mathematical formulation, before you can write down, this is the equation that discards the temperature within the object. And if I solve this equation, which is essentially an unsteady state equation, unsteady state because it is time dependent, and it is a three dimensional equation that we are considering about, we are talking about heat flow in that system, to be talking about thermal energy conservation.

So, therefore, before we can formulate the equations actually, whether in the equation, in which coordinate system we will write the equation, how many times will this equation contain, all these things, we have to do little bit of conceptual modeling, which is very important part.

For example, in conceptual modeling, you will try to address. As well, I have the slab geometry, what is the coordinate system that I will like to use? For example, equations could be derived may be in terms of, depending on the geometry, may be you have spherical geometry, may be you have a cylindrical geometry, may be you have Cartesian geometry, so based on that we are going to derive the equation.

So, you can say that is the object, really two-dimensional object, or it is a threedimensional object? Hardly, you can say that, well, how do I solve this problem? I mean, do I have to really solve the temperature here also, within the furnace environment and within the solid also? Or it is just that I can assume that the furnace temperature everywhere is 1200 degree centigrade, heat is being transported to the surface of the solid slab by radiation and then I formulate the problem?

So, series of questions need to be asked in order to get complete picture of what you are really looking at. And it is only when you have done this, you are in a position to write the governing equation or you can formulate the problem mathematically. For example, I can say, I can go in the reverse direction, if I write down that well equation, it goes something like this. This is the heat conservation equation, and then I write this particular equation, these are the transport of heat through conduction within the solid. So, this is conservation of heat energy within this.

This is the unsteady state term, because the temperature initially is 0, it is gradually changing and attending towards, approaching 1200 degree mark, so this is the unsteady net rate of accumulation, this is the conduction along the x direction, this is the conduction along y direction, conduction along the z direction, and then we have the source term.

So, this essentially, once I write this equation, you will be immediately know that I am considering an unsteady state phenomena, I am considering a three-dimensional phenomena 1, 2 and 3, and I am have also considering that there is a finite heat source term in the system. For example, this may be initially ferrite pearlite steel, plain carbon steel containing ferrite and pearlite, and we know that is going to heat the sample, the ferrite pearlite will combine inside proportional to give rise to austenite.

So, solid state phase transformation reaction may take place within the solid, and that solid state phase transformation reaction may produce some amount of heat, and that heat must be accounted for in the governing equation. So, looking at the equation, you can backtrack and find out for what conditions we have written. Similarly, if you are trying to say now that I would like to consider, I go on a reverse direction unsteady state equation, I will consider three-dimensional equation phenomena, I consider Cartesian coordinate geometry, I say that this conduction which is taking place only within the solid, I am not bothered about the surroundings I am looking at, because my objective is

what I want to find out the time, what for the slab to attain a temperature of 1100 degree centigrade.

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So, this is not of my immediate interest for surrounding, it is the solid object, which is of my immediate interest.

So, therefore, I can say that 12 Cartesian coordinate three-dimensional source term unsteady state problems, and then the task become simple. It is unsteady state three-dimensional heat conduction with a finite source term. Of course, as a modeler, you have to now profile **it** expression for steel. You must understand this represents only one equation, and hence we can at the most have one unknown. And the unknown here is the temperature, which as the equation indicates a function of x y z and E.

So, we have to have some expression for the source term, and this source term itself may be temperature dependent. If you remember the iron carbon phase diagram, there you can see, you can look at the tie lines and find out that the proportion of austenite will change depending on hot temperature. Therefore this is a steel, is actually a function of time, therefore this is a non-linear heat source term, because unless you know the temperature, you will not able to predict, or solve this particular equation.

So, as far as the formulation is concern, we can now formulate the problem, I am going to come back to it. But note that because we are talking mechanistic model, and that I say

mechanistic models are going to give rise to partial differential equation always, we have to not only write down the equation, but we have to write the boundary condition also.

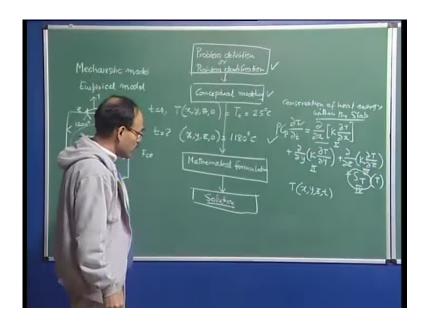
It is the formulation, which essentially implies that we have to write down the governing equation, we have to write down the corresponding boundary condition, and then only the statement of the problem is complete, then only we can say that yes we have been able to formulate the problem correctly.

This exercise can be taken for example in the context of ladle metallurgy. You can say that I have ladle gas injection procedure, and what I want to model there? Well, I have added a ferro niobium solid, and I want to find out that after first time ferro niobium gets completely dissolved in the steel bar. So, your problem definition is complete. Now, you have to look at well this salg (()), next we go to conceptual modeling. It is a cylindrical system, but the flag is located away from that this sub symmetry sources three-dimensional problems basically.

But, it is a multiphase problem, I have gas injection, I have a slag layer and I have melt, do I solve it as a multiphase problem? Do I solve it has the unsteady state problem or an unsteady state problem? So, all these question we are going to ask, and then we can say that I am having conceptually modeled the situation, I can say that because dissolution is a mass transport phenomena, I have to have you know the flow in the system known, so, therefore, I have to solve the fluid flow equation, having up end the fluid flow equation, I will solve the mass transport equation, because dissolution is a mass transfer phenomena.

And that is the way we going to formulate the problem. Once you write the conservation of momentum or fluid flow equation, conservation of mass or the mass transport equation, we will write the boundary condition, and then we will say yes, we have been able to formulate the dissolution of ferro niobium in steel, in a realistic manner, because we have been able to write down the corresponding equations, but differential equations, which you called as a governing equations, together which is the bounding equation.

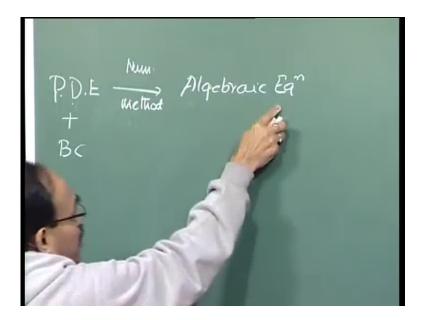
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Once you have mathematical model formulation, and then we are going to solve this equation solution, this is very important for us, and we will discuss this in detail later on. Solution, because this partial differential equations in most of the cases, you will not able to solve analytically. Analytically means, suppose you have to solve many differential equations, non homogenous equation, you have then written complementary functions, and simple ordinary differential equation, even unsteady state equations also through products solution technique, error function solutions techniques can be solved by hand, but these equations cannot be solved by hand.

So, you will not able to solve analytically, which means if you given a pen and paper, you will not able solve these equations, you have to resort to numerical technique to solve this equation. What does a numerical technique do? A numerical method basically converts an algebraic differential equation into a set of algebraic equation, and then we solve the equation.

So, the purpose of the numerical method is to convert a given differential equation into the corresponding algebraic equations, And then we require, once you have a set of algebraic equations, large number of algebraic equations, then off course we require a digital computer, otherwise we will be taking very long time to solve it. So, the set of algebraic equations can be solved with the aid of a suitable solution algorithm or a computer platform or computer program, whatever you may like to say.



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So, partial differential equation – PDE, and this gets to algebraic equation and solution, not just one algebraic equation, many algebraic equation it gets through, and between the two, what connects the two is the numerical method. One important point I would like to say that there are enumerable numerical methods available, by which you can translate a partial differential equation into set of algebraic equation.

So, therefore, the form of the algebraic equation is going to be different depending on what time does numerical methods you have applied to translate this differential equation, and this equations. For example, if I take a differential equation, use Taylor series approximation, I will get one type of algebraic equations; if I use different kind of algorithm, for example, if it is an ordinary differential equation, there are enumerable algorithm, like, I can consider Runge-Kutta method. I can consider Runge-Kutta gill method, second order method, fourth order method, Adams-Moulton method, so many, and in each case, the final algebraic equation looks different.

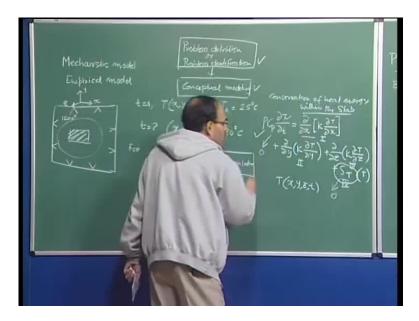
So, therefore, the solution of the algebraic equation are also going to be different depending on that sort of a numerical methods we apply, but we must understand that the

partial differential equation, together with the boundary condition, defines a problem explicitly, which implies that we have to have an unique set of solutions.

Now, since I have said that with different numerical methods, it will produce different algebraic equations and hence different solutions; this seems to contradict my statement, but mind it, that eventually, an algebraic solution, because when you write down the algebraic equations, from that the differential equations into algebraic equations, we solve the equations at different location in the system, which we called as a various nodal point.

So, when the large nodal points are used in the system, or the resultant number of algebraic equations is very very large, in that case, the solution is just quiet having different numerical methods, going to be identical and in that solution basically we will talk about the great independent solution. So, when you do mathematical modeling, it is very important for us to establish the great independent solution, which is the meaningful solution, which is the reflection of solution of the differential equation, together we did boundary condition.

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So, we solved the equation, and we can try now to get certain asymptotic solution, for which, I have analytic solution. For example, I have a three-dimensional equation that I have treated, now if I set this term to be equal to 0, consider that this term is going to be

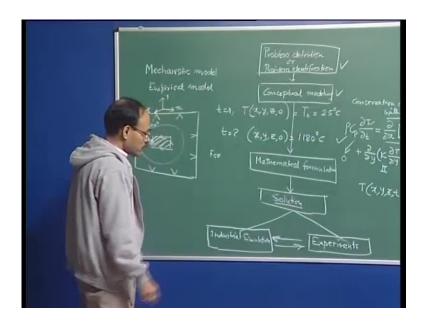
is equal to 0, that is the steady state phenomena with no heat source, then I have a pure steady state heat conduction in three-dimension, I refine all state that well, all surfaces are fixed at certain temperature a bench mark situation, which has nothing to do with the problem, I am going to... because, I solve the equation, but I have to understand, and I have to know also that these conversion is correct. The algebraic equation and the solution that I am going to get is a meaningful solution, is a correct solution. How do I test it? So, under certain limiting or asymptotic situation, I am going to solve these equations.

And a typical bench mark solution or a limiting situation is that we write down the complete model for this, based on the conversion and the solution techniques. And we say that in the computer program or platform, let us say this source turn to be 0, let us consider a steady state situation by considering this to be is equal to 0, we then find out that problem confers to a steady state three dimensional scenario. And then if it is further assumed that all surface are maintained at a constant temperature, need not be same, in that case, an analytical solution to this particular problem is possible, which has is given in when standard heat conduction takes place.

So, I can run my computer program, I can solve the equation, and for this particular limiting equation, I can compare with my text book solution. So, then I can say there is a reasonable certainty, that the programming that I have done is correct, because it is able to replicate many such solutions, which has given by other people or other sources. We can also say that I am not the first person to solve this equation; there may be many another people who have to solve these equations for certain different cases, or different boundary conditions.

So, I can also run my computer program, or run my solution, or the differential equation for certain situations, which have been relatively well investigated and reported by pervious investigator, and then I can say that I have developed the computer program, the computer program produce estimates of temperature, that very well agrees with those reported by various other investigators. So, I have some confidence that the computer program and the solution methodology that I have developed following problem definition, conceptual modeling and mathematical formulation is did correct, and gives reasonable estimate.

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So, having up to in some confidence about the numerical solution technique now is I am going to bifurcate. And at one stream, I am going to now run this program in its entire, now no more asymptotic solution. So, I am going to use the exact equation which I have written, I am going to write down the exact boundary condition, what type of boundary conditions are there. I am not talking it talking about it, but you know that every differential equation will have certain boundary conditions and initial conditions. For example, if you know a simple second order ordinary differential equation, if you integrate that, you will get two constants of integration, how do you derive those constants of integration? You obtain this constant of integration from the boundary condition.

So, similarly, we will have seven boundary conditions attach to it for each, there is a second order term, two conditions on x, two conditions on y and two conditions on z, and one condition on T. So, with seven conditions embedded, I should be able to solve that exact problem that I have been able to identify in the beginning. So, I can solve the problem industrial simulation, at the same time I have been able to, when I do industrial simulation I have been able to find out that, well, the inside temperature is this, outside temperature is this at various time.

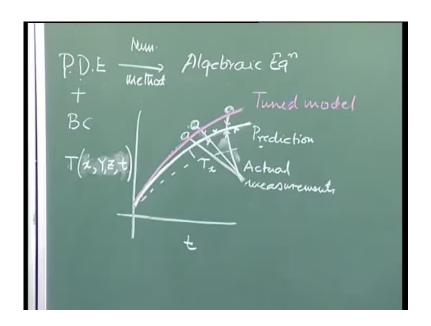
So, I want to check weather my results are correct or not. So, parallelly, along with industrial simulation, I can carry out some experiments. So, I can use techniques to

determine the surface temperature for example, and then I can compare the two. So, these experiments can be carried out for example, may be with a contact pyrometer or may be with an optical radiation pyrometer. So, I can measure from this distance what is the surface temperature at certain time. So, model can also predict, once you do the exact simulation of the industrial process at the time temperature history at various locations, this comparison will give us an idea that whether our model has been correct or not. As I have mentioned to you, there may be many assumptions involved in formulating the governing equation, in formulating the boundary conditions. For example, this heat source term, we can make various kinds of an approximation to formulate that how the heat is liberated because of the solid state phase transformation equation.

Also, I can formulate as I say that the transport of heat from the surrounding to the surface, whether it is by convection, whether it is by radiation, what is the heat transform coefficient? What is the emissivity of the surface? Many assumptions may be there, many idealizations may be there, so we are not expecting that there is going to be one to one correspondence within the experiments and industrial simulation. So, there is going to be some amount of uncertainty, and therefore which is always desirable as I have mention that models without measurements are incomplete.

So, therefore, in any sensible investigation, we have to have parallelly experiments and simulations, experiments as well as model simulation, and these will give us some idea that how realistically we have been able to formulate the model in the model simulation. At this particular point, we may find that some tuning may be necessary, because we have done, we have made some assumptions and some idealizations and therefore there is some discrepancy or differences between this and this. So, in order to match the predications with the experiments you want to, we said, force fitting.

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So, now, we find out that at given location, this is time T, at certain location capital X, say Y, Z and certain time t 1. So, at certain specific location, capital X, at certain time t 1, suppose I find that, or I can say that it varies as a function of, as a location of X, capital X, as a function of y z and t, so I can say that at this particular location, suppose the temperature varies something like this, this is my prediction.

I would write it like this, let us say, let us fix Y, let us fix z as a function of x, and as a function of t. So, that is why it is the fixed value of Y, fixed value of z as a function of x, and t, I am plotting the temperature, so this is actually T at a location x as a function of time.

So, this is my predication, and I can find that my measurements are going like this. In that case, it may be possible to do some tuning and bring all the points a little bit closer, and I can find out that now my points have become like this.

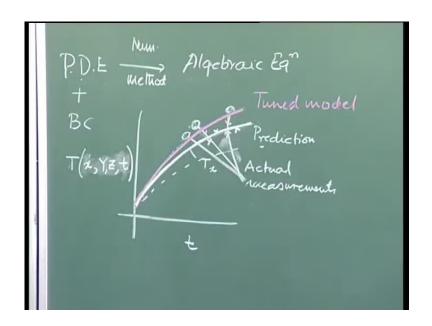
So, I am forcing the points, these are actual observations, and I am forcing the points to corresponds to actual predication, and these differences happens between this measurement this is the actual measurements, and what are the reasons for this difference? That my actual measurements is here, while the line goes something like this, and the actual measurements are different from the line, because of the fact that we have some idealizations and simplifications in formulating the model.

So, I wish to now this is not going to be a very realistic model, say therefore, so this are the actual measurements, so I want to bring... this, for example, I can say that this basically I can move this line little bit here, let us see, draw like this.

So, these are my actual measurements, and this is the line that I have predicted. So, there is evidently some difference between the experimental points and the model line, now I cannot change, these are actual measurements, so these points cannot be altered, but I can change the model. From here the line can go here, I can bring it here, but changing certain parameters, for example, changing thermal conductivity, changing boundary conditions, changing source term, I should be able to change the prediction, take it up and take it down.

So, this stage of bringing the predication closer to the reality is termed as the force fitting, or this is the tuned model. So, in many cases, model tuning is going to be necessary, because we have made certain amount of idealizations. So, once where the tuning necessary or not, these two steps are going to be determined, when you do industrial simulation, when you carry out experiments, we find out that there is large difference between the two, in that case, we will tune the mathematical model, introduce some patch factors, adjust the heat transfer coefficient, adjust the thermal conductivity value, manipulate the source term in order to get better correspondence with the experimental measurements. And this purple color line shows that now with model tuning I have been able to get the trend of the actual variations fairly realistically. So, I say it is an industrially now tuned mathematical model, and this mathematical model is now all set for implementation on the shop floor, for doing process analysis design optimization, whatever you would like to say or consider.

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Now, coming back to this particular issue, so I have how the models are formulated that has been discussed, now you understand that in many cases tuning of the model is going to be necessary, I want to elaborate this. Now, there are many cases where we can formulate the equations exactly. For example, you all know that if I have to pipe flow problem, flow of fluid through a pipe, I can write the exact equation, I can write the exact boundary conditions, absolutely no problem, I can get an analytic solution. And in that case, I will find out that analytic and numerical solutions will match exactly with the experimental observation.

Suppose, I measure the velocity with a laser Doppler velocimeter, if it is a transparent tube, then I can measure the velocity with a VIV, I can measure the velocity with the hot film anemometer, I can compare with my numerical model, fully developed flow through a pipe, there is not going to be even a little difference between predication and experiment. Predication and experiment will match in photo, there is no question of middle model tuning terms.

So, therefore, we can see that there are certain physical situations for which we can formulate the problem exactly, we can write down the equation correctly, we can write down the governing equations correctly, and there will be no problem as far as with matching of predication and experiment. So, this is the first category of the problem.

The second category of the problem is that were our understanding is not complete; therefore, many idealizations are under approximations, it will go in model building. In that case, what happens, we will invariably except that the model predications and experimental observation will not match, because our understanding of the physical scenario or physics of the process is not totally clear.

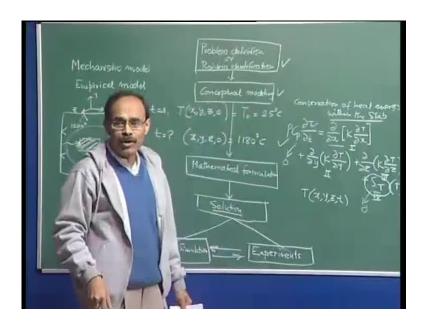
For example, how does when you inject gases, argon gas through the ladle, how a gas envelop forms around the orifice? How it devolves into a flume? How it breaks up, the big gas envelop breaks up into a number bubbles? How the bubbles coalesce during their rise or disintegrate? This complete picture we understand it reasonably, but the picture is not absolutely clear as of today.

So, therefore, if I have to write model equations for this particular phenomena, that an injection of a gas, its formation of a big gas envelop, its break up into a number of bubbles, the qualisys of the number of bubbles into a bigger one, and all these incorporating phenomena, all this phenomena we have to write down, that model for bubbles is for distribution, we will see that there is going to be lot of idealizations and terms.

In the secondary cooling zone of a continuous caster, for example, if I have to say that I want to develop a mathematical model for heat transfer coefficient from the first principle, there also we will see that our understanding is far from being complete, because in the impinging, we have a nozzle, through which you have spray mist, which is coming, a spray mist is what? It is air in water, so the spray mist comes and this is the surface of billet, and the spray mist impinges on the surface, and in the process, it is break down into number of droplets, it impinging in the hot surface, removes heat from the hot surface.

So, therefore, these forces is extremely complex, and our understanding of droplets formation, breaking of the continuous stream into droplets, impingement of the droplets, evaporation etcetera at the surface, these are part from being complete.

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So, as a result of which, when we try to formulate problem for this scenarios, we will see that considerable amount of idealizations and approximations will go into model building. Unfortunately, steel making processes fall in the second category, steel making, mathematical models of steel making are not going to be like pipe flow problems, that we can write down the equation exactly, that we can formulate the governing boundary condition exactly, no, we will never have this kind of a scenario in steel making, because of the inherent complexities of a subject, because of the nature of the manufacturing technique.

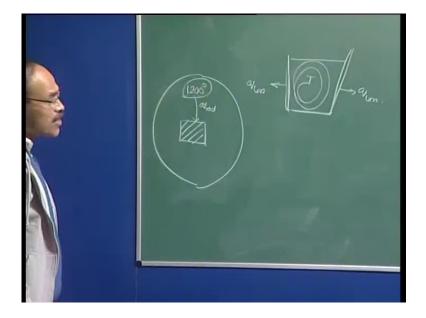
So, therefore, it is to be appreciated at this stage that when you build models for steel making processes, inherent problems, inherent idealizations are going to be there in the formulation of governing equations. Idealizations will be there in terms of formulating the boundary conditions and the initial conditions, and all these are going to be a matter of concern, and all these are going to give us some deviation from the experimental observations. People in industries today say that if your model can predict within plus minus 20 percent, 25 percent of what we see in industry, we will consider your model into the reasonably good.

So, that is the kind of range or the tolerance we are given because of lack of complete understanding of the subject. Now, we can say that once we have identified the problem, we have solved the problem, we have tuned the problem, now we have we are in a state

that yes we have a mathematical model that has demonstrated performance. Of course, following tuning, tuning will always be necessary in optimization, in industrial design.

So, the mathematical model developed cannot be in straightforward way applied. So, without these companion experiments, we will never know the extent of tuning, which is necessary. Therefore, I have been repeatedly telling that merely solving governing equations and boundary conditions are not enough; this is called as a black box modeling. We have to have companion measurements in order to demonstrate the applicability or suitability of the mathematical model that we have been able to formulate.

Now, let us look at the issues of uncertainty. As we have tried to indicate so far that the formulation of the model, as well as the boundary conditions are going to give raise to some amount of uncertainty.



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Now, I will explain very briefly in the context of uncertainty in boundary condition. For example, if I have to model that scenario, which again I will draw here, so there you may require that what is the main temperature. Now, I can say that the surrounding temperature is 1200 degree centigrade, and its radiation, which is giving heat to the slab through itself.

Now, therefore, somehow I have to know what the furnace temperature is. So, I have to have measuring devices, and based on that I will be knowing that what is the surface temperature. So, there is going to be some amount of uncertainty, when we rely on empirical measurements from the boundary. This surrounding temperature, when I am solving for temperature within the solid, surrounding temperature is needed as the boundary condition. And how do you know the surrounding temperature, by carrying out certain experiments, or with some temperature controller or pyrometers etcetera.

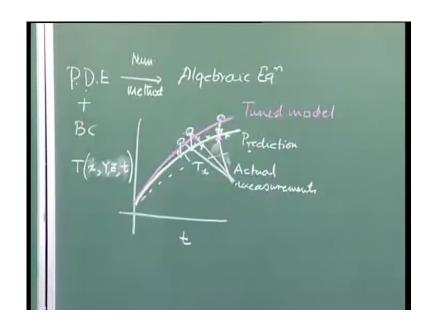
So, therefore, those measurement devices also have some inherent errors, therefore, certain amount of error and uncertainty is going to be associated with this. Similarly, when you are talking of heat flow, modeling of heat transfer in the melt, and then we require that what is the difference in temperature here is a result of q loss.

So, we must know that at what rate through the refractory lining in the ladle, heat is being lost outside. How can you know this? There is no way of determining it theoretically, we have to put a probe here, either we measure temperature or we directly measure the heat flux through a heat flux transducer, and then tell you that this is range at which the heat is being lost.

So, when I write the governing differential equation for heat flow within the melt, these informations will appear as boundary conditions with a governing equation, where from this boundary conditions, will be given these, will be directly based on experimental observation. So, that is why when we measure these quantities, either the temperature within the furnace, or the rate of heat loss through the steel shell in a ladle, these are all empirically determined parameter. So, they are called prone to error because of inherent problems associated with the measuring devices. So, we understand that the uncertainties are going to be because of number 1 uncertainty, is formulating the governing equation, correct, that I have explained. We can have all uncertainties that we do not know, for example, the thermal physical properties of the system.

Thermal conductivity, how it varies as the function of temperature and composition, we may not exactly know. If you do not know, in that case, we can say that the properties of the system are not known high temperature properties and kinetic data are not always available, so they are also bringing certain amount of uncertainty.

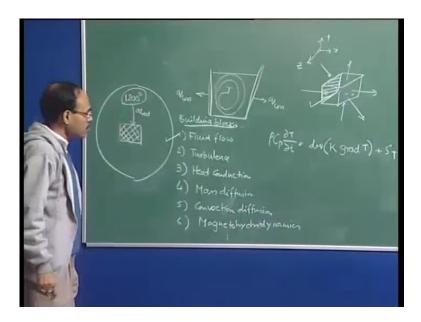
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So, uncertainty in boundary condition, uncertainty in governing equation, uncertainty in high temperature properties and kinetic data are the major sources of such differences that the difference that we have seen here.

So, these three can be considered to be the major reason, we must understand also the numerical methods are also prone to some error, but eventually, when you heat that grid independent solution, then that can be forgotten, or we may not consider that, because grid independent solution have to be identical, but for certain numerical techniques, if you do not take precautions, that give you some erroneous result also. So, numerical error is also one reason. So, we have to have a robust numerical method in order to derive solution from the governing equations. Now, I have written that heat conduction equation very clearly, how do you formulate these equations?

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So, having understood, let us go back to this particular issue, having said that I want to find out the temperature within the solid, then I have to understand that is the thermal energy conservation, because that I know that the temperature at a point within the location is because of the heat which is being received, and the flow of heat within the solid itself, it is governed by that.

So, it is basically conservation, the rate of change of temperature at this particular point is a function of the rate at which the heat is being lost. Heat is coming from the embind to the surface, and I should able to write an equation for this.

So, this is going to be an energy conservation equation, now the question is how do I write those equations? I mean, do we every time derive an equation from the control volume base? For example, we have seen Navier-Stoke's equation, for example, you know Laplace equation. So, how do you derive those equations, we take an elementary control volume, and in elementary control volume, we can say that through this, I can say heat is coming in, heat is going out, this is x y and z direction. And then we have heat which is coming in this particular direction, leaving in this particular direction, heat which is coming in this direction.

So, the net rate at which heat content of this control volume increases or changes is a function of the net flux of heat elementary. So, I can setup an elementary control volume,

and then I can find out what is the characteristic for the conservation equations for this particular control volume.

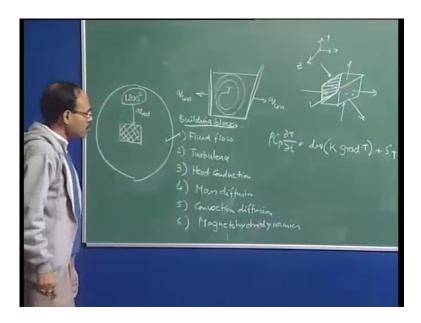
Imagine, if I take this control volume to be one of the phase margins with the boundary, then that is how through the boundary control volume I will have the boundary information getting into the calculation domain. This may be an internal control volume, and within an internal control volume I know it is the conduction of heat, therefore, I would say conduction of heat along x direction, conduction of heat out along x direction, conduction of heat along y direction, and so on, because this is a ladle control volume. And one of these surfaces, and very well become the boundary surface, once upon through whole is located here, and through that particular surface, we will have the boundary information getting into the system. So, one way of deriving the model equation is that we set up the control volume, formulate the governing equation. And whatever equation I have written earlier, that can be derived very easily by control volume equation.

But for every problem, it is not justified to write down the control volume formulation and derive the equations. Because, I am not the first person to investigate heat conduction phenomena, there are innumerable other people who have to study heat conduction phenomena. These equations are documented in literature; these equations are documented, which I have written for example. Now, I write it in a little bit conservative divergent K, gradient T, are plus S T. So, if you expand it in three dimension, you get that derivative, second order derivate along the x, second order derivate along y, second order derivate.

So, these equations are well documented in the literature. So, once I understand the phenomena, I can pick up that particular equation from a previous one, at the most I can examine that am I not perhaps blindly copying this equation, is the equation dimensionally consistent, is the equation correct, is the equation why I am going to (()).

So, how can we pick up, there a innumerable equations of heat conduction, I have said two-dimensional study sate, two-dimensional in cylindrical polar coordinate, threedimensional in Cartesian coordinate, one-dimensional in spherical coordinate, so there are lot of heat conduction equations that is in front of me.

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So, how do I pick up the right equations, which is suitable to me? And that is the status by the conceptual modeling. At the conceptual modeling itself I am not able to see that what is the differential equation I have to pick up from the literature, so I can all together avoid this control volume formulation part, you can of course do this if you have time, but it is not necessary always, equations are available in the literature, you have to understand the physical scenario and then (()).

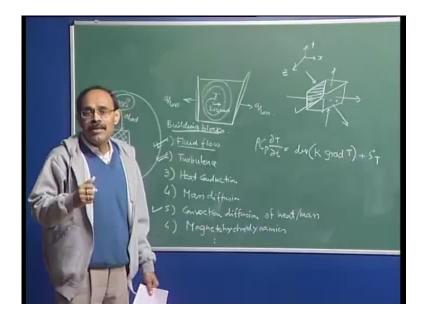
So, there we say that there are building blocks available, so this is called, I would say heat conduction building blocks. So, in heat conduction building blocks, what I am going to have? I am going to have heat conduction in three dimension in spherical coordinate, heat conduction in three dimension in spherical coordinate, heat conduction in three dimension Cartesian coordinate, steady sate. Similarly, I can have other equations, unsteady state heat conduction in Cartesian coordinate, unsteady state three dimensional heat conduction in cylindrical polar coordinate, unsteady state three dimensional equation in spherical coordinate. So, I have a series of equations in the heat conduction module or heat conduction building blocks, and I am going to pick up with my own understanding.

So, therefore, we have summarized various building blocks necessary for mathematical simulation of steel making process, and these building blocks could be, want to be fluid flow building blocks. So, building blocks.

What do the building blocks contain? Building blocks contain the differential equations or the mathematical models of fluid flow. Why so many mathematical models? Because, they contain a set of equations retained in unsteady state mode or steady state mode, retained in two dimension or three dimension, retained in various coordinate systems, so that is why we have a range of choice available to us.

We have to only understand that we got to simulate the fluid flow; therefore, we have to go to fluid flow building blocks and pick up the correct equation. We can have building blocks for turbulence also, there may be a various types of turbulence model as I have mentioned earlier in one of the lecturers. So, we can have various kind of models, you have a choice there, what I kind of a model you would like to use? So, according to that we can pick up the right equation that is applicable to your case. You can have heat conduction module, which I have just now discussed, and the counterpart you can have is mass diffusion also.

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You can have convection diffusion, you can have magneto hydro dynamics, and there are all modules or building blocks available to us, and there are many such building blocks which are available. So, we have to understand the problem. We have to see what we are trying to model and accordingly... So, I want to... for example, I will just illustrate, I want to model heat flow within a ladle. Now, I understand that in the ladle there may be some fluid flow involved, because of the transport of heat..., this is a liquid, so within

the liquid, or in the melt, transport of heat is due to convection and diffusion. Diffusion means it is heat conduction here, so diffusion is a generic term, for all we have momentum diffusion, we have mass diffusion, you have heat diffusion. Therefore, heat diffusion is heat conduction, momentum diffusion is viscous diffusion, and mass diffusion is mass diffusion.

So, the movement of temperature, heat will flow from one point to another point, because of presence of the fluid flow as well as the diffusion or heat conduction itself. Therefore, I will say that I need not a heat conduction module, but a convection diffusion of heat or mass, this is the building block, these are one and the same, equations one and the same type of equations, because temperature and this is the scalar quantities. Therefore, I can say that I can pick up this, and then I will have to model, because I am talking of convective heat transport. This convective heat transport I will not be able to address unless I know fluid flow. Therefore, fluid flow I will have to use, and if the first turbulence, in that case, I will use turbulence also.

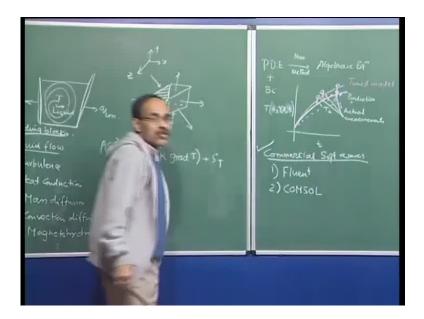
So, these three building blocks will formulate the mathematical framework, or provide the mathematical framework for prediction of temperature. Of course, when you talk of this convection diffusion of heat, in that case, we will require thermal boundary conditions to be prescribed.

So, all boundary conditions have to be prescribed, whatever is necessary, but as far as writing the governing equations are concerned, we will select fluid flow, will select turbulence, we will select convection diffusion, and then identify what is the dimensional analytic problem, whether it is a steady state or not. So, from the innumerable number of equations that are available under each of these categories, we have to pick up the right equation.

So, the last point that I want to make here is we have many phenomena, which we may to calculate simultaneously. We may have to calculate, for example, electromagnetic driven force, for example, if you want to simulate fluid dynamics in continuous casting slabs, for we may have elector magnetic breaking, electromagnetic studying, so we may fluid flow turbulence heat transfer; many phenomena can be simultaneously or interrelated. We have multiphase flows, if we are injecting gases, we have slab metal mixing, we have heat loss, we have fluid flow, we have turbulence and if you go to continuous casting, you may have magneto hydrodynamic. So, you may have many such phenomena. It is not just one, in the case of slab reheating, it is just heat conduction, but when you are talking of liquid street processing operations, you may have many different modules, interconnected modules to be used simultaneously to predict the given phenomena.

So, we are talking about solving the series of non-linear partial differential equation, the equation is not going to be only one, there may be several 10's or 12's of such equations you are going to solve, and these equations are interlinked with one other, these equations are non-linear, and there may be the non-linear, boundary conditions may be complex. So, solving this equation is horrendously complex task, and if you have to write a computer program, this is going to take many months before you can write the computer program to solve the 7 equations and derive a meaningful result of temperature (()).

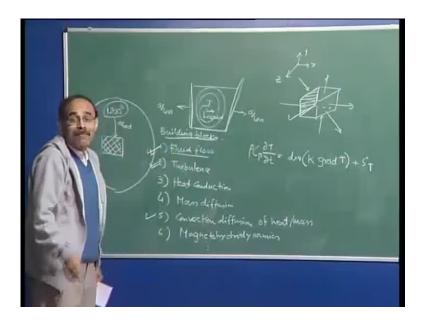
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So, what is the option? There comes the role of commercial software's, so today engineering trainee, and particularly, people who are involved with steel making and process formulae, they have to know that what are the commercial software platforms available, which they can effectively use to solve the equation.

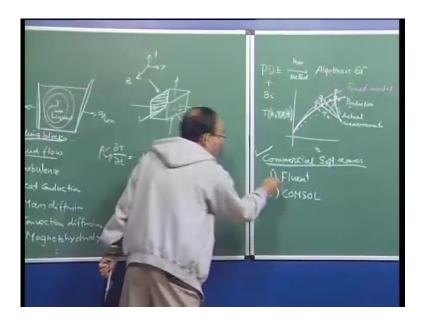
So, your job ends at formulating the problem, and then you configure the problem with the aid of commercial software's. The solution, when I talk of the solution, solution essentially encompasses writing up the equations, then coding the equation in a computer and then getting the numbers.

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So, this coding part can be totally eliminated, if you can reserve two commercial softwares. And two commercial softwares, which are very popular today are fluent and COMSOL. So, most of the mechanistic models that we use, that can be solved, mechanistic model for steel making can be solved by using fluent. Fluent has all these modules, all these building blocks build up; you have just say that it is three-dimensional, unsteady state, turbulence fluid flow and fluent is going to, pick up the correct equations for your system from the building blocks that is in the (()).

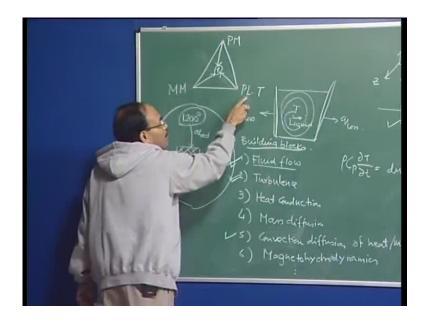
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So, using these commercial software's, you will able to get the numbers or generates the results in particular no time, and then you can pushed process; that means, you can plot and you can scientifically visualize the results, how the temperature is changing as the function of time, or how does the velocity is changing in the function of time. In later, all these features, you can draw it nicely, you can scientifically visualize and obtain useful information of the process.

So, to conclude the discussion on physical and mathematical modeling, I would say that mathematical modeling is a very important part today, because physical modeling does not give us complete picture.

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So, mathematical model and physical modeling should be applied together. And typically, and finally, I would say that if you draw a vertex triangle, and then in this vertex lies the physical model, in this vertex lies the mathematical model and in this vertex lies the planned scale trail. And there is the truth, and so in order to heat the truth, you have to have these three components, and then only possibly you can do process analysis, process design optimization correct.

So, any comprehensive investigation on steel making must therefore embodied physical modeling, mathematical modeling, and plant scale in different proportion depending on your resources. And when these three are combined together, you are in a position to get some meaningful results, and that is where the truth or the real process lies, and it is with this comprehensive knowledge you will be able to do process design, process analysis, process optimization and control analysis.