Steel Making

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Module No. # 01
Lecture No. # 38
Modeling and Measurements

We continue our discussion on modeling and measurements.

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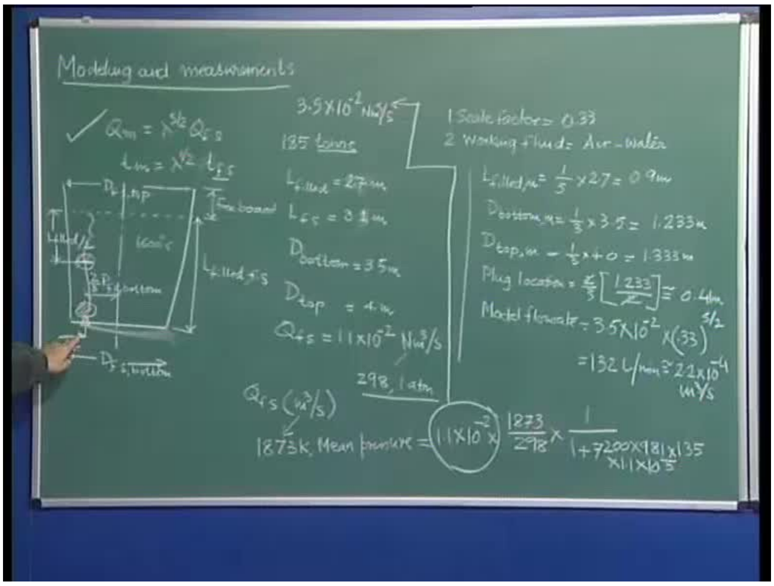

By the by, some people write modeling with one l; this is an American way of writing; British way of writing is double l. So, what we have seen so far is that following geometric similarity, we consider dynamic similarity. And, in the context of metallurgical processing operations, I have indicated that we can consider metallurgical processes or steel making processes to be dominated by inertial and gravitational forces in most of the situations. And therefore, we can assume that the flow phenomenon… or, we can get dynamic similarity in steel making system on the basis of Froude similarity criteria. And, flow rate scaling criteria, which I have mentioned in the context of a gas stirred ladle system, that is, Q model. This is equal to lambda to the power 5 by 2 Q f s. I have also indicated and talked about kinematic similarity. And, I said that if there are geometric similarities between the two ladles for example and if the flow rates are scaled in accordance with this equation, then the corresponding time scale in the model and the full scale are going to be related in proportion to lambda to the power half. This also we have talked.

Now, let us take a typical example to show you that how do you really proceed to model for example, a gas stirred ladle system. Suppose it is given that I have a ladle, which is cylindrical shape. This is the line of symmetry. So, this distance is the D full scale. Let me draw it horizontally than better. And then, we have this much is L filled. So, the height of the liquid, that is, filled full scale. This I will say as (Refer Slide Time: 02:37) D f s, bottom and this is D f s, top because it is a tapered vessel. And, let us say that the porous plug is located somewhere here. And, this distance is essentially 2 over 3 into R f s, bottom.

Let us ascribe some typical characteristics value. Let us say we talk about a hundred… Let this ladle be 185 tonne (Refer Slide Time: 03:24). This is the metric tonne, 1000 kg size ladle. And, let me see if I can give the values. So, L filled – let us say. And, this is full scale system and I am saying this is about 3.1 meters. Then, we will have L full scale, which is the total height of the ladle and suppose this is about 3.5 meters. Then, we have… This is 3.1; that is better. And, we will put it as 2.7 (Refer Slide Time: 04:07). And then, we have D bottom and we have D top. And, D bottom – let us put a value of 3.5 meters; and, D top – let us put a value of 4 meters.

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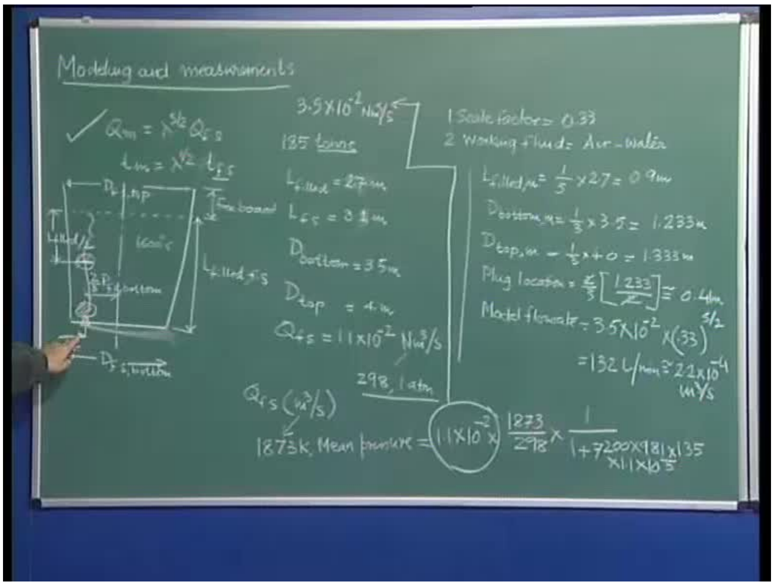

In the plan, most of the time, the flow rate is going to be given in terms of normal meter cube. So, the gas flow rate Q full scale, which is given; and, this is 1.1 into 10 raise to the power minus 2 normal meter cube per second. These are the characteristics, which are given and of course, the plug is located. So, if I have to now consider to build a model, working model and suppose I make a decision, what are the decisions that I am going to make? I am going to make now two decisions before I go into modeling. The first decision being decision 1, which is scale factor; and, let me say that I choose a scale factor, which is a very common value. This is a fairly representative value of the scale factor that researchers use in laboratories. And, number 2 – I make a decision about the working fluid. And, let I consider that while the steel system has steel, molten steel and argon here, I consider air-water system. So, the bulk liquid is water instead of steel. And, as I have indicated earlier, that once we use water… And, that the kinematic viscosity of water and steel are identical. So, we will not be able to respect Reynolds and Froude similarities simultaneously.

And, we have assumed already or we will consider in general that phenomena in steel making systems are Froude dominated. So, in this case also, that condition will hold good and therefore, the flow rate scaling equation will… This is the flow rate scaling equation that will be used. So, now, I can say that you have geometrical similarity; therefore, I understand this distance is redundant as far as you can see. This is (Refer Slide Time: 06:32) called the free board actually. This is the free board. It is, what is this distance, is very meaningful to us, but this distance is not meaningful. So, the ladle can be… The overall total dimension of the ladle, that is, that will be built in the laboratory may not have any geometrical similarity or in terms of the characteristic total length. But, what is important is the depth of the liquid. So, that needs to be scaled. So, I would say that therefore, L filled model is going to be roughly 1 by 3, which is the scale factor multiplied by 2.7 and that gives us 0.9 meters. So, I know that.

I will take a vessel; I will construct a vessel in which 90 centimeter is going to be filled up with water. Therefore, the total height I can take it to be (Refer Slide Time: 07:30) 1 meter, 1.1 meter, 1.2 meter. That hardly makes any difference, because this part does not take… This portion does not take part in the… as far as molten steel flow is concerned. Similarly, I can say that D bottom, D top – by the by, m essentially represents these are model values – one third of 3.5, which is roughly about 1.233 meters. And similarly, this is top, which is 4 meters. Therefore, it is one third of 4 meters and that is going to be 1.333 meters.

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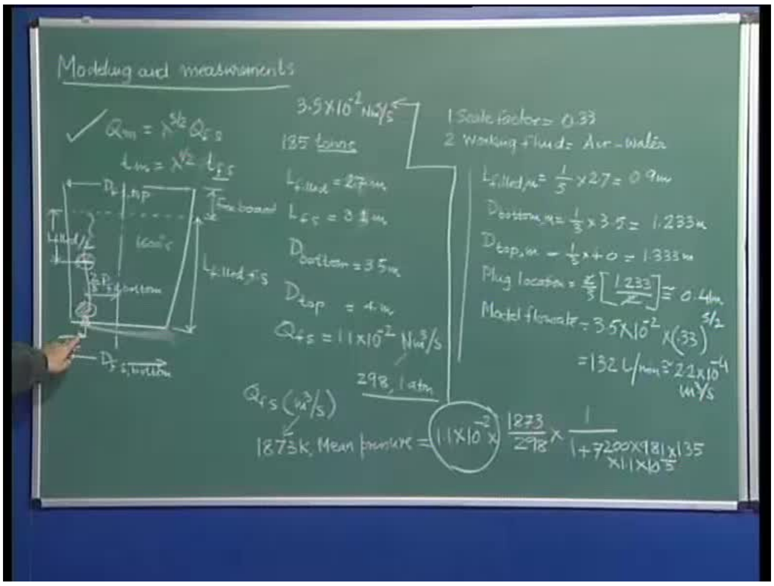

We must understand now. So, geometric features are scaled. So, plug location – two third of R, which is equal to bottom; so, which is 1.233 divided by 2. R is diameter divided by 2. So, this this cancels out. And therefore, this is roughly going to be 0.4 meters. So, that is the plug location in the model itself. From the axis of symmetry, I have the plug located at 41 centimeters. Either in the left or in the right, we will have no consequences, because we are talking of axis symmetric geometry.

Now, as far as the gas flow rate is concerned, this is given in the terms of the normal meter cube. Therefore, this is the reference to 298 kelvin and 1 atmosphere pressure. But, actually, when the gas is injected here into the molten steel, molten steel at a 1600 degree centigrade. So, the volume of the gas expands, increases quite a bit, because of this temperature. And also, because we have 1 atmosphere pressure it is injected at. And, the pressure here is much higher than the 1 atmosphere pressure, because pressure here is (( )) 1 atmosphere plus the ferrostatic head corresponding to this height when the gas is going to be released here. On the other hand, when the gas bubble is going to be here, the pressure on it is going to be 1 atmosphere plus the ferrostatic head corresponding to the this particular height. Therefore, the gas volume actually seen by molten steel is not equal to this, but it has to be corrected taking into account the Boyle's and the Charles' law. So, I would say, the actual flow rate first has to be calculated.

And, that actual flow rate in the full scale – that will come out to be in terms of meter cube per second. And now, I have its reference to actual state. So, I am going to say that this is therefore, is to now 1873 kelvin and mean hydrostatic pressure – mean pressure. Mean pressure means the pressure at the mean height. So, if this represents now the L filled by 2; L filled by 2. That is the mean height. Then, I can say that the pressure at this particular level is going to be atmospheric pressure plus the ferrostatic pressure corresponding to this. So, that is the pressure and this is the temperature 1873 with which I am going to correct this normal meter cube. So, the expression will look like 1.1 into 10 raised to the power minus 2 into 1873 divided by 298. And, it is inversely proportional to pressure. So, 1 divided by 1 plus rho – I take 7200 steel density into g, which is 9.81 – and then, L filled by two; and, the L filled is equal to 2.3 – it is going to be 1.35. And, I have to convert this into atmosphere. So, I have to multiply this roughly by 1.1 into 10 raised to power minus 5. So, because of this component, the gas volume is going to be increased by a factor of 6. Because of this component, it is going to be decreased by a factor of 2. So, the overall contribution of pressure and temperature is going to be…

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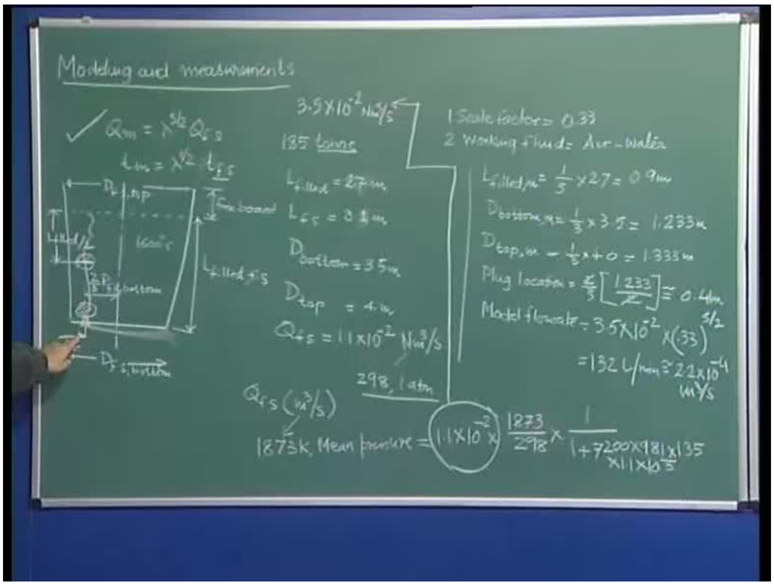

This gas flow rate will fill like approximately 3.5 times 10 raised to the power minus 2. That is the gas flow rate. So, this gas flow rate needs to be scaled down in order to find out the model flow rate, because this is the actual flow rate experienced by molten steel. Therefore, now, I can say that model flow rate is equal to full scale flow rate into lambda to the power 2. And, that full scale flow rate – 3.5 into 10 to the power 2, but it is going to be this. And, this flow rate is going to come out to be this multiplied by 0.33 into 5 by 2, which corresponds to roughly about 132 liters per minute; or, approximately, 2.2 into 10 raised to the power minus 4 meter cube per second. So, all the parameters now in the model have been calculated. So, I can go and start the ladle; take it 1.2 meters; fill it up with 90 centimeter of water; locate the plug at a distance of 41 centimeter from the left and then start injecting gas at the rate of 132 liters per minute, which is going to be dejected.

Note that when you injecting water, water does not have a high density. So, because of pressure temperature difference, that a water temperature – mostly, it is going to be room temperature. If it is the hot weather, maybe water temperature is going to be 26. If it is a cold day, the water temperature maybe going to be 10 degrees or 12 degrees. But, it is not going to be this significantly different. So, even if I inject in the normal meter cube gas flow rate, the normal meter cube and meter cube per second will have no consequences when water models are concerned, because of two facts that the temperature is not very high and that the density of water is not as high as that of steel, which is seven fold larger. Therefore, pressure-temperature correction of the flow rate in water model system is going to be insignificant. So, whatever we have (( )) injecting in to the system, whether we talk it in terms of normal meter cube per minute or meter cube per minute, it does not have any bearing as far as water modeling is concerned.

So, we talked about geometric similarity; we talked about dynamic similarity. And, I had indicated in the beginning that when we talk of modeling of steel making system, we are concerned about the four different stages of similarities: geometric, mechanical, and dynamic, and kinematic – are the parts of mechanical similarity.

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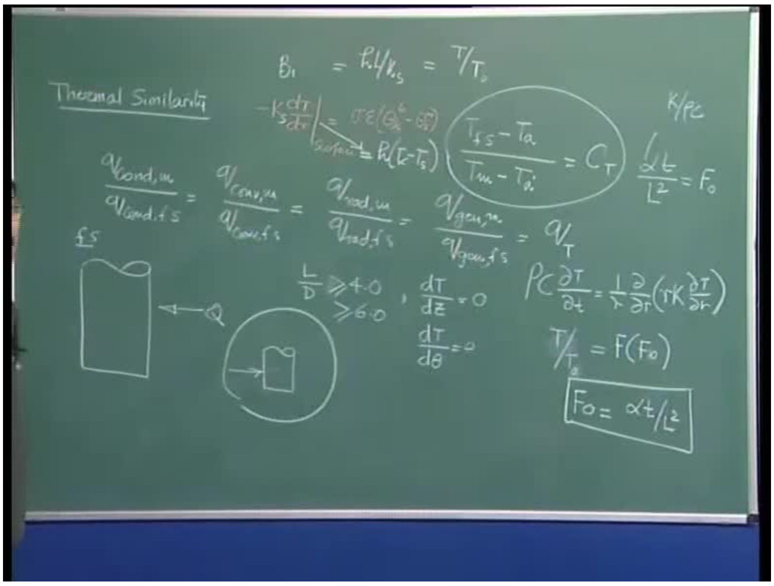

And now, we will talk about thermal similarity. In the previous case, the systems considered were essentially isothermal. I assumed that the molten metal, which is contained in the ladle, uniformly, it has a temperature of 1600 degree centigrade similar with the water model system. And, the water was going to have the temperature of 25 degree centigrade. So, we were talking about modeling of isothermal system. Now, when we talk of thermal similarity, obviously, we understand that we are talking about non-isothermal systems.

Now, before we can satisfy thermal similarity, we must understand that we have to address mechanical similarity or dynamic similarity. Unless we can talk about the similarities of fluid motion in the system, we cannot talk about thermal similarities. That is why, when I started talking first about modeling and measurements and outlined the four states of similarity, I did mention that the orders are very important; geometrical similarity; then is dynamic similarity; then is thermal similarity, because if you do not address dynamic similarity, in that case, we will not be having similarities of the flow patterns or similarity of flows in the system.

Now, similarity of flows is very important as far as flow of heat in the non-isothermal system is concerned, because we know that heat transfer occurs from one point to another point by three different mechanisms: conduction, convection and radiation. And, in molten steel system, convection is going to take an important part. So, unless we can talk about the similarities of the flow field, we will not be in a position to address convection. The discussion of dynamic similarity must therefore precede the discussion of thermal similarity. Heat flow depends on the convection currents generated in the system.

Now, we are talking about non-isothermal systems. And, I am going to say now the statement of thermal similarity is this – temperature difference at corresponding locations and at corresponding times will always be at a fixed ratio. If such a condition is satisfied, then I am going to say that the systems are thermally similar. I define again. Thermally similar systems are those in which temperature differences at corresponding location and at corresponding times are going to be at a fixed ratio. What is temperature difference? Temperature difference with respect to what? The temperature difference is with respect to any arbitrarily chosen datum. So, we can say, let the room temperature be the datum; and, with respect to the time, I am considering the temperature difference. So, I can say that the ratio of the temperature difference with respect to an arbitrarily chosen datum must be at a fixed ratio. And, we must understand that I am talking about corresponding location. So, once I say geometric similarity is there between the model and the full scale, I can visualize that yes, for every point in the full scale system, there is a corresponding point in the model.

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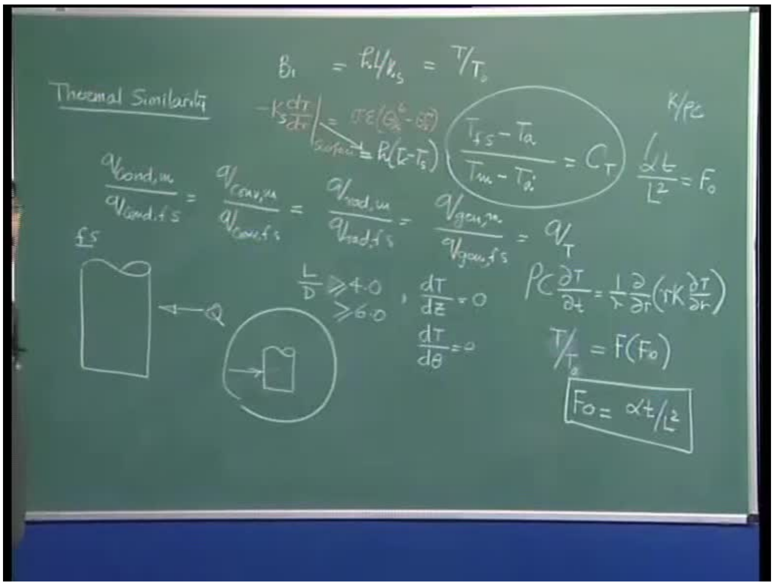

And also, we have talked about at least kinematic similarity or similarity of time scales. And, I have shown that the similarity of time scales corresponds in proportion to lambda raised to power half particularly in those systems, where flows are Froude dominated of the inertial and the gravitational forces control the flow. Therefore, I also know that if it is 10, 5 plus 10 at model or 5 minutes after the process, initiation of the process in the full scale system should be maybe 1 minute after the initiation of the process in the model system. So, there has to be some correspondence between the time scale also. And, as I repeat again that at corresponding locations and at corresponding time, you should be able to visualize as I say make this statement – the temperature difference with respect to any arbitrarily chosen datum must be at a fixed ratio.

Now, how can you achieve this condition? I have made this statement, but how can this condition is going to be fulfilled? I can say that temperature in the model, temperature in the full scale minus an arbitrarily chosen datum and temperature in the model at corresponding location with an arbitrarily chosen datum must be at C T. What I have to do in order to get to these results? That at corresponding location at corresponding time, this is going to be satisfied. How am I going to ensure that the theorem is now… That at corresponding location at corresponding time, if you can make this equal, that the rate of heat flow by conduction in model to rate of heat flow by conduction in full scale is equal to rate of heat flow convection in model to rate of heat flow convection in full scale is equal to rate of heat flow by radiation in model to rate of radiation in full scale and is equal to q generation model by q generation full scale. And, if I say this is equal to (( )) T, then I can say that yes, if this condition can be maintained, then I should be able to…

Say that systems are thermally similar. And in that case, we will see that yes, indeed the temperature profiles, temperature at corresponding location ratio, temperature differences at corresponding location and corresponding time are going to be equal in case I have been able to successfully maintain this quality. Now, we know about these three components, because heat is transferred from one point to another point by three different mechanisms. In steel making, molten steel making systems or in vessels containing molten steel, we may not have… Within the bulk, radiation may not be important. Radiation is important only from the surface perform (( )) loses its temperature.

What is this generation? This generation is in fact some heat evolved maybe because of chemical reactions for example, or some heat consumed. So, the generation can be visualized in terms of dissipation. Also, I did not write that. So, generation, if it is a positive generation, then that means it is produced. If it is a negative generation, that means it is going to be consumed. And, this consumption of heat in the system can be because of for example, a chemical reaction taking place in the system. So, this ratio has to be fulfilled and then the corresponding similarity in temperature profile can be obtained.

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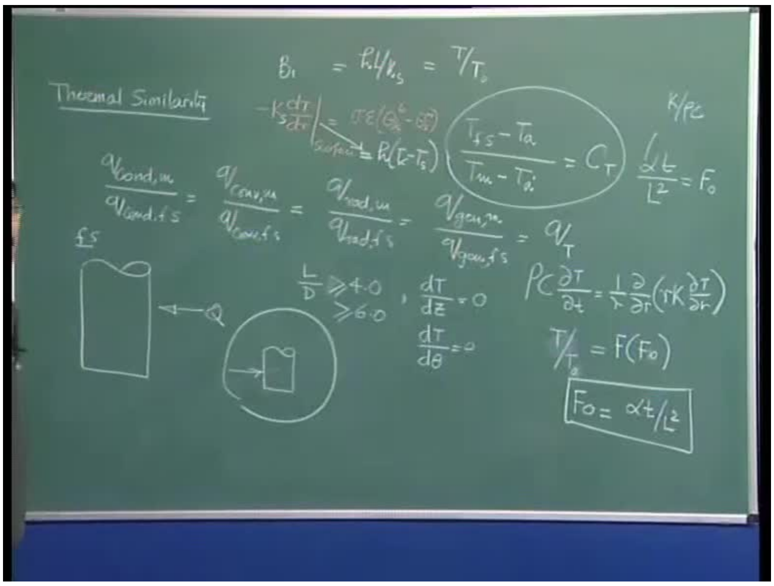

Now, the same procedure can be followed just like the way to derive similarity numbers. For example, as I said that, what you have seen in the case of dynamic similarity, we have seen in the case of dynamic similarity that few numbers governed heat flow. For example, in the context of dynamic similarity, I have shown Euler's number, Reynolds number, Froude number. These are the numbers that control. And, where from these numbers have come? These numbers have followed from two different routes. One was the governing equation; I have taken Navier-Stokes equation if you remember. I represented it in non-dimensional form and I have shown that yes, one over Euler is the function of Reynolds (( )) Froude. Then, the other stream was I considered the fundamental definition of dynamic similarity that the ratio of the forces has to be equal. And, there from, I have obtained that Reynolds number must be is equal to Reynolds number, Froude number… So, two different stand points are there.

From the fundamental definition, we can get the similarity criteria. From the governing equation also, we are going to get the similarity criteria. For example, if I say that look there is a cylinder. And, this cylinder for example, is subjected to some Q. It has been heated. And, I am interested to find out the temperature here. And, suppose this is the full scale system and then I have a small scale (( )) I want to… My objective is this is an industrial system; I know what is the size of the cylinder; I know what are the rates at which heat is being… It is being heated; maybe it is contained in a furnace and maybe exposed to 1100 degree or 1200 degree centigrade; whereby, it is getting continuously heated up. So, initially, the object is at room temperature. Then, I put it inside the furnace and the object gets heated up.

I know the heat path; I know the dimensions; I know the thermo-physical properties. I want to carry out a simulation in my laboratory with a small-scale miniature ingot. And then, on the basis of this, I am going to find out that what the temperature is, which is the scaling of operation actually. So, now, the question is that I can geometrically scale this down if the L by D of this cylinder is exactly equal to the L by D. Means L by D is known as the geometrical aspect ratio. If the geometrical aspect ratios are similar, in that case, I can say this is geometrically similar.

Now, the question is I have kept it in an enclosure; what should be the furnace temperature? At what rate the heat should be flowing into the solid, such that the temperature profile here and the temperature profile where… That means this condition is satisfied between the model and the prototype. Therefore, I can understand. Now, go back to say that in this case, heat is being transported by convection and radiation from the furnace wall into the solid; and within the solid, the heat is being transported by conduction. So, I have to consider ratio of conduction to convection in the model ratio of conduction to radiation in the model. And thereby, you will be able to formulate the criteria. Or alternatively, I can also find these numbers from the governing equation. For example, if I consider the equation here, if this is an infinitely long cylinder, that means the definition of an infinitely long cylinder is L by D ratio is greater than 4. Suppose this condition applies. In that case, I can assume that the cylinder can be treated mathematically as an infinitely long cylinder. This is the definition of an infinitely long cylinder. Some people say 4. It is a well-accepted criteria. 6 perhaps is a much more accurate criteria. But, this is also reasonably accepted number as far as characterizing a cylindrical object as an infinite or non-infinite in (( )) direction.

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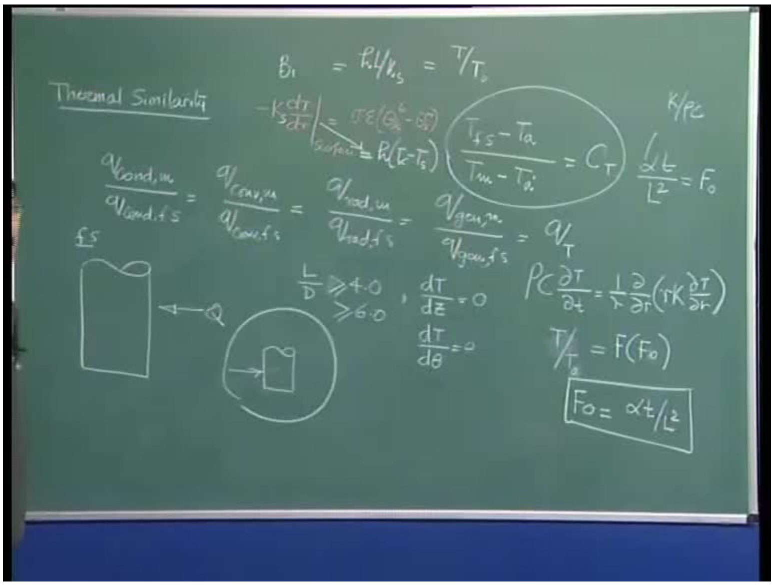

It is a very long very long cylinder or an infinitely long cylinder, I can say that this is basically a case of one dimensional heat conduction; the heat is flowing in a radial direction. Flow of heat along this direction is not important; flow of heat along this direction is not important. Why not? Because the object is cylinder. So, it is symmetrical. For every theta, you see the scenario is absolutely identical; it is enclosed in a furnace. So, it is exposed to the same surroundings. So, this is theta symmetry and that it is infinitely long. And, the consequence of infinitely long is if z represents this direction, then this is going to be equal to 0. And, the actual symmetry is this (Refer Slide Time: 26:46) that is equal to 0. Therefore, I am going to characterize this cylinder in terms of R theta in z-coordinate system. If these two derivatives: one along z, one along theta is equal to 0, which essentially tells me that the transport of heat within the solid is going to be dictated by a uni-dimensional heat flow equation.

Within the solid, what is the mechanism of heat flow? It is purely conduction. Within the solid, there is no convection; there is no radiation; it is only conduction. Initially, the solid was at room temperature. And now, progressively, the temperature of the solid is increasing, because it is gaining heat, resuming heat from the furnace wall. Therefore, it is a case of unsteady state heat transfer equation in one dimension. So, I would say the problem, the governing equation that will characterize heat flow within the solid is going to be an unsteady one dimensional heat conduction equation in R direction or radial coordinate. And, if I write that equation, that equation looks like… (Refer Slide Time: 27:51) This is the conductive transport or the diffusive transport term and this is the unsteady state term. There is no theta deduction term; there is no z deduction term, because I have approximated this.

Now, if I represent this equation in one dimensional form; one can very easily do that; I have also shown you that if you take the Navier-Stokes equation in dimensional form, convert it into non-dimensional form, you get some number. And, this equation will show you that we have Fourier number; the temperature profile is going to be the function of…. That is what is going to come out. With respect to… I can say this is a non-dimensional equation or you can write it (( )) So, the non-dimensional temperature filled within the object is going to be a function of the Fourier's number. This is the dimensionless number that is going to follow from this particular equation. So, the Fourier number is very important. So, we can understand that the non-dimensional temperature within the objects will be equal provided the dimensionless Fourier number – Fourier number all of you must be knowing from your second year transport phenomenon course – and, this is going to be alpha t by L square. That is what is the Fourier number. L represents the characteristic length and alpha represents thermal diffusivity, which is nothing but k divided by rho times C; rho is the density and C is the specific heat. So, this is the definition of Fourier number.

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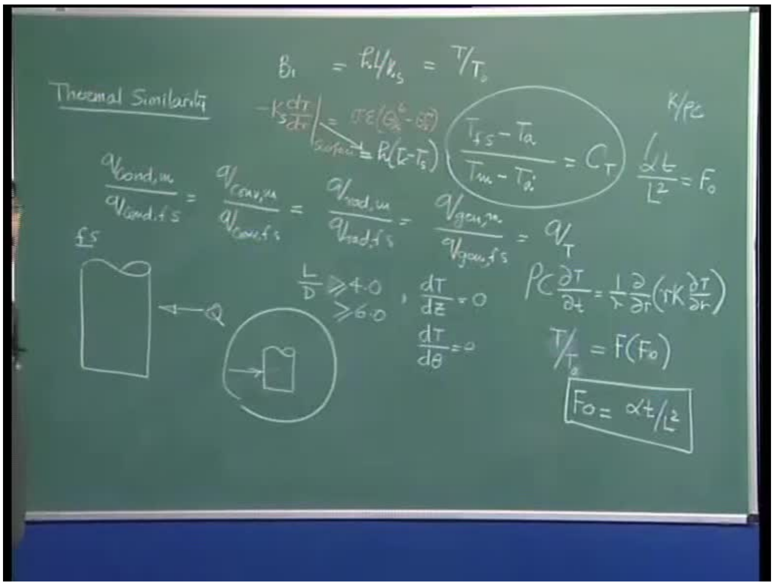

Note that the solution for this equation depends on the governing boundary condition also. Therefore, just by merely making this equation identical or this number identical will not give us the required result, because a differential equation without the boundary condition is meaningless. It can have an infinite number of solutions depending on what kind of a boundary condition we use. For example, we solve for our steel making systems, C dimensional Navier-Stokes equation. You go to the aerospace engineering department; they also solve Navier-Stokes equation. You go to mechanical engineering when they solve design turbines, flow of fluid over the turbine blade, etcetera. They also solve turbulent Navier-Stokes equation. You go to chemical engineering; everybody uses the same equations, but the solutions differ. And, the solutions of the same set of equations depend on the geometry of the system as well as the boundary condition. So, geometry we have taken into account; governing equation gives us this boundary, this criteria that the Fourier number in the model and the prototype must be identical.

And now, the most important part, that we have to consider the boundary condition in order to complete the analysis. So, fortunately, in fluid flow problems or dynamic similarity, we do not have to consider the boundary condition, because if we have a wall in the model also, at wall, the velocities are going to be 0; in the full scale also, the velocity is going to be 0. But, here at this surface and at this surface, the rate at which heat will be coming in maybe completely different, because they may be enclosed in a different furnace or a different turbine atmosphere. What is the boundary condition here?

The boundary condition – suppose if I assume that heat is flowing from the (( )) to this purely because of radiation, then I can say that at the boundary condition exposed, boundary condition is going to be something like heat flux. In the radial direction at the surface, is going to be… I am making an assumption that it is going to be purely… So, this is the famous expression, Stefan-Boltzmann constant, emissivity of the surface; theta infinity is the furnace temperature and theta s is the surface temperature. So, this is the radiation flux and this is the heat flux, which is being received at the surface itself. These thetas and T's are one and the same thing; only difference is this theta is in absolute scale. So, it is 273 degree plus t infinity that is equal to theta infinity; that is the only difference.

Now, from this number also, we can get a boundary condition. On the other hand, if we say that it is convection that is… that controls the heat flow, it is not the radiation, but it is only convection. Alternatively, you can say that it is mixed mode – both radiation plus convection, which transports the heat. One way of saying it is total radiation; the second limiting way of saying that it is heat flux received at this particular point is going to be the convective heat flux in which h is the convective heat transfer. So, if I assume that, in that case, I can show that if the heat flux is due to convection, then I will say this will give rise to hL by k as a number, which is the… This is k solid; this is k solid. And this is… We have Fourier number. This is equal to alpha t by L square. So this will tell us that the dimensionless temperature T over some value here – T naught I have written – is going to be a function of h L by k s, which is the heat transfer coefficient characteristic length.

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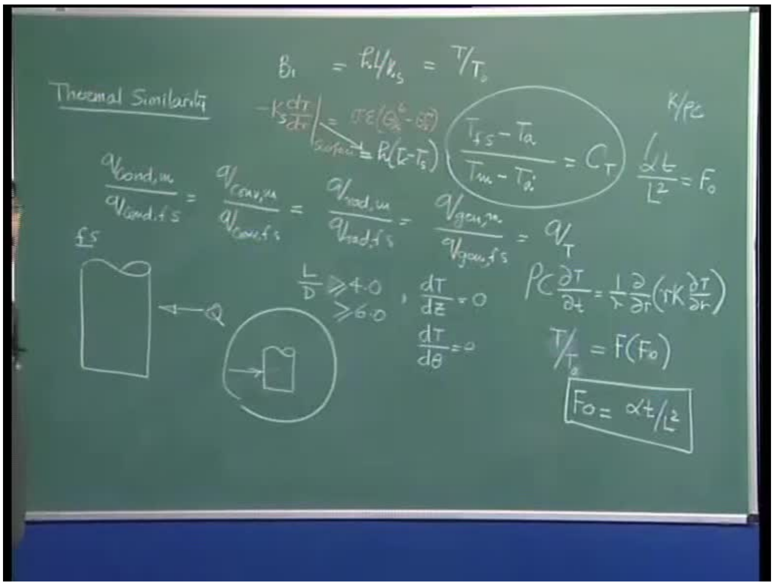

What is the characteristic length? In this case, maybe the radius of the diameter. And, k s essentially represents the thermal conductivity of the solid under consideration. So, if I now say that look if this number, biot number in the model is exactly equal to biot number in the full scale system; if Fourier number in the model is identical between the model and the (( )) system; in that case, what we are going to see? We are going to see that this condition is going to be fulfilled. Note that this number essentially represents the heat transport by conduction; convection to conduction. So, I am taking into account convection. So, fundamentally, this number would have been derived by taking the ratio of the conductive to convective heat transport in the model without considering the boundary line equation whatsoever.

Similarly, this number tells us the heat flow, because of heat conduction only. So, we could have gotten this number by taking into account the equivalence of the conductive heat transfer rates between the two systems itself. So, the analysis tells us that thermal similarity between these two cylinders can be obtained provided this number and this number is going to be maintained. Therefore, based on this number, I will find out that what is the h that is required. Based on this number, I will find out that what is the corresponding time equivalence between the two. And then, it will tell, if such heat transfer coefficient we applied in this particular system, then at this particular time, this condition is going to be valid.

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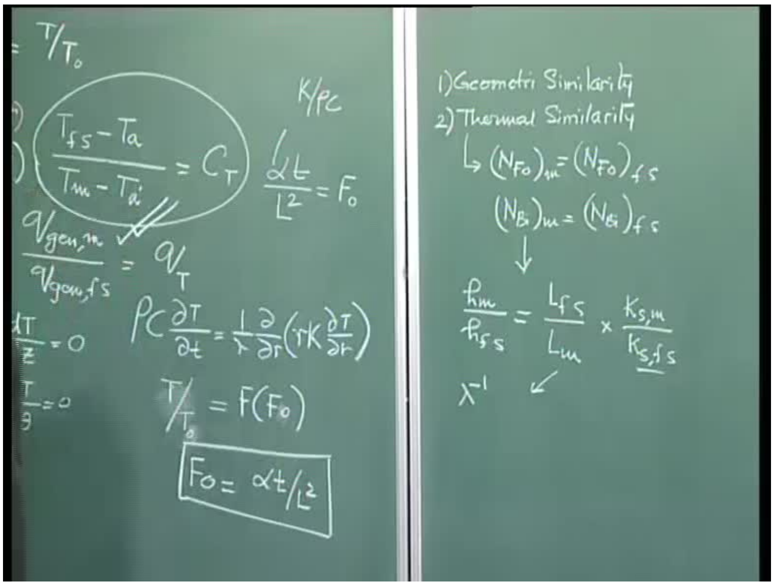

So, when such a problem is to be investigated, we must understand that we are talking here about geometric similarity and straightforward, we will talk about thermal similarity. In this case, because of the domain we are trying to get thermal similarity between this and this and that these two objects are solids. Therefore, there is no question of any fluid movement. Therefore, the dynamic similarity does not come into the picture. So, the geometric and thermal similarity will entirely govern the problem. And, I have shown you, that if N Fourier model is equal to N Fourier full scale and N biot model is equal to N biot full scale, in that case, this particular condition is going to be maintained in the two. Now, therefore, I can say that look the biot number equivalence gives us to adjust that h model by h full scale is equal to L model by L full scale and then we have k solid model and k solid full scale.

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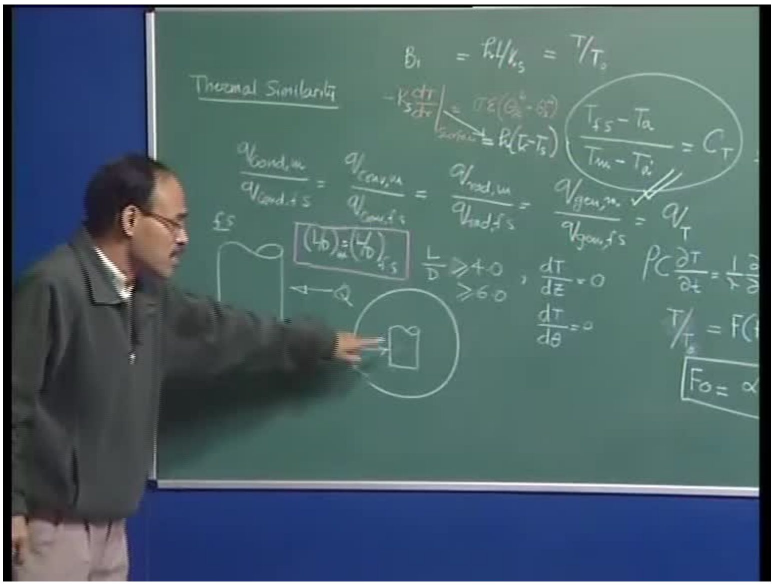

This term L f s by L m is actually lambda raised to the power minus 1. This is the scale factor that we have already decided when we are talking about the geometric similarity that we have maintained. Say L by D. So, we have L by D model – this is equal to L by D full scale. That is the geometric similarity. And, once we have done it, the scale factor is known. So, lambda is known to us. This may be a steel cylinder; this may be an aluminium cylinder or a copper cylinder; does not matter. There is no requirement that it has to be steel; I can use a different material.

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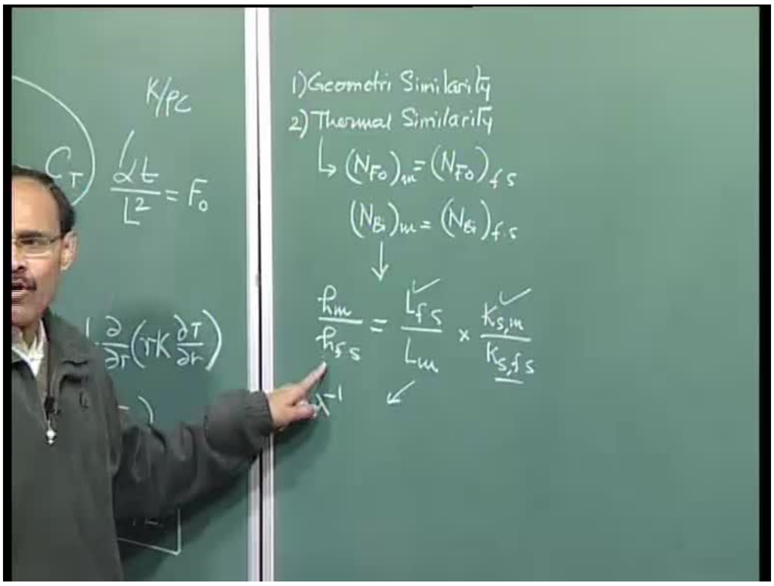

So, I know the ratio of the thermal conductivities of the two solids. If I have the solid, which is steel in full scale system, I know the thermal conductivity of steel. If I have taken copper, I know the thermal conductivity of copper. So, this term is known; this term is known.

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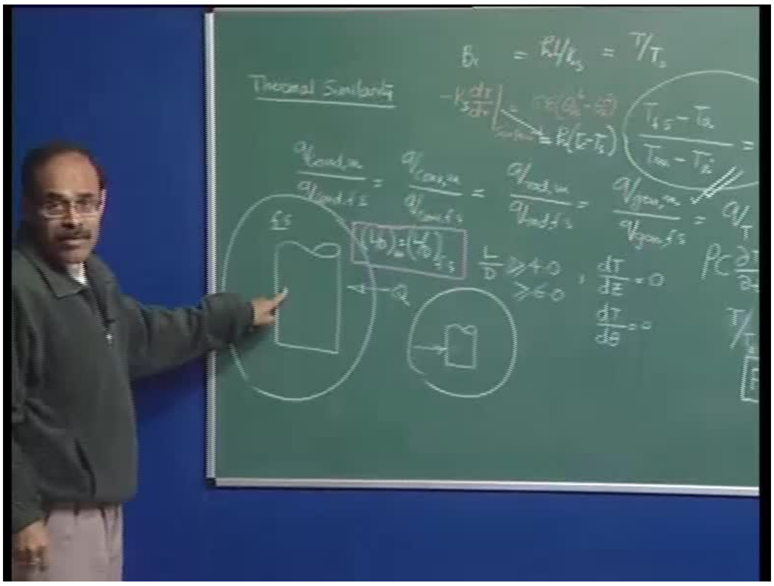

The heat transfer coefficient in the full scale – that means the container at the furnace in which it is there. So, I know the heat transfer coefficient already in the furnace and now, I have to find out the corresponding heat transfer coefficient.

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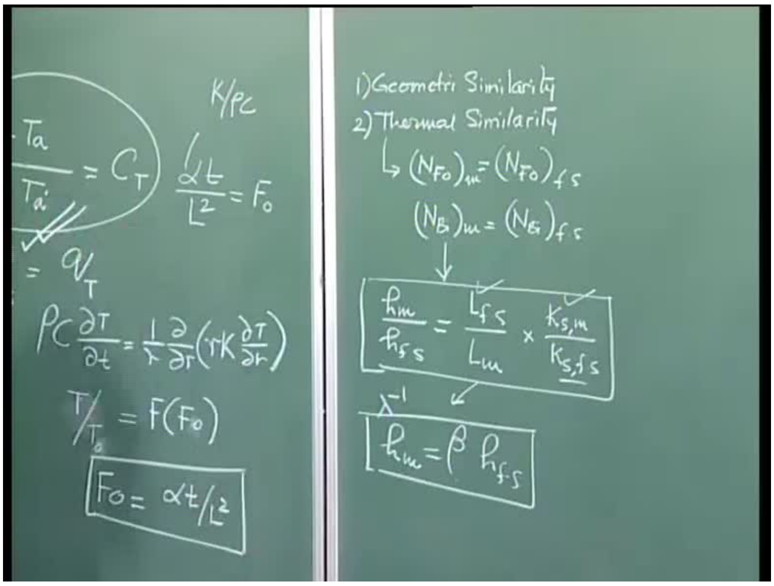

Therefore, this particular expression will give me that look h model is equal to some constant beta into h sub f s in which beta is nothing but lambda inverse multiplied by k s m divided by k s, f s. So, having known the value of heat transfer coefficient in the full scale system, I should be able to adjust my heat transfer coefficient in the system in such a manner that this condition is going to be maintained.

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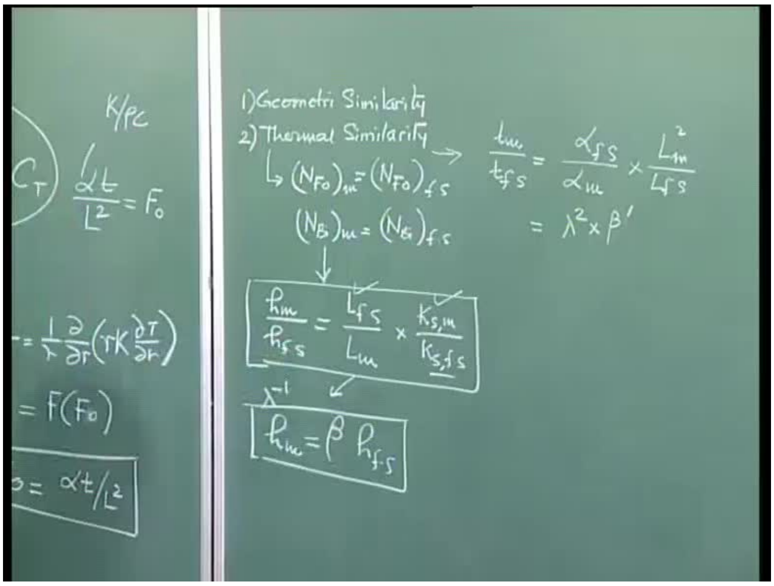

Similarly, the Fourier number equivalence will tell me that look I have t model by t full scale. That is going to be alpha model alpha full scale. So, alpha t and then L square; so, this is going to be L model square by L full scale, which is equal to lambda square into some constant, maybe the biot. Thermal conductivity – this is k divided by rho C. So, these are material-dependent properties; thermal diffusivity.

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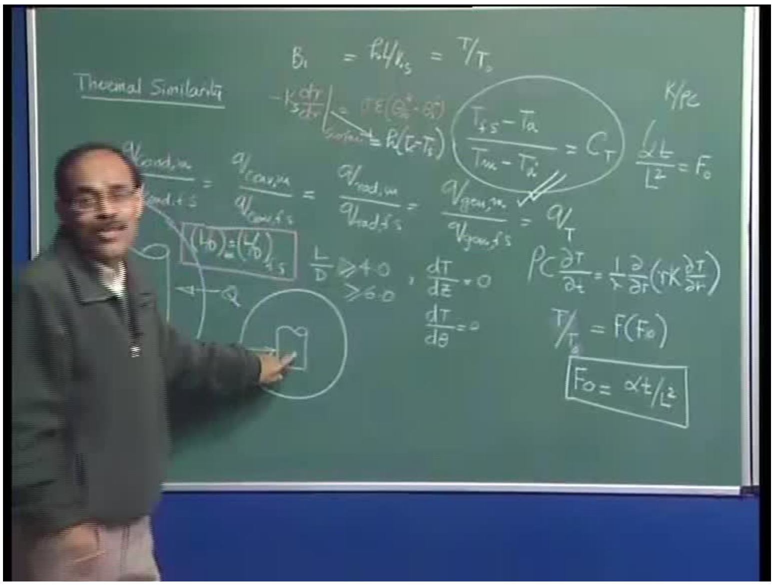

So, once we have fixed the material… because what are the two steps? I said the first step is deciding the scale factor; second is decide on the material.

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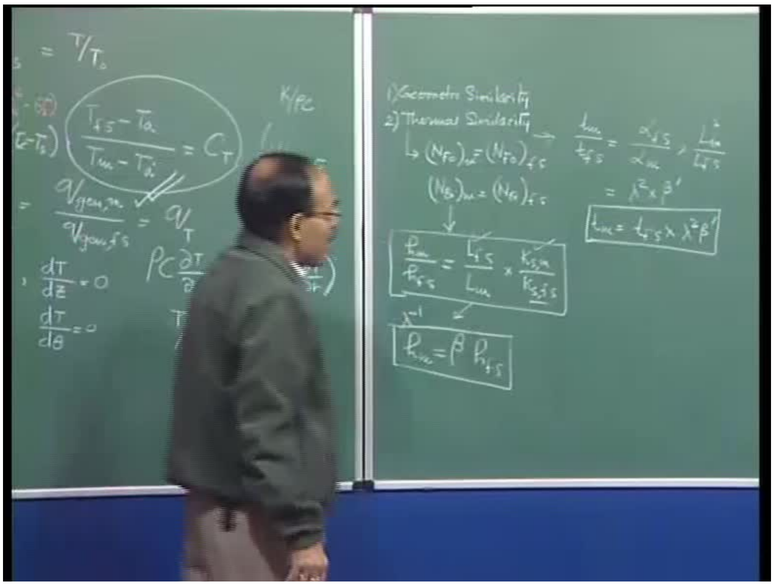

So, the moment you have decided on this particular material, this parameter alpha m has become fixed. You know alpha f s already – what is the material in a full scale system. So, this parameter is also known. So, now, we can say that the corresponding time – so, t f s into lambda square beta prime. Therefore, if time sum t is equal to 0… Again, when you talk of time in the full scale, that means you choose an arbitrarily datum. So, the point at t is equal to 0, you have 0 plus; you insert the sample into the furnace. So, from that point onwards, if you say after 10 seconds; in that case, you put 10 seconds here and find out what is value of alpha square in beta prime and then find out that what is the corresponding time. And, this will be also with reference to the point at which you insert the sample in the laboratory scale.

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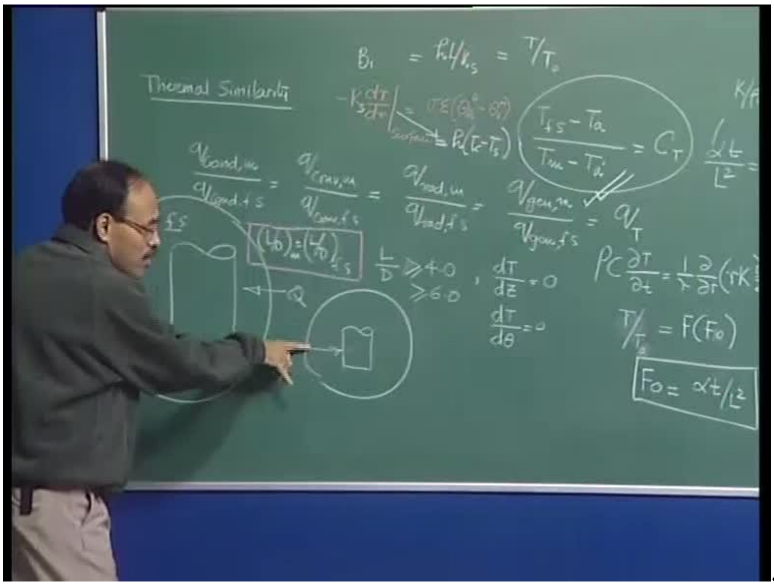

So, this will tell you the correspondence between the time scale. This gives you the (Refer Slide Time: 41:12) correspondence between the…

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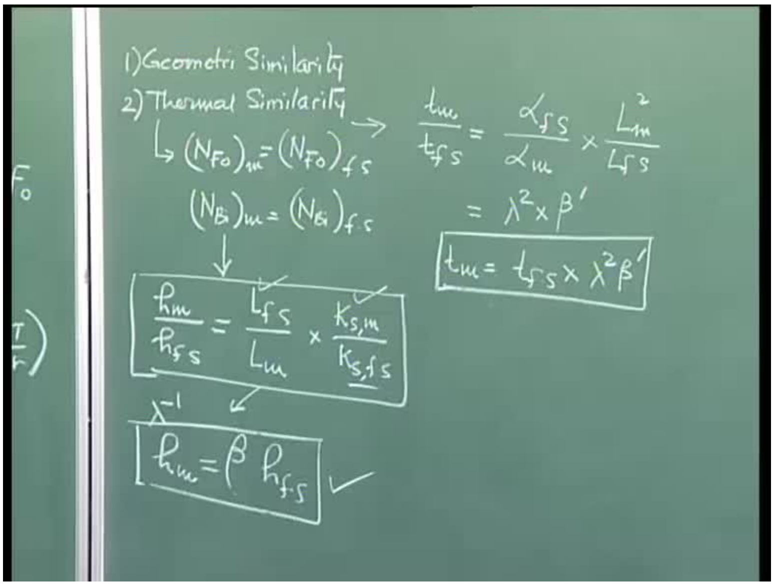

So, if we select the material, if we select the scale factor, then first job is to ensure that only the heat transfer coefficient has to be in accordance with this equation. And, once heat transfer coefficient is in accordance with this equation, (Refer Slide Time: 41:29) I can map the temperature here at different times; and then, say that corresponding to this time, if this is the temperature measured here, the corresponding temperature in the full scale system is going to be such and such, which will be on the basis of (Refer Slide Time: 41:44) a time scale governed by the Fourier number quality. Therefore, when you talk of heat transfer thermal similarity, we must understand that we will not only talk about the governing equation or we will not try to derive the similarity criteria based on the governing equation, but we will have to consider the boundary conditions as well.

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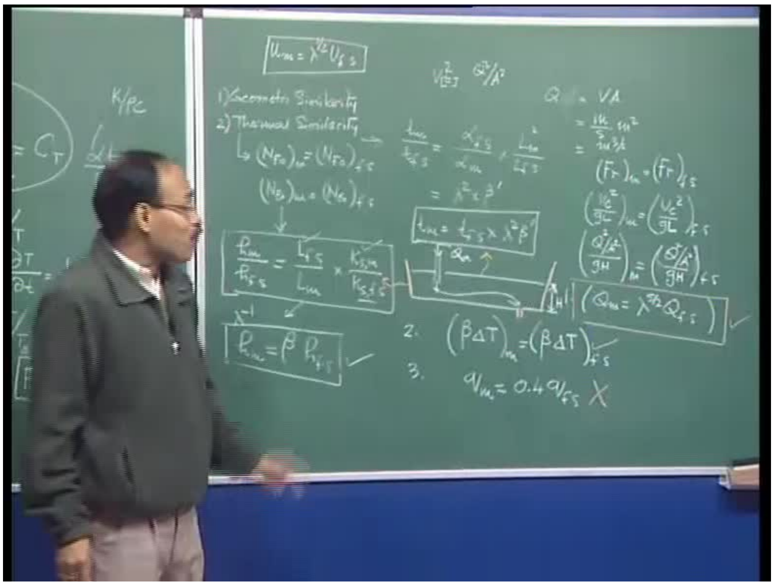

Now if you look at – for example, take a practical case; this is a very simplified scenario. If you take a heat flow in a tundish for example, a single strand tundish – I am showing it. And, I want to carry out a laboratory scale model; I have chosen a scale factor. I want to do the experiment with water, for example. And, suppose I have made a glass vessel tundish shaped and I wish to carry out an experiment; I want to see the flow pattern also. But, I want to consider non-isothermal flow. Now, in this case, although I can scale down on the basis of the governing equations, get the numbers and maintain the equivalence, it will be very difficult for me to maintain the equality, which will come out on the basis of the boundary conditions; because you can imagine that if you have a 1600 degree molten metal in a vessel, tundish, you have some finite amount of heat loss.

Now, water – I can only use up to 100 degree centigrade or 95 degree centigrade. And, at 95 degree centigrade, if I use a glass vessel, I will see that the rate at which the heat will be lost, that similarity I will not be able to maintain with the actual steel making system in the shop floor, because they are the rate of heat flow that is going to be significantly larger. So, it will be very difficult for us to maintain the similarity of heat loss behavior in laboratory scale system by using a glass vessel of ladle or tundish or mold, etcetera. So, thermal similarity studies are going to be really difficult.

Now, for example, if I consider the boundary condition and the governing equations together, I find out that, if this phenomenon… Now, this (Refer Slide Time: 43:59) is a… There is a geometrical similarity; there is going to be dynamic similarity; there is going to be thermal similarity, because we are talking about non-isothermal system. And, because of the tundish, there is going to be some fluid flow involved. Therefore, as I have mentioned repeatedly that before we can talk about thermal similarity, we have to talk about dynamic similarity. So, we will talk about dynamic similarity and then we will talk about thermal similarity. So, dynamic similarity essentially in this case, is going to be the same number. So, it is the Froude number equivalence. Froude model must be equal to Froude prototype full scale. And therefore, on the basis of this, what is Froude number? We have seen U C square by g L model. This is equal to U C square by g L full scale.

If I consider the characteristic velocity as this particular velocity… And, what is the velocity? Velocity is nothing but volumetric flow rate by cross-sectional area. So, I take the volumetric flow rate, Q inlet divided by the cross-sectional area of the nozzle. Then, I get what? I get the velocity. And that velocity I can ascribe it to be equal to C. In this case, the meaningful length scale is actually the depth of the liquid, which governs the flow rate out of the tundish under the steady state condition. It is not orifice dimension, but it is the depth of the liquid, which is H (( )) So, I can say that from the basis of this, we can write down Q by area nozzle divided by g H in model; and, Q represents what? Q represents the volumetric flow rate. And therefore, I can say this is going to be (Refer Slide Time: 45:52) is equal to Q divided by A over g H full scale.

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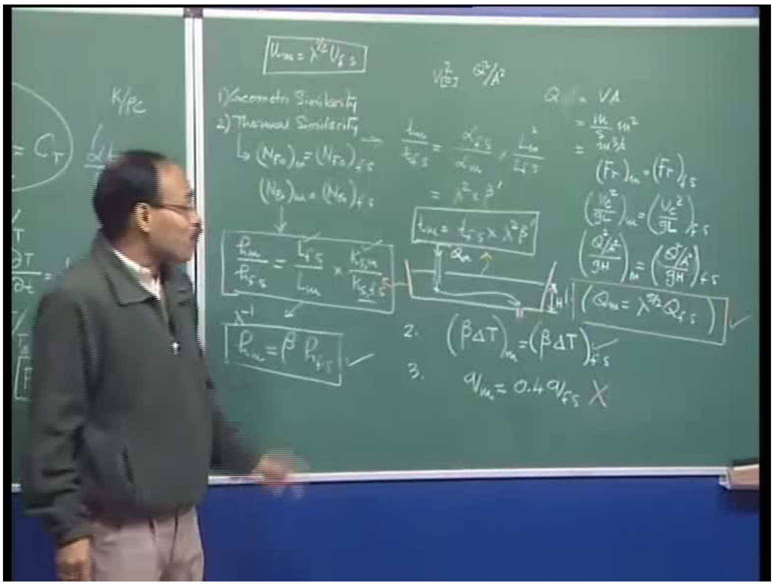

Now, area model by area full scale is lambda square; H model by H full scale (Refer Slide Time: 46:11) is equal to… So, this is – you can see here it is U C square. So, one U C is equal to Q by A. So, this is going to be actually Q square by area square; Q square by area square. So, what is Q? Q is equal to volumetric flow rate into cross-sectional area, is equal to velocity. So, we have velocity is meters per second and this is meter square (Refer Slide Time: 46:48). So, this (( )) Therefore, we can say V square has a dimension. So, this is the equality of dimension; that means, dimensionally, V square is equal to Q square by A square. And, that is what I have basically written, that U C square is equal to Q square by A square. So, A square model divided by A square full scale is equal to lambda to the power 4; H model by H full scale is equal to lambda to the power 1 and that is equal to 2. Therefore, I can say, this equation will follow very easily that Q model is equal to lambda to the power 5 by 2 into Q full scale. So, that means, again you see this 5 by 2 correspondence comes; and, the 5 by 2 correspondence essentially comes from the definition of Froude number, which tells us that U model is equal to lambda to the power half into U full scale.

So, the velocity is in two systems, which are dominated by Froude number, varies in proportion to lambda to the power half to the… Consequence of this is the (Refer Slide Time: 47:58) 5 by 2 relationship, which we have also seen earlier. So, when we talk of tundish… Therefore, to maintain dynamic similarity, we can say that the model – if I know the full scale flow rate, 3 tonnes per minute, I should be able to find out that what is the corresponding model flow rate in terms of the scale factor itself. So, this is on the basis of the dynamic similarity. So, we have considered the Navier-Stokes equation and this is the limiting form, (Refer Slide Time: 48:23) because we have ignored Reynolds number.

Now, you have to consider no boundary condition for Navier-Stokes equation, because boundary conditions are all alike in the full scale in the model system. So, to derive the thermal similarity criteria, now, I visit the thermal equation and also I know that with the thermal equation, thermal balance equation I have to consider the boundary conditions. Now, the thermal similarity tells us that this (Refer Slide Time: 48:50) condition… I am not going to derive this as beyond the scope of discussion here. So, this is going to follow from the governing heat flow equation, which is applicable to the flow of liquid in the tundish; so, the condition 2; and, this is the condition 1 (Refer Slide Time: 49:07).

And, condition 3, which will come out from the boundary analysis of the similarity of the boundary condition is that q model is equal to 0.4 into q full scale. In which, q represents the rate at which it is being lost from all the surfaces of the melted (( )) – this condition (Refer Slide Time: 49:30). Only when these three conditions are simultaneously satisfied, we can say that yes, the model tundish that we have fabricated in the laboratory will exhibit not only geometric similarity, but also simultaneously dynamic and thermal similarity. Now, as I mentioned, therefore, we imagine typically in industrial surfaces, we can have 5 to 10 kilo watt per meter square as the heat flux. So, if it is 10 kilo watt per meter square, in that case, what we require in the model is going to be 4 kilo watt per meter square. And, if I have a glass vessel and 100 degree centigrade temperature of water, I cannot get 4 kilo watt going out of the melted self going out of the liquid water bath through the glass vessel itself. So, this condition can be satisfied (Refer Slide Time: 50:14).

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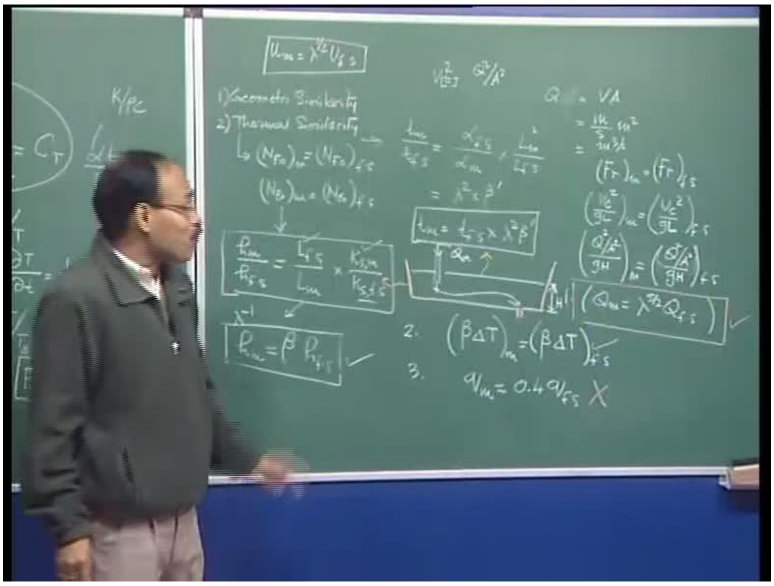

What is beta? The coefficient of volume expansion. And, delta T again is a differential temperature – bath temperature may be with respect to the inlet temperature. This difference may be the delta T. We can assume, the difference with respect to any arbitrarily chosen reference temperature; the reference temperature may be the inlet temperature; you may choose wall temperature or man temperature, whatever you are convenient with. So, this condition (Refer Slide Time: 50:40) can be fulfilled, because beta in steel and beta in water are going to be known to us. So, if we know the delta T value in the case of steel, we should be able to find out that what should be the corresponding delta T, because we are trying to find out. The fluid is coming here; why is the issue of thermal similarity coming here? Because steel is coming here at 1600 degree centigrade.

The temperature of steel within the tundish is going to change significantly, because lot of heat are going to go out of the tundish through the tundish wall itself. So, that is why the non-isothermality comes into the picture, because of continuous loss of the heat from the molten metal. And therefore, the metal comes here (Refer Slide Time: 51:26) at the 1600 degree centigrade, but it does not go out through the stranded 1600 degree centigrade. This goes out at a relatively lower temperature. There is a temperature gradient in the system. So, that is why we want to find out, we want to study that differential temperature of non-isothermal behavior. So, we are also going to use a vessel, in which we are going to put in water at 95 degree centigrade. But then, there has to be some kind of a heat loss also in order to replicate the exact behavior of steel flow in the tundish system. And therefore, we consider the governing equations of fluid flow to get to the dynamic similarity criteria.

We consider the governing equation of heat flow to get to the thermal similarity criteria. And, with the thermal similarity governing equation, we also consider the boundary conditions to thermal similarity. So, three sets of number come out from the analysis. So, on top of geometrical similarity, we derive this (Refer Slide Time: 52:22) on the basis of our assumption that the flow phenomenon is governed by the Navier-Stokes equation and it is Froude dominated. This is from the thermal similarity; this is (Refer Slide Time: 52:30) from the boundary condition. And, as I have mentioned it, that this condition can be fulfilled; this condition can be fulfilled in the laboratory scale model very easily. But, using transparent glass vessel and water as the liquid, there will be no scope to fulfill. Therefore, thermal similarity studies of steel making systems in laboratory scale modeling are going to be really difficult.

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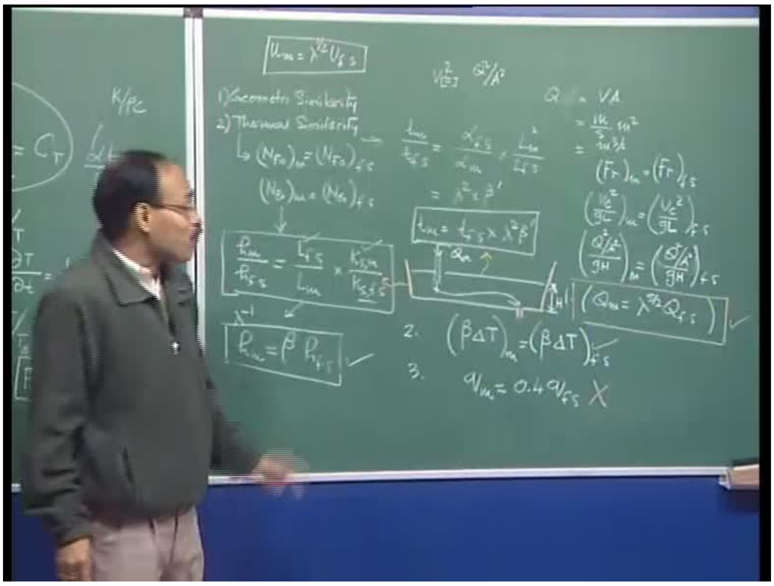

How do you get out of this then? How do you investigate process? As I will discuss later on, in such problems, where there is lot of uncertainty investigating a process, we resort to mathematical modeling and then we address steel making problem, get some numbers, scientifically visualize those. We may not physically observe them, but we can certainly scientifically observe them, observe the numbers and then conclude that at which point temperature is more, which point temperature is less and so on. So, wherever physical modeling is not going to be appropriate or accurate, in that case, we have an alternative way and that is mathematical modeling that we are going to do.

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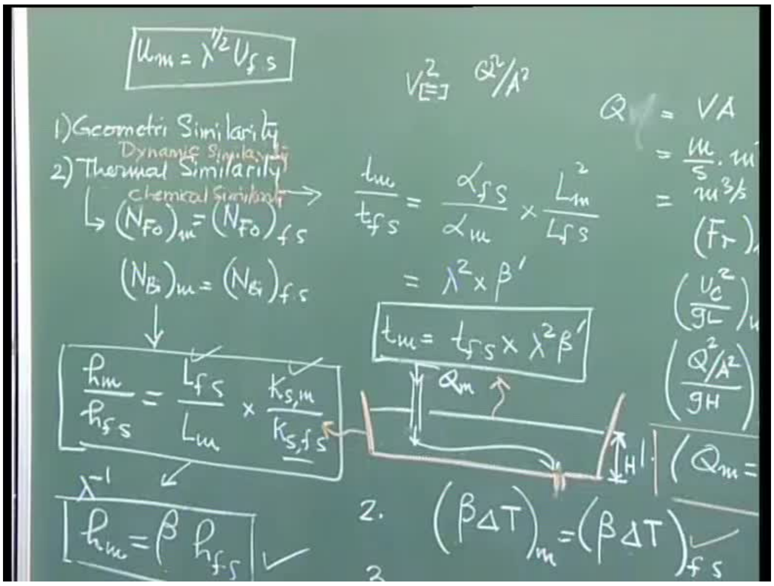

Now, the last state of similarity that we talk about in addition to geometric; and then, we have here, which we did not consider in this problem is the dynamic similarity. And, the last state of similarity is what is known as a chemical similarity. I have to briefly mention you before I conclude the discussion particularly on the states of similarity. The pretty exhaustive discussion really I have given and you should be able to understand the potential of physical modeling and some of its limitation and appreciate the beauty of the subject, because if I do not give you a sufficient background, maybe you will not be able to see, our lectures not going to be meaningful to you. So, these are the four states of similarity actually – 1, 2, 3 and 4. And, we have talked about the 3 and the last if I wish to make a few points on chemical similarity.

Now, when we talk of thermal similarity, we talk of temperature, heat flow similarity. When we talk of dynamic similarity, we talk of flow similarities and we talk of chemical similarity. That means we are talking about similarities in concentration profiles. Now, chemical similarity comes into the picture, because our systems may be reacting systems. (( )) they are all reacting systems indeed. We have carbon oxygen reaction, we have desulphurization reaction and so on. So, we would like to stimulate. Therefore, in physical modeling, chemical similarity to get similarities in concentration profiles of a particular species.

Now, just like the species transport, is like heat transport, heat is also energy. So, it is a scalar quantity. Flux is a vector quantity; heat is a scalar quantity. So, the species transport or mass fraction of species, which is moving from one place to another place, is also scalar. So, the same type of argument is going to hold good in the case of chemical similarity also. So, I will say now… Remember the definition that I gave you in terms of thermal similarity. I will map a one-to-one definition for chemical similarity also. Chemically similar are those systems – in place of temperature, I am going to say – where concentration difference with respect to an arbitrarily chosen datum maintain the fixed ratio at corresponding location at corresponding time.

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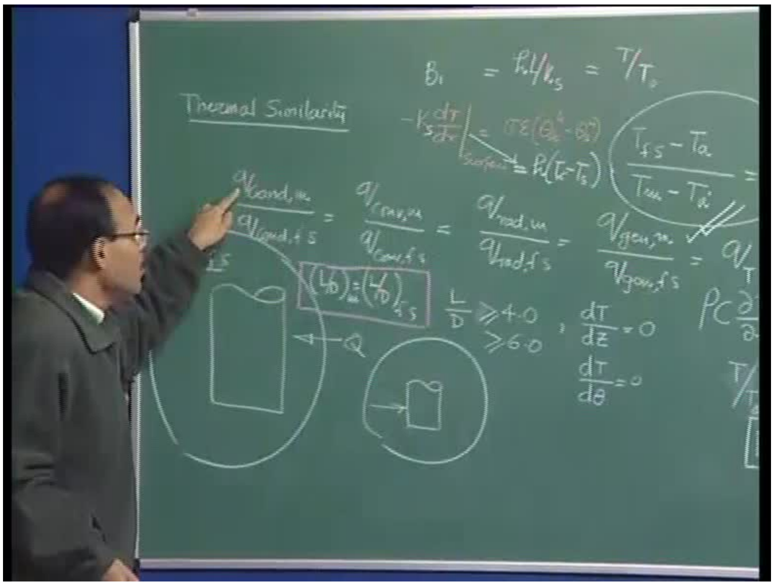

So, replace that word of temperature by concentration. Whatever I have defined, replace this T's by concentrations or mass fractions; you will get the conditions, the results of the chemical similarity. These are the reasons why heat was transported from one point to another point. What are the corresponding mechanisms? Mass transport by diffusion in the model to the diffusion in the full scale; mass transport by convection in the model to convection in the full scale; radiation does not take place in the case of system. There may be mass generation or depression because of chemical reaction. So, if you know thermal similarity and if you understand the definition, you can immediately formulate the criteria for chemical similarity.

Thank you.