Steel Making
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Module No.# 01
Lecture No. # 37
Modeling and Measurements

So, what we have seen is that the motion of the fluidor the flow of the melt in a steelmaking reactor, such as ladle tundish, mold or a converter is the consequence of various kinds of forces, and consideration of these forces, give usthe necessary framework to develop on the dynamic similarity criteria, and I have indicated thatin systems; where inertial forces, gravitational forces, viscous forces and pressure forces are only relevant there.

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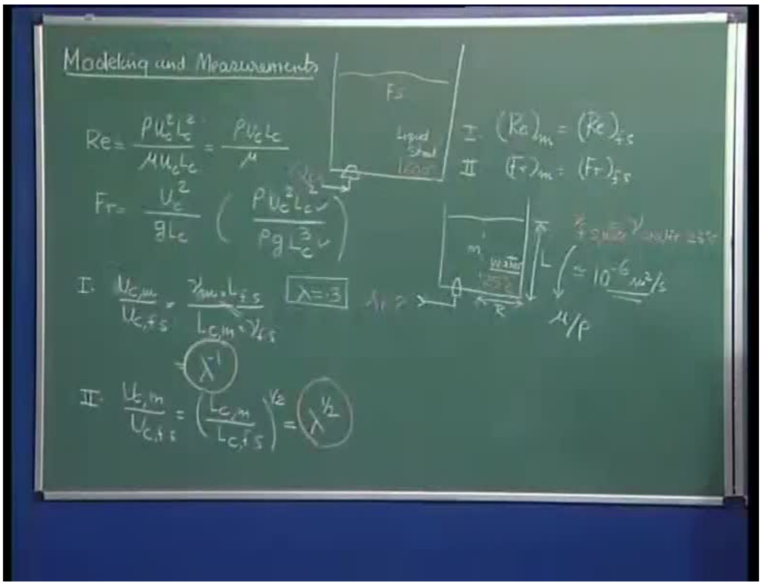

The similarity can be expressed in terms of two limiting consequences, that is the Reynolds numberin the model is equal to Reynolds number in the full scale, and Froude number in the model is equal to Froude number in the full scale.

So, if you can maintain this, additional numbers are going to follow, if you have more number of forces that i have already discussed.

So, if you can maintain these two equality, in that case we can say that, yes the systems are dynamically similar.Of course before you can do this we have to respect the geometrical similarity. So, geometrical similarity plus satisfying Reynolds similarity as well as the Froude similarity, will ensure that the two systems are going to be dynamically similar.

Now, let us look at the definition of Reynolds number, Reynolds number as we know is the inertial force by viscous force, now inertial force as an expression like L c square and viscous force as an expression like u c into L c. So, this is a dimension of Newton, this will also have a dimension of Newton, and if we simplified this then we get rho u c L c, this is a traditional definition of Reynolds number as we encounter.

These two are therespectively, density and viscosity of the melt; u c is the characteristic velocity L c is the characteristic length.

Now, what is a characteristic velocity and a characteristic length scale, these are the velocity and length scale of the system that are relevantfor the present analysis. So,I am going toexplain thatbit more, when I write down.Similarly, we can say that the Froude number definitionaccordingly we will come out to be U 2c by g L c.

So, you can write down gravitational inertial force, in the denominator we have gravitational force, so rho times g times volume and L square L square, actually how it comes, let me say it comes out like this, this is the same expression of inertial forces, and gravitational forces is rho g into L c cube.

So, we have rho rho cancels out, L c square and L c square cancels out, so we have L c, L sub c in the denominator, and that is how, so this is inertial force and this is the gravitational force, and the definition of Froude number is the characteristic velocity square divided by the acceleration due to gravity into length scale.Now, the characteristic velocity coming back to the issue of characteristic velocity and thecharacteristic length scale, these are the velocities and the length scale which are known to us.

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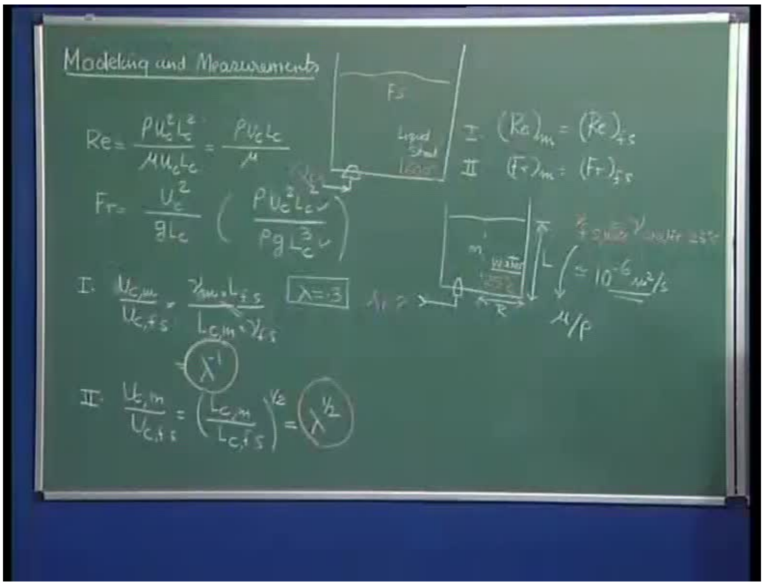

Now,for example I have a gestured ladle system I cansay, where I have a porous plug located at the bottom. You can say I have depth of liquid L, and I have radius of a diameter of the vessel and if it is, whether L this depth of the liquid should represent the characteristic length or the radius of the vessel or the diameter of the vessel should represents the characteristics, that is an important issue to decide, because it is not usually forthcoming,for example in tundish you may have three different length scale, length of the tundish, width of the tundishand depth of liquid in the tundish, and of this which one has to consider in order to.

So, as you have seen that the forces acting in the systems, so this is the inertial forces which is acting on the system and the inertial forces is as a result of the fluid motion, and we have to understand that which corresponds or which is a more meaningfulentity, eitherthe length scale or the radius of the vessel which should come to the consideration of characteristic length.

Similarly, characteristic velocity, what is that velocity. Is it this velocity through the orifice, which should come into this particular expression or it is some other scales of velocity. So, it is some amount of thinking about the process is necessary, before one can explicitly quantify, that what is a characteristic length and what is a characteristic velocity scale.

Because there are multiple number of characteristic length scale and characteristic velocity, or length scale and velocity scale which are there in the system, and of these only one you can take or consider as a characteristic length scale and characteristic velocity scale which can be used here, to determine the relative magnitude of thetwo dimensionless groups.

Now, one point you can note here that the Reynolds number depends on thermo physical properties of the fluid, density comes into the picture, viscosity comes into the picture. On the other hand, Froude number does not depend on the thermo physical properties of the liquid, it is independent.

So therefore, if I can say somehow that a phenomena is Reynolds dominated, one of the limitingconditions, that if we if we can approximate the system as the Reynolds dominated system, a Reynolds dominated system means the Reynolds number is not very large, therefore we can say that the viscous force is very small, and in that case viscous force is very large, and in that case the Reynolds number is actually very small.

So, we can say it is a viscous force dominated situation that essentially is Reynolds number governed flow. On the other hand if we say that it is inertial and gravitational force dominated system, in that case I can say that it is a Froude dominated system.

So,I repeat again, so if I say that we have inertial force and viscous forces are important, but gravitational forces are not important then I can characterize the system to be Reynolds domination.

On the other hand if I say that the system is governed by or dominated by inertial forces, and the gravitational forces and viscous forces is not important, in that case I can say it is a Froude dominated systems.

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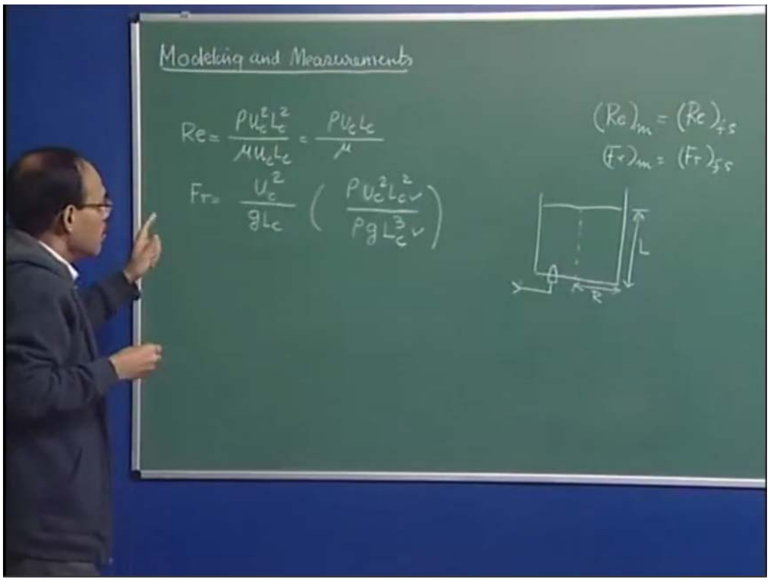

So therefore, if the system is Froude dominated, if this number governs the process, in that case we must understand whatever is the fluid that we used has no consequences as for as dynamic similarity criteria is considered ok.

So,therefore we will come back to this issue in a minute, but now let ussay that what is the fluid that we want to use here, and then this issue of Reynolds dominated flow versesthe Froude dominated flow will be more clear to us.

But, in a sense for a accurate modeling, we would like to respect both Reynolds equality and Froude equality, in those situations where inertial viscous gravitational and pressure forces are important, but we may see that it may not be always possible to maintain both these equality simultaneously, as these going to be evidentfrom our analysis.

Now, suppose the full scale system which contains molten steel as this, and this is the model system and this is the full scale systemand we have.Similarly, we have a porous slag here or through the porous slag, we have argon injection and this is liquid steeland this is. Suppose, for the time being I say that I use a fluid which is water, so I want to make a transparent model, because I want to visualize the flow.

Now, I canof course as you will see later on,I canmathematically model the flow in the system and find out that what is the velocity here, what is the velocity here, what is the velocity here,I can map the velocity, this is called scientific visualization, and on the other hand if I create water,I can physically see how the flow is going on, provided the vessel is made out of some transparent material like, phosphates or gas.

So, this is called physical visualization, so when I construct the physicalmodelI am considering or I am examining the possibility of physical visualization of the flow, and therefore I can find out that, well if I use water then in that case I can visualizethe flow very nicely, if the entire ladle is fabricated out of plexiglas ortransparent plastic sheets.

On the other hand,I would say if you say sir I want to use mercury in that case,I would say that physical visualization of the flow is not possible, because mercury is nottransparent, so using water gives us many advantages. One of the advantages of the first very advantage is water is the cheap liquid, the second advantage is that it is non-corrosive, it is non-hazardous liquid.

And the third, from the view point of studying the process, it is extremely important to recognize that water is transparent,therefore we can very conveniently physically absorb the flow pattern,for example if the flow is there and I drop,I add a drop of potassium permanganate, then I can see how does the color distributes, and thereby I can very clearly see that what is the flow pattern in the system, because of these kind of a gas injection through a porous plug.

So, physical visualization is possible with water, and that is why water as a replacement for liquid steel is, you know has been very popular, and as I have mentioned already using water as the bulk fluid makes this process, what we call as a or known as aaqua’s modeling or water modeling.

So, for the time being, let us assume that we make novel laboratory for the sake of convince, we would like to use water. Now, one important point which all of you mustnotethat water at room temperature, if you are using at 25 degree centigrade steel at 1600degree centigrade, they are kinematic viscosity is approximately equal, I think why should I say approximately, it is only one percent actually the value, it is for as first approximation I can say, that is almost equal to nu water.

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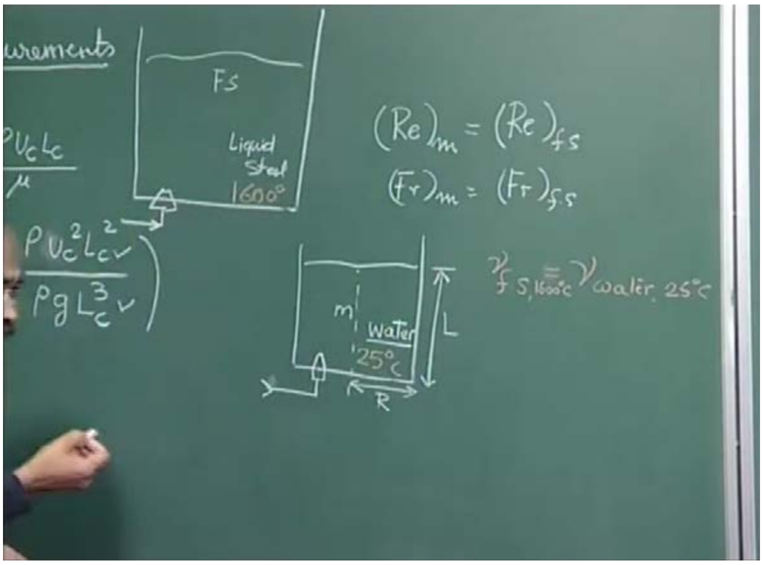

So, must note that these are state properties,therefore I must mention that they are at the room temperature, so this is 25 degree centigrade and this is 1600 degree centigrade.

And what is the value, the value is in S I unit, it is approximately 10 raise to the power minus 6 meter square per second, this is the value.Now, if you release water and now I have constructed a water model,I have chosen a scale factor, suppose I say that lambda is equal to point 3, this is what I have chosen. Now, I have to consider these two equalities in order to, so I have geometricallymade the vessel similar to a full scale system,I havescale with down in size, in terms of the chosen scale factor.

For the sake of convenience I had used water, and now I want to find out thatthe systems have to be dynamically similar. So, this equality, and this equality has to be respected. Now, you must remember when I know the argon gas flow rate in the full scale, the flow rate of air here, if you are usingas a replacement is really not known to us.

Everything else I have scaled it down,I have scaled it down,I have used water everything is set except for one single parameter that what is this organ flow rate or airflow rate that we are going to use through this, while this flow rate in the full scale is known to us.

This is not known to us, and this will be known by considering that these two equalities and the necessary flow rate, which will have to be injected in the model, and which will generate a flow pattern in the system that will make this system dynamically similar with full scale system.

So therefore, by consideration of the dynamic similarity criteria, the flow rate is going to be derived, so let us try to make the Reynolds numberin the model is equal to Reynolds number in the full scale. So, we say, as you see that what is kinematic viscosity; the kinematic viscosity is actually dynamic viscosity divided by density that is the definition of kinematic viscosity, so I can say that Reynolds model criteria one and criteria two. So, criteria one would give us u characteristic model by u characteristic full scale, is equal to L characteristic model, nu characteristic or nu model and L full scale into nu scale.

But, by definition L characteristic ratio of the characteristic length is equal to lambda. So,I will write down that, this is equal tolambda raise to the power minus 1, because this characteristic length of a model divided by characteristic length of the full scale is equal to lambda.

So, the lambda is in the denominator and then, because nu model is equal to nu full scale,I have already said that I am using water in case of steel, their kinematic viscosity being identical, so I can say that this term and this terms will cancel out, they are approximately equal, and therefore the characteristic velocity scale in the system are inversely proportional to the geometric scale factor.

This is the consequence of Reynolds equality. Let us look at the Froude equality, and the Froude equality will tell us that well u c model by u c full scale is equal to then you take L c model by L c full scale characteristic length, full scale raise to half.

Because, it is u c square, so ratio of the velocities are going to be square root of the ratio of the characteristic length, that relation due to gravity will vanish from both the sides and this tells us lambda.

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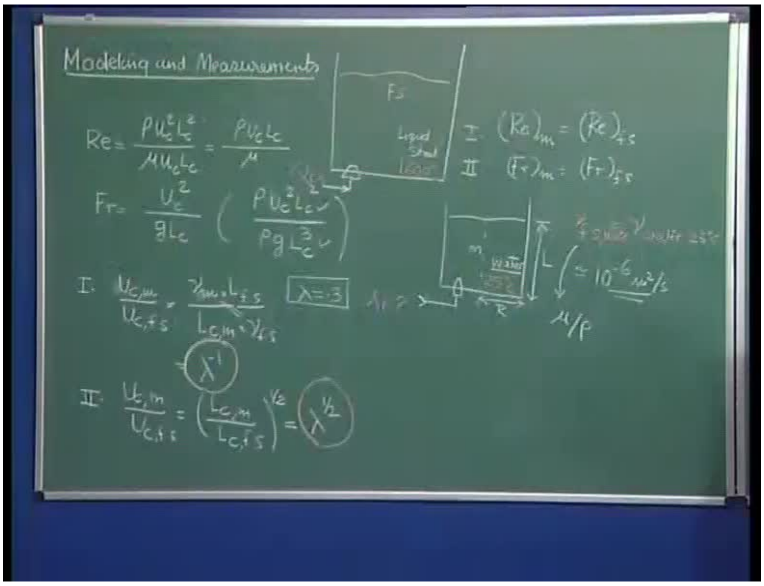

Now, one criteria says that the ratio of the characteristic velocity scale is inversely proportional to the Reynolds, similarity demands that the ratio of the characteristic velocity scale should be inversely proportional to lambda, the Froude equality criteria demands that the characteristic ratio of the characteristic velocity in the two system to be proportional to lambda raise to the power lambda, raise to the power half.

So, these two criteria, the Reynolds criteria and the Froude criteria, therefore cannot be respected simultaneously, we cannot meet them, you see that the one in the same equation, but giving us the different functional relationship.

So,therefore, we will not be able to maintain these two equalities simultaneously, where we use kinematic viscosity of the model. We use a fluid whose kinematic viscosity is identical to the kinematic viscosity of the full scale system. So, we make a very important deduction at this particular stage, in reduced scale modelinggoing back to this equation, both theseconditions, both these equation can be respected only for one single value of lambda, and that lambda is going to be equal to unity,

So, in full scale system, if the size of the model vessel is exactly equal to the size of full scale system, in that case we can say that I can respect both these Reynolds and the Froude similarities simultaneously, even though I may be using the fluid which has the same kinematic viscosity as that of a steel. Alternatively if I say that no I am talking of a reduced scale modeling in that case,I will make this particular assumption.

This particular assertion that in reduced scale modeling employing a fluid having the samekinematic viscosity as that of the full scale system, it is impossible to regard both Reynolds and Froude similarity simultaneously.

So, then what is the way out, so we have to make now one assumption, and this is the limiting assumption we have to say, and I come back to that point which I have just now mention, that of all the forces which are acting in the system are all the forces equally relevant, if they are equally relevant in that case we cannot make any simplification. My objective now is to analyze the process, and throw out one of these two criteria entirely from the analysis.

But,I have to rationalize this, now these are the consequences of inertial viscous and gravitational forces in steel making system, we note that the size of the reactor is very large, so L c is going to be the characteristic length which is large.

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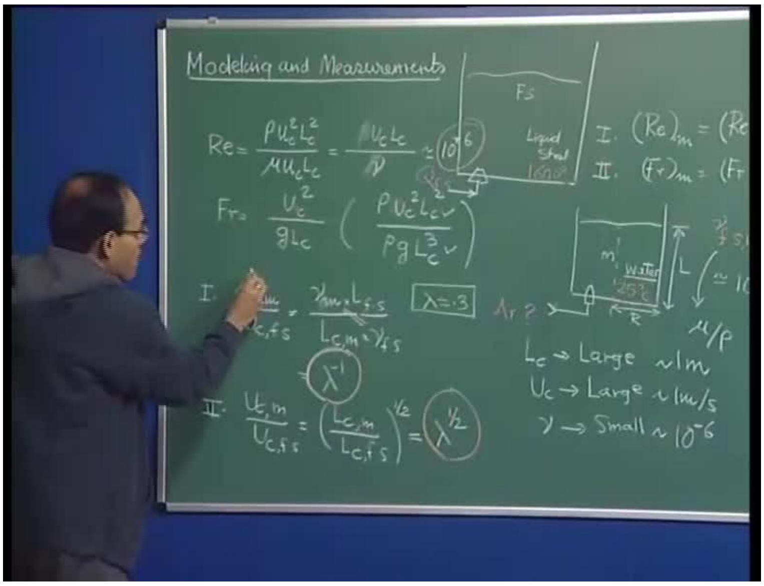

Similarly, thelevel of agitation in steel making system is also going to be very large, so the velocity is also very large, this is also expected to be large in steel making system, and the kinematic viscosity is going to be extremely small.Therefore,I can look at this, look at this parameters mu by, if I now consider this as thisone, and the same thing, so I have removed density and viscosity and replaced viscosity by the kinematic viscosity, which is viscosity divided by density.

So, now if you look at thisorder of these valueswhat could be the value of L c, may be of the order of one meter that means the reactors are three meter in size, four meter in size, they are not one centimeter or five centimeter in size.

Characteristic velocity is very large, it depends on what you are talking about, if you are talking about a furnace tapping operation, in that case the extent of stirring in the ladle can be of the order of tens of meters per second.

On the other hand, if you are talking about gestured ladle system in our gas injection, in that case the order of the velocity could be one meter per second, so as a first approximation, let us say that we are talking of one meter per second and then the kinematic viscosity is small, and the value is already given.

So therefore, we find that the value of the Reynolds number will come out to be on basis of this order of magnitude, analysis of the order of 10 raise to the power minus 6, what does this number indicates. This number indicates that the inertial force in the system is one million times to larger than the viscous force.

Similarly, if you analyze the ratio of this numberwhich comes out to be Froude number, which is equal to u c square by acceleration due to gravity, so this is oneu c, is of the order of one, this is L c characteristic length is of the order of one, so this Froude number will come out to be of the order of 10 raise to the power minus 1, because g is approximately 9 point 81,which I am taking it to be is equal to 10.

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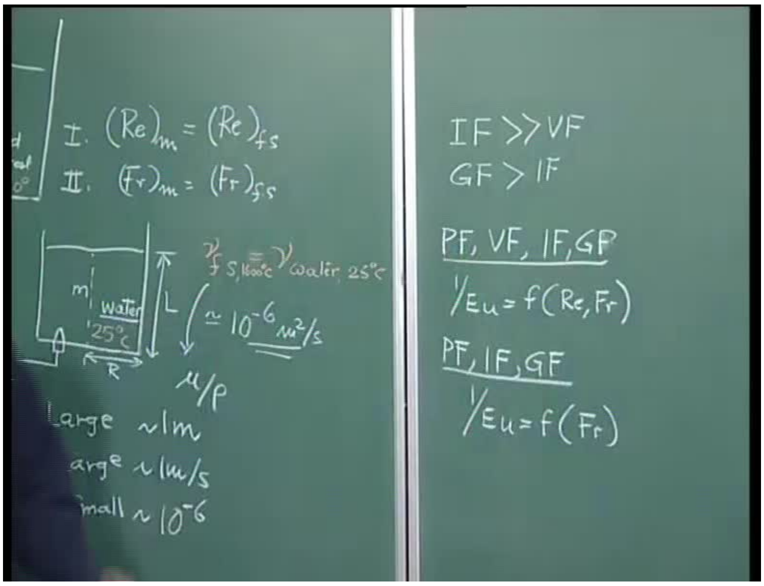

So, it is this order of this magnitude will come out to be 10 raise to power minus 1, which will essentially tell us that the gravitational force is 10 times larger than the inertial force itself.Therefore, Reynolds number tells us that inertial forceis much larger than the viscous force. The Froude number order of magnitude value tells us that gravitational force is larger than the inertial force.

So,therefore I can say that well, the viscous force ininertia gravitational force is larger than the inertial force, and the inertial force is much larger than the viscous force which tells us that of the various forces, which are acting in the system.

The viscous forces are least; they have no consequences as far as the flow of the fluid in the system is concerned.Therefore, we can say that well, let us forget about the viscous force, the similarity of viscous force is not an issue as far as deriving the dynamic similarity criteria is concerned.

So, if you throughout the viscous force, in that case what are the forces we are remain with. Now, we havehere,for example as I said,I started the discussion by assuming pressure force, viscous force and then you have an inertial force and we have gravitational force.

And based on that I have derived this criteria, and I have also shown that, when these are the four forces I can say that well in non-dimensional force, this functional relationship holds good, but now if we say that well, the pressure force is there, the viscous force is not there, inertial force is there, gravitational force is there, only three forces are there in that case, the functional relationship will boiled out to.

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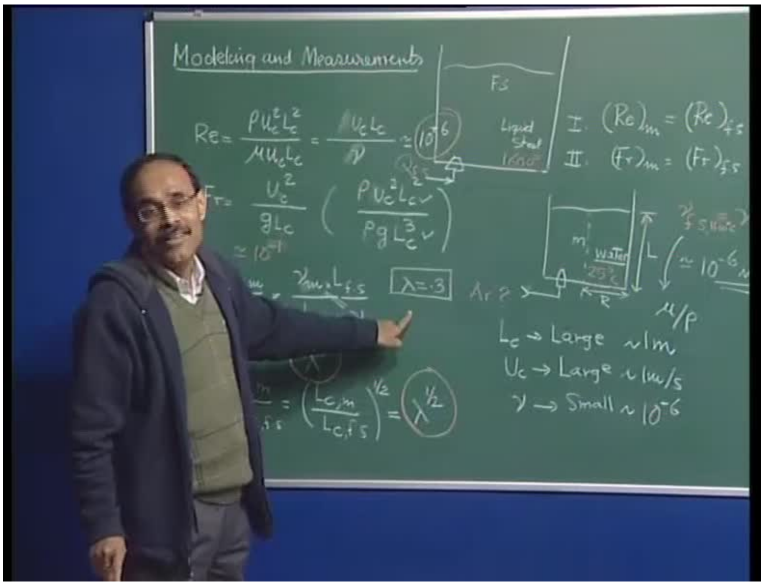

I have thrown out by doing this analysis; Reynolds number, and why I have thrown out, because I want to use reduced scale modeling in my laboratory,I want to use water and I cannot respect both Reynolds and the Froude similarities simultaneously. So, as a result I have to disregard one of these two numbers, and now I am doing in induct analysis of the process,I am trying to assign some order of magnitudes and thereby I come to this conclusion.

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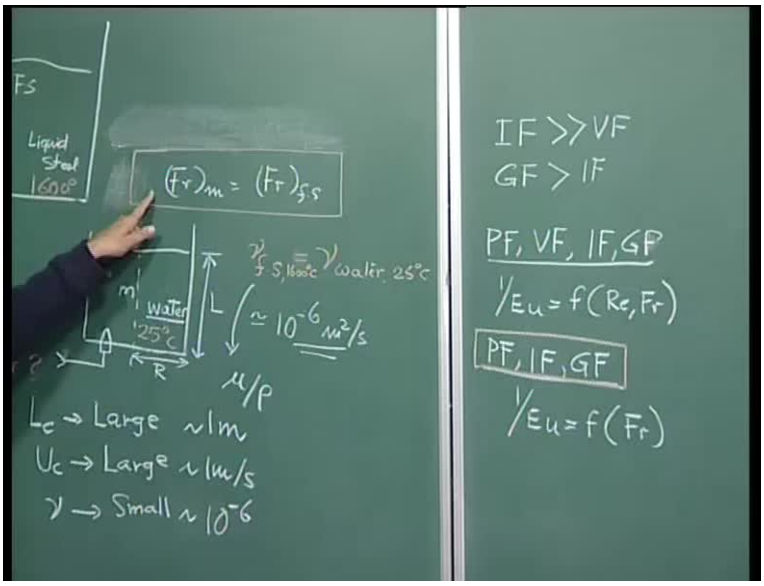

Then the viscous force in the system is perhaps not so important, and therefore you can ignore it and therefore, the forces which are relevant in the system are actually this under the assumption, while you have started the analysis with this, and this was the dimensional non-dimensional representation.

Now, a pressure force, inertial force and gravitational force are there, there are the relevant forces in that case, this is the limiting expression, and therefore I can say that well. Now I am going to remove this, that Reynolds criteria in reduced scale modeling using water is not so important, and hence we can consider that dynamic similarity of steel making systemare going to be defined only by this single criteria, providedsurface tension and other forces are not relevant itself.

So,therefore we will make this assertion that steel making systems are Froude number dominated systems, they are the systems which are dominated by inertial as well as gravitational forces.

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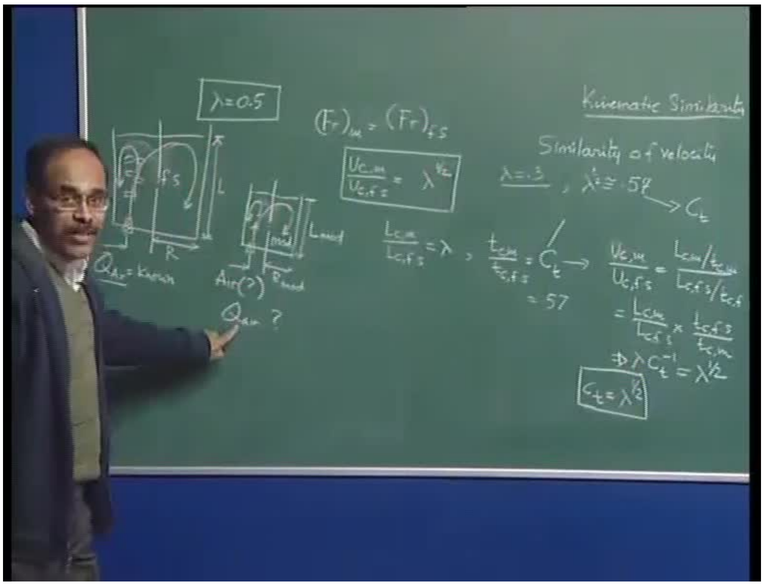

So, the consequence of stating that the Froude number equality holds good or this represents, the modeling criteria or dynamic similarity criteria, in reduced scale modeling is in water, we can say that, therefore the characteristic velocity scale in the system are going to be related in proportion to lambda.

So, the characteristic velocity in the two systems are going to be related in terms of the scale factor, in accordance with this equationprovided, we say that a system is dominated by problem.

So, we will have geometrical I will tell you now, that on the basis of this, howthe argon flow rate can be calculated or knowfrom the full scaleflow rate is given to us, but before you do that I am going to also say a few words about kinematicsimilarity.

So, dynamic similarity between model and full scale is going to be maintained, when the systems are Froude dominated with this particular criteria. So, geometrical and dynamic similaritiesare criteriacan be derived very easily,or known to us, you know on the basis of the analysis that I have just now presented.

Now, kinematic similarities so this is the similarity of forces, and kinematic similaritytalks about similarity of velocityother is kinematic, it is define like corresponding particles in model and full scale; describe the same type of path or street lines in corresponding intervals of time.

So, the consequence if the systems are kinematically similar then you can see that well, if this is my full scale ladle, and suppose in the full scale ladle the flow is going something like this, and if it is my model ladle and in the model ladle also you will see that, the similarity of flow pattern is an essential reflection of kinematic similarity, kinematically similarare those systems, which exhibitsimilarity in flow fields, and we define it like this, that corresponding particles in model and full scale system trace out similar for geometrically similar path in corresponding intervals of time.

Now, if I say therefore, let us look at the consequences we know that l c model characteristic length in the model and characteristic length in the full scale is going to be always equal to lambda, and kinematic similarity says that weare going to have characteristic, suppose time scale t c characteristic time in the model by characteristic time in the full scale, and that is equal to some constant c sub t,I do not know the constant value of the constant.

But, we must understand thaton the basis of this I can find out that what is the corresponding ratio of velocity, the consequence is going to be,for example v characteristics,I can say therefore, on the basis of this model by u characteristic full scale is going to be is equal to u characteristic model is nothing, but l characteristic model by t characteristic model, and this is going to be l characteristic full scale divided by t characteristic full scale.

Length by time is nothing, but velocity. So, once these proportionalities are maintained I should be able to find out that what is the corresponding ratio and therefore, I can say that this is going to be is equal to l characteristic model by l characteristic full scale into p characteristic full scale by t characteristic model.

And this I have seems to be is equal to lambda, and this is equal to c sub t inverse. So, this is equal to c sub t inverse.I can say that well, if the system is now Froude dominated for example, then I already know what is this particular ratio right, what is this particular ratio, this is equal to lambda to the power half provided the system is Froude dominated, because I have shown you that the characteristic velocity scales are in proportional to lambda raise to power five.

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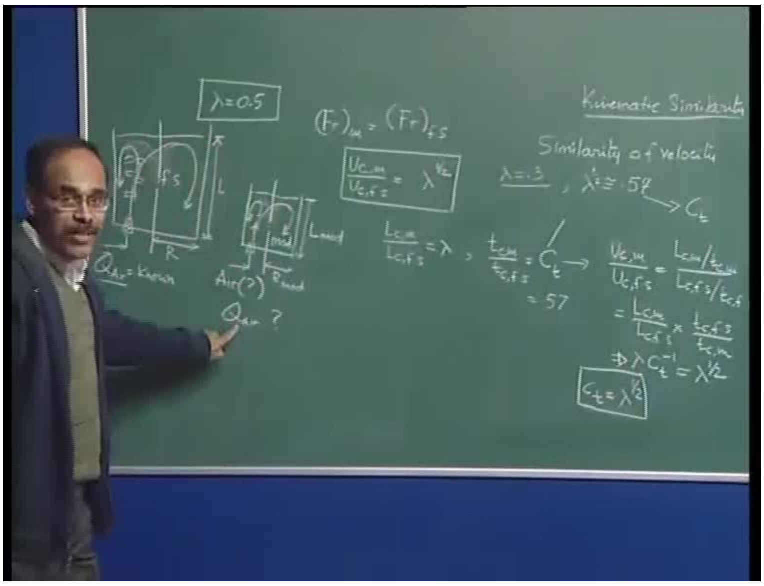

So,therefore I should be able to say that what is the corresponding and this comes out to be therefore, c t is equal to, I take it to the other sides, so this becomes c sub t lambda divided by lambda to the power half is equal to lambda to the power half, so the corresponding time scales in the system now is known in terms of the scale factor lambda itself.

So, the concept of thatas I mentionedthat corresponding particles in that in model length full scale system trace out geometrically similar paths in corresponding interval of time and that corresponding interval of time will be govern by this particular relationship which gives us that the time scale in the model and the full scale system.

The ratio of the time scales in the model and the full scale system is equivalent to lambda raise to the power half.Therefore, in one case, in the full scale case if it is for example, c t is equal to,I can say roughlylambda, suppose if I take lambda is equal to point 3 in that case, so lambda to the power half is going to be is equal to roughly about saypoint 5 approximately.I am saying this is going to be 2 point 5 may be 57 or something like this.

So therefore, if I know thatthis valueis going to be is equal to c sub t, so if I put this point 57 here,I will be immediately able to know that, if I am talking about one second time in the full scale system, that one second time actually corresponds to point 5 seconds 57 seconds time in the model system.

If,I am talking about ten seconds time in the full scale system,I will be talking about 5 point 7 seconds in the model system, and in this way I should have an idea of the corresponds between the time scales in the two system itself.

So, let us consider nowthe dynamic similarity and show you thathow can you really model the argon flow rates, based on the consideration, because one issue needs to be now sorted out at well, we have consideredthe flow pattern,of course now is not like this a flow pattern will go something likeyou know, it goes something like this and then it goes something like this, that is displaced towards something like this.

So, you have rising velocity here, because the gas is rising up and then the gas leaves surface and that is how the surrounding goes, and the systems suppose they are dynamically and kinematically similar, so that is why I have.

But, as I mentioned to you while this dimensions are known to me, this dimensions are known to me, this is the central line, this is the central line and note that I amalways drawing the porous plug displaced from the central line, because you remember that I said that this is to be converted later on to a ladle furnace, and in ladle furnace we do not want the porous plug to be located immediately below the electrodes, because it is going to otherwise, let to electrodehunting or electrode consumption that is why if this porous plug is always displaystowards the one, now this dimensions l this is the depth of liquid,I am not saying this is l l sub c, this is l and this is r, so this is r sub full scale, this is l sub full scale, this is l sub model and this is r sub full scale, r sub model.

So, these dimensions I have obtained once,I have selected that well lambda is equal to 0 point 5, the normal diameters are not so important to us, you again remember I said that in the context of gesturing in ladles that conditions at the nozzles of the orifice are not critical to the flow rate circulation, because these are the systems which are driven bypotential energy rather than the kinetic energy of the injected gas.

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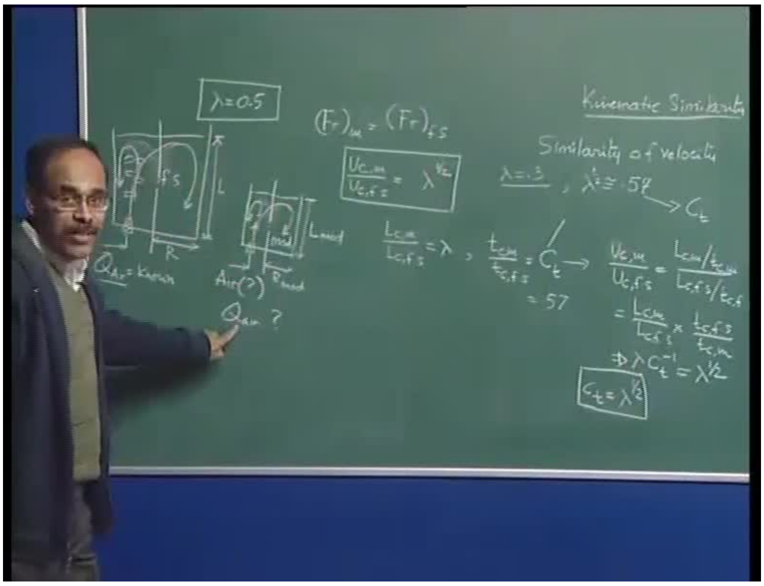

The kinetic energy of the injected gas depends on the nozzle diameter, but the potential energy or the buoyancy energy which is supplied by rising bubbles does not have anything to do with the porous plug or the nozzle, it is depends on the equilibrium size of the bubbles or the distribution of bubbles in the system itself, which is govern by thermo physical properties of the fluid.

So therefore, whether the injection of gas in the model in the full scale system is by porous plug or I use a nozzle here or a orifice here, it is hardly any consequence, but matters here is that, what is the flow rate of air or argon which I say q air,for example q is the volumetric flow rate is the usual representation, and I say that this is not known on the other hand q argon is known.

So, having similarity for the nozzleis does notmake any difference, we can use anything what you want to have, we have we want to what we find out that, what is a the flow rate, so this case therefore, it becomes somehow distorted model, because I am not scaling the nozzle dimensions.

But as I mentioned that my understanding is like this, as I mentioned to you also that hydrodynamic conditions at the orifice or nozzle is not critical to flow the circulation

So therefore, I may do not try to scale down these dimensions,I can disregard this so to someone else it must look like a distorted model, but to people who understands the ladle hydrodynamics well they can say no, even this model would not a porous plug, but some kind of atuyere you know give us the right kind of a simulation or right kind of acurrent dynamic similarity provided we can (( )) scale out the scale, the flow rate which is needed here.

So, let us make few idealizations as wego on to derive that what is an appropriate volumetric flow rate needed in the model, in order to have this dynamically similar with this, becauseI told you thatmetallurgical systems or steel making systems in particular are extremely complex. Therefore, we have to make certain assumptions and certain idealizations.

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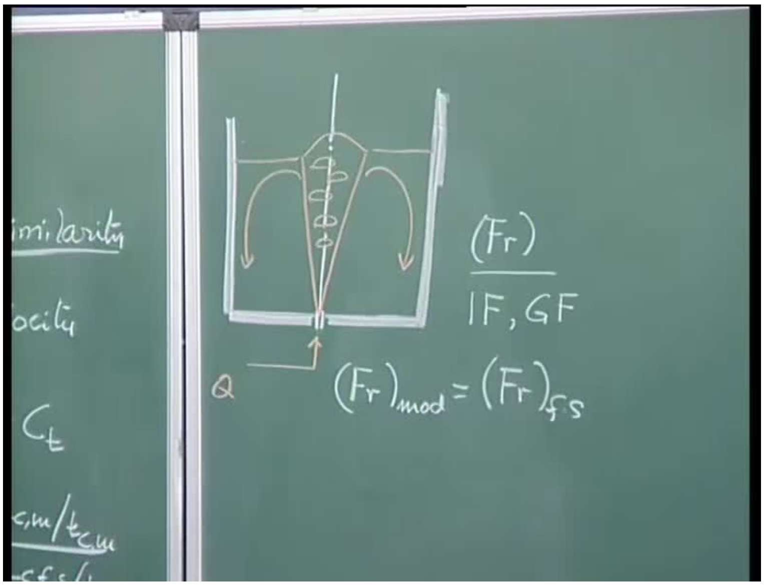

And even then we may see sometimes that the analysis does not producemuch convincing result, so let us for the time being demonstrate to you,I considering a central gestured ladle system by central gestured ladle system,I mean that well I have a porous plug or a nozzle which is located at the central line.

The theories are well understood about central gas injection, even though the central gas injection is not practiced in the industry, so we have something like this a two phase plume which develops here, and growing an idealized plume boundary and that is howthe gas is injected from the bottom, and once gas is injected it creates bubbles here, the bubbles raised and as a result of which the fluid moves in this particular fashion, so I can say that well this represents my vessel.

Now, in this system as per ourearlier assumption that we are considering Froude number is the onlyrelevant dimensionless number which governs the modeling or the dynamic similarity criteria, and let us assume that the system is geometrically a similar and now our objective is to find out that what is that flow rate.

So therefore, in this system we have inertial force and viscous forces which are inertial force and gravitational forces which are importantand the modeling criteria is only expressed in terms of Froude number, in the model must be is equal to.

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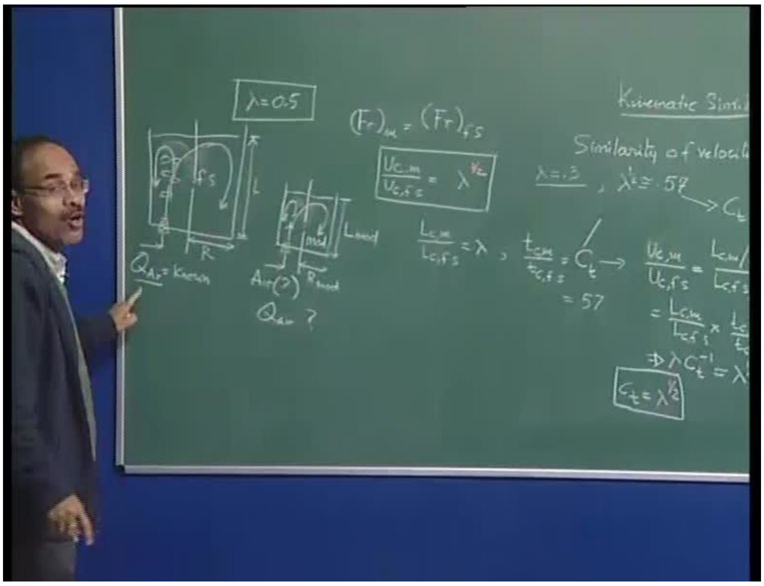

I am going to show that by considering this similarity number, the similarity of the Froude number or equivalence of Froude number between the model, and the full scale and expression for model flow rate of gas can be found out in terms of full scale gas (( )).

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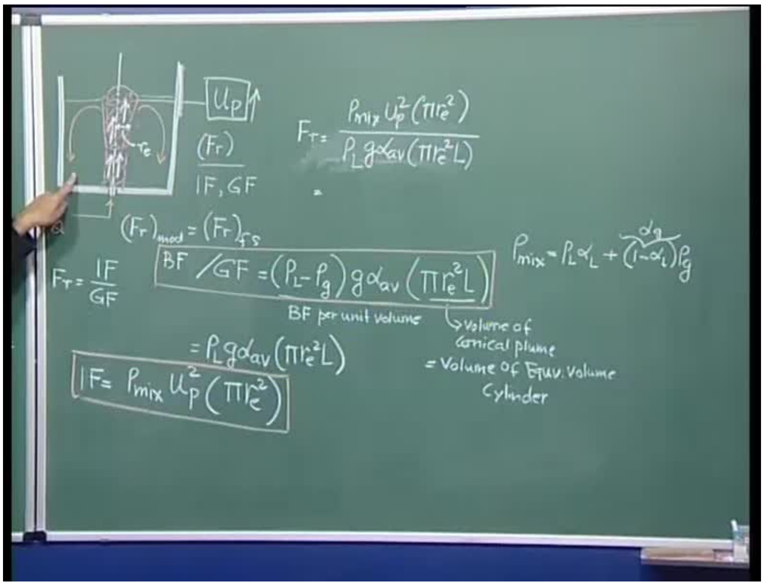

Let us now, what is the definition of Froude number, we have is inertial force by gravitational force that is equal to Froude number, now let us this gravitational force here, essentially is the buoyancy forceor we can say thatthe net force net gravitational force is the buoyancy minus.

I am going to explain this to you, so this is the net buoyancy force that is a density difference, rho liquid and rho gas, so one is the weight of the bubble and other is the related with the weight of the liquid, which is the equivalent to displaced volume or the mass of the displacement.

Now, this represents the buoyancy force per unit volume, buoyancy force per unitin this system where are the buoyancy forces acting, the buoyancy forces are acting only within the two phase plumed region, there is no buoyancy force here, there is no buoyancy force here.

So,I am going to find out now a ratio of the inertial to gravitational forces or buoyancy forces within the fluid region itself, and if I say that let me now idealized this plume in terms of a equilibrium cylinder, what they mean by an equivalent cylinder,I mean by an equivalent cylinder is that the volume of the cylinder is exactly equal to the volume of the idealized two phase plume, which is the gas liquid conical region that I have drawn here.

So, the cylinder has and this therefore, represents the volume of the cone conical plume volume of conical plume which is equivalent to volume of equivalent volume cylinder, which I haveequivalent volume at r sub i,therefore represents this; e represents the equivalent volume, it is the radius of the equivalent volume, these essentially represents what is my r sub e, the radius of the equivalent volume cylinder.

So, buoyancy force per unit volume multiplied by the volume of the two phases region gives us the gravitational force or the buoyancy force which is acting this,I can say buoyancy force by gravitational force.

This is associated with the acceleration due to gravity, and this is the total buoyancy force which is actinggravitational force which is acting within the two phase plumed region, and now considering that the gas density is very small,I can say that I can also write that this is.

What is alpha average means. The alpha average means that we have some gas volume fraction here, so may be 10 percent,20 percent or 5 percent of the gas volumegases are occupied, so the plume has a volume. So, these alpha average multiplied by the total volume pi r e square represents, what this represents the volume of gas which is present in the two phase region.

In reality as I have mentioned earlier in the context of secondary steel makingdiscussion, on secondary steel making or ladle metallurgy that is going to be variation of gas volume fraction, but as I said we had finding out a modeling criteria based on some sort of idealizations and assumptions.

So, for that sake we have assume that, let the volume fraction of gases be distributed, eventually there is an uniform average gas volume fraction which is nothing, but alpha average even though we may have variationin gas volume fractions, we can integrate that over the plume and find out an average or a representative gas volume fraction. So, these are representing the total amount of gas which is present in the two phase plume region. This is the expression of buoyancy force.

Now, let us look at the inertial force within this, so visualize this to be like you have a cylindrical region, and you have a two phase mixture which is flowing, you know it islike a you can visualize this cylinder like a pipe, and then the flow of fluid through a pipe only thing is that you are having flow of a gas liquid mixture.

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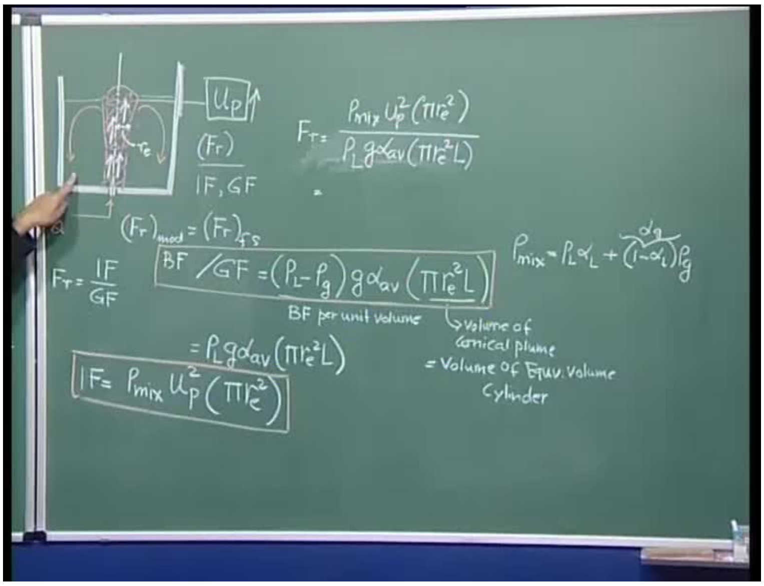

So, by definition again the inertial forces as we have seen, it is rho mixture, because that is the density of the fluid, you remember what I have written for inertial force expression.

Inertial force expression is nothing, but density multiplied by the velocity square multiplied by the cross sectional area, so inertial forces within acting, within the two phase region is equal to u sub p square, and I am assuming that the gas liquid mixture is raising with a constant velocity, which is normally called the plume raised velocity and this also I have discussed in the context of secondary steel making or ladle metallurgy.

So, the plume raised velocity essentially is the upward rise velocity of the gas and the liquid mixture, so if that is thevelocity and it is constant across the length, again this is an idealization and then I can say a pi r e square is a cross sectional area,therefore this now represents the inertial force.

So,I have got an inertial force which is acting in the plume region,I have got the buoyancy force which is acting in theplume region, now if I take the ratio of the two, then I can find out, let me just quickly erase thisand then show it to you that inertial force by gravitational force which is the definition of Froude number,therefore the Froude number will come out to be rho mixture u p square into pi r e square, and here we are going to have rho liquid alpha average into pi r e square.

So,I can say that now coming back to this, if you remember I also mentioned that gas volume fraction within the plumes are going to be very small, because the injection flow rate in the ladle metallurgy steel making is small,therefore we can say that the density of the liquid, what is the mixture density, the mixture densityactually as per the continuum approach is rho liquid into alpha, liquid plus 1 minus alpha l or which is equal to into rho g.

So, alpha g this is equal to actually 1 minus alpha l, alpha l is the volume fraction of liquid, alpha g is the volume fraction of gas alpha g plus alpha l is equal to 1, because in any control volume.For example, here alpha l is equal to 1, alpha g is equal to 0, here we have finite value of alpha g,we have finite value of alpha l.

So, the weighted average of the gas density, a liquid density plus gas density is equal tothe mixed density, just like the way you say, if you have a matrix of different materials.For example, we have pearlite, which consist of ferrite andcementite.

So, how do you say the hardness of pearlite, so you sayvolume fraction of ferrite multiplied by hardness of ferrite plus volume fraction of cementite, plus into hardness of cementite, and then you find out which is called a volume fraction or weighted average hardness of hardness based on a volume fractions.

So, it is based in the continuum approach, and this is the way, the same way you define the hardness for acomposite material, you can say that the mixture density is going to bethe fraction of the liquid density, plus the fraction of the gas density and that is what it is.

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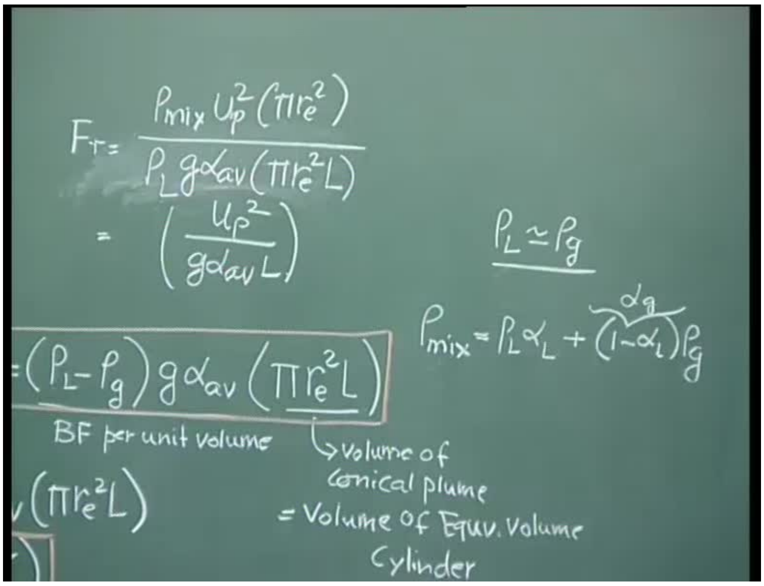

That, if the gas volume fraction is very small, in that case we can say that, well I can approximately considered that rho liquid is approximately is equal to rho g, and this is going to be true provided that the gas volume fraction is very small, and as I have shown in the context of ladle metallurgy that two to five percent of gases are there in the plumed region which tells us that thiskind of a simplification is indeed possible.

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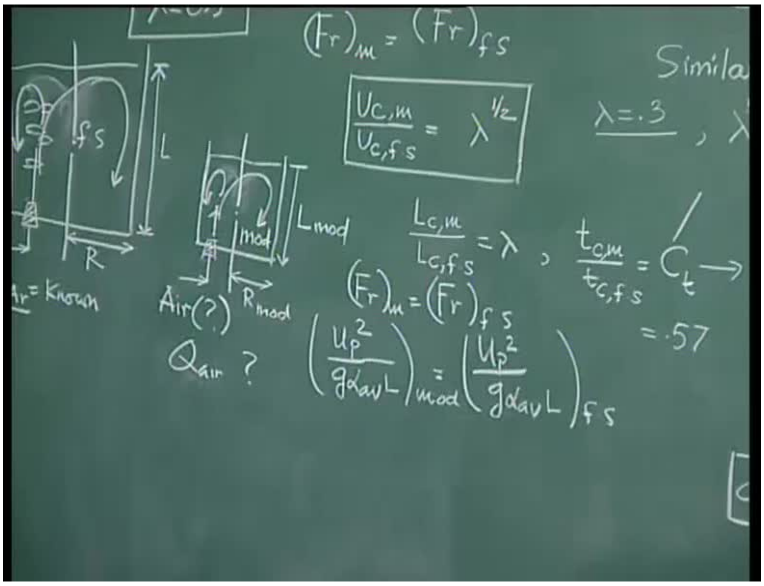

So, then we can say that this equation will boil down to g into alpha average into l, sonow this is the definition of the Froude number, and therefore I can say Icomeback here,and show you,therefore from the equation that Froude model must be is equal to Froude prototype or full scale,I can say that u p square g alpha average into l in model is equal to u p square g alpha average full scale.

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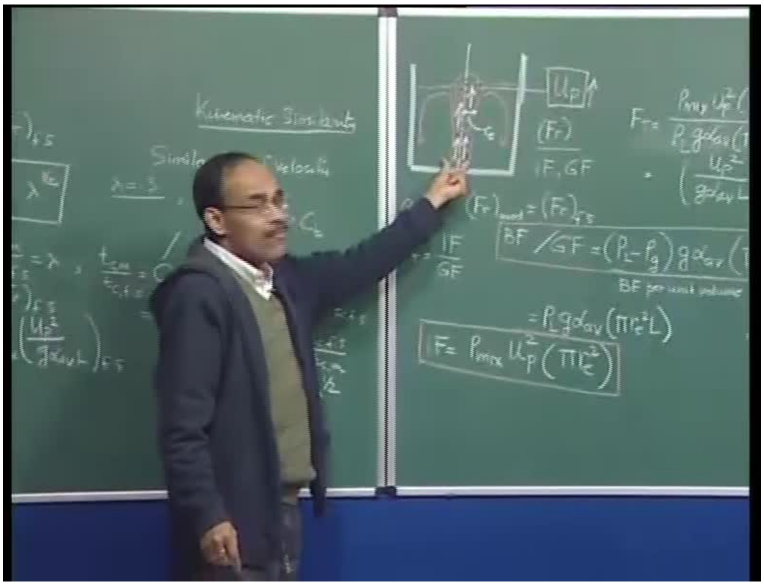

I make one trust assumption, but not to unrealistic if the plumed angles are not to different, and the flow rates are small, I would expect thatif the gas volume fraction inmodel is two to five percentage, similar kinds of gas volume fractions will also be there in thefull scale system, because we are talking of the similar flow rate ranges in between in the model as well as in the prototype.

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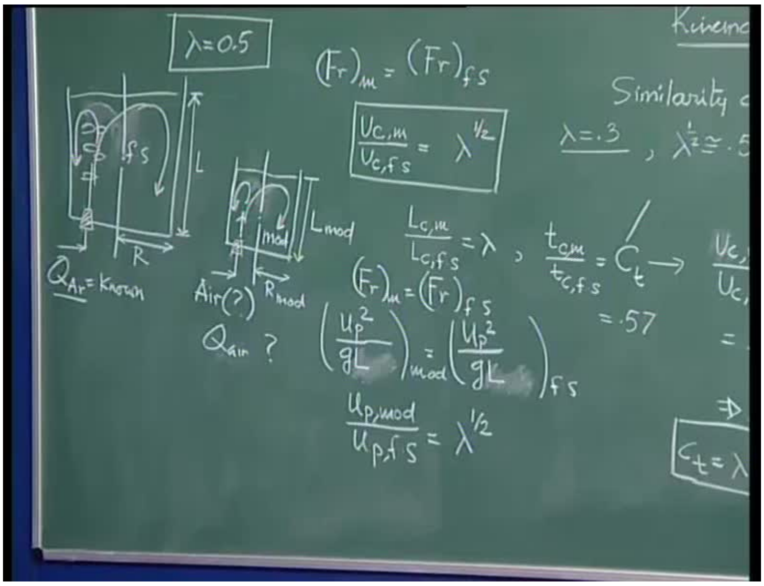

So, as a first approximation I can say that well,alpha average in the model is equal to alpha average in the full scale, if I make that assumption in that case it turns out that now u p model, by u p full scale will come out to be lambda, so I can erase this now, there is no necessity of, so I can say L.

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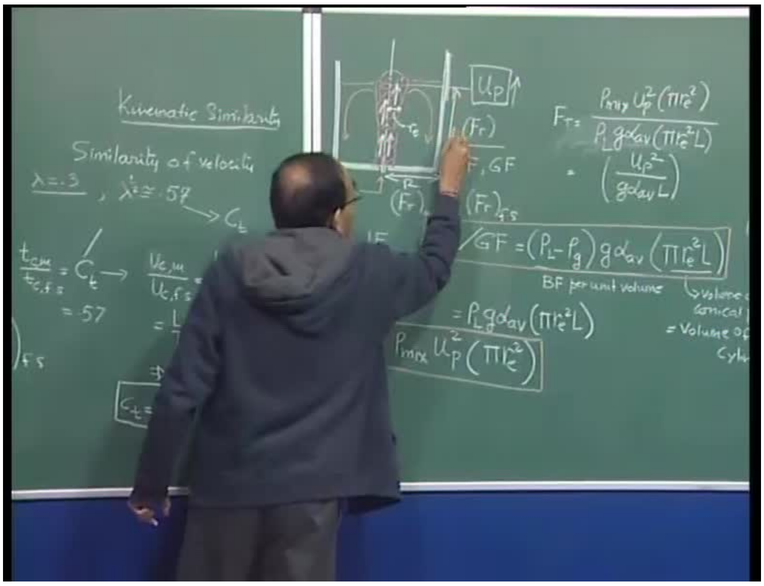

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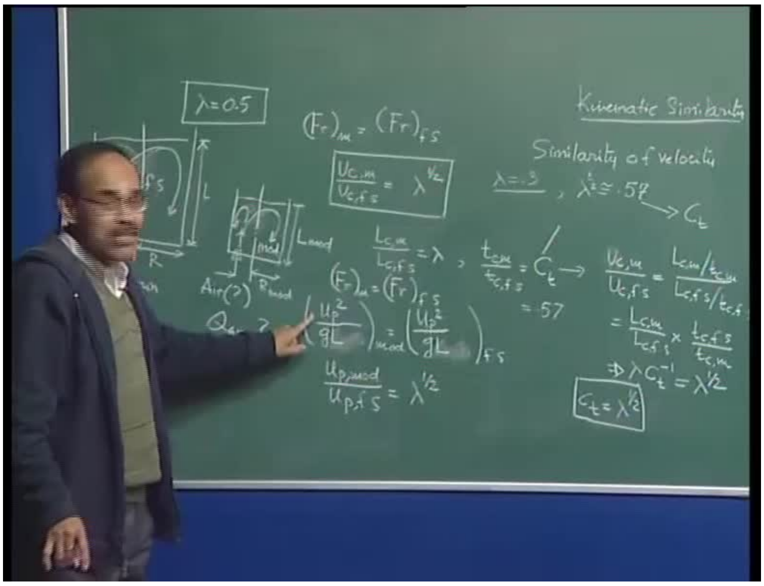

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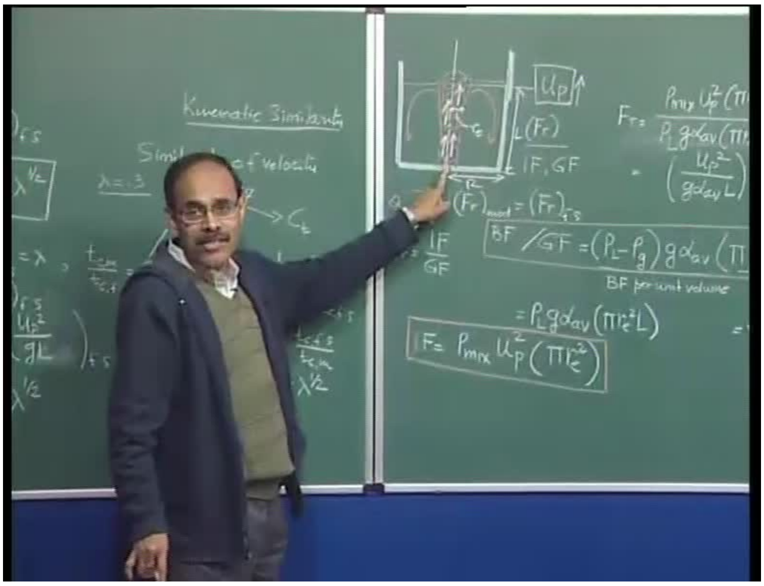

So, once I do this, one important thing comes out here that although the system has both radius as known asthe depth of the liquid, my analysis tells that the characteristic velocity in the system is the plume raised velocity, it is not the velocity through the orifice that has nothing to do with the system.

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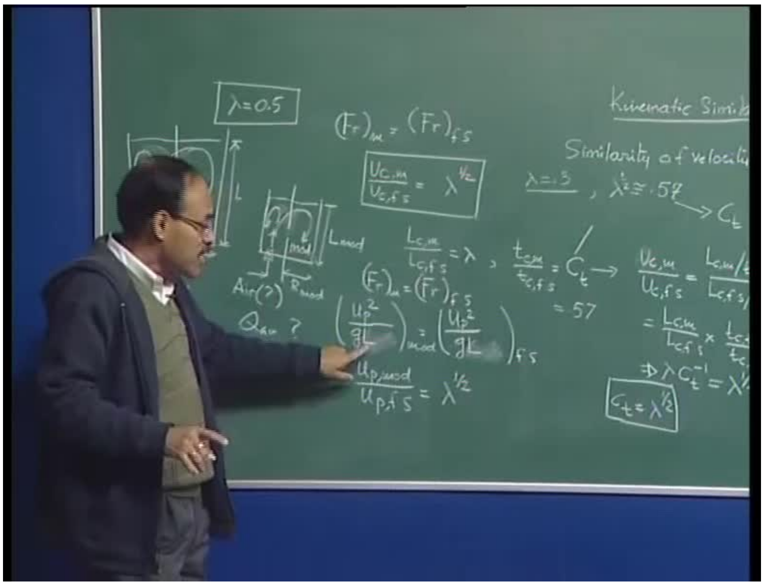

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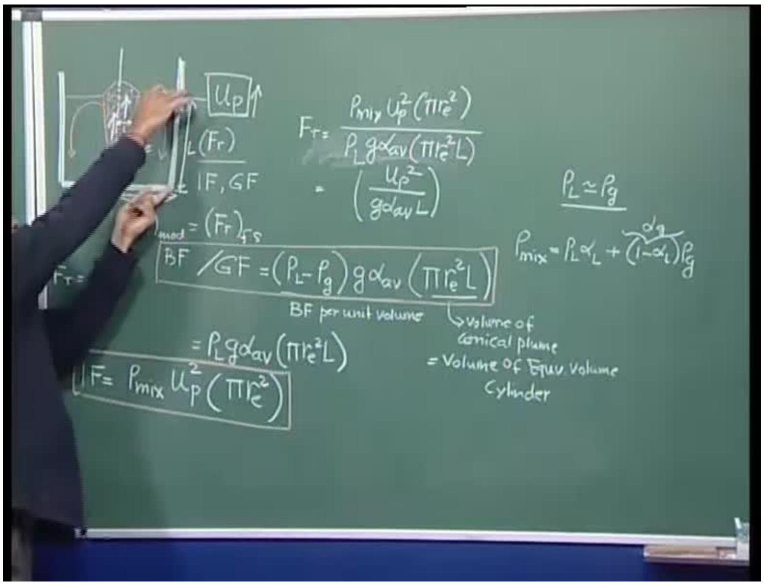

Similarly, this analysis tells us that the characteristic length you remember, the Froude number was characteristic length divided by characteristic length square divided by g into characteristic length, and therefore in place of l c, what has come here, l has come which is this gap.

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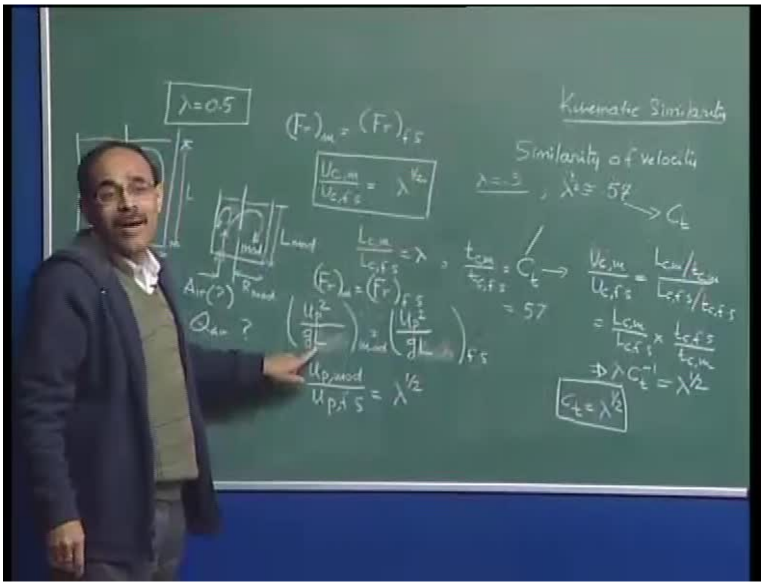

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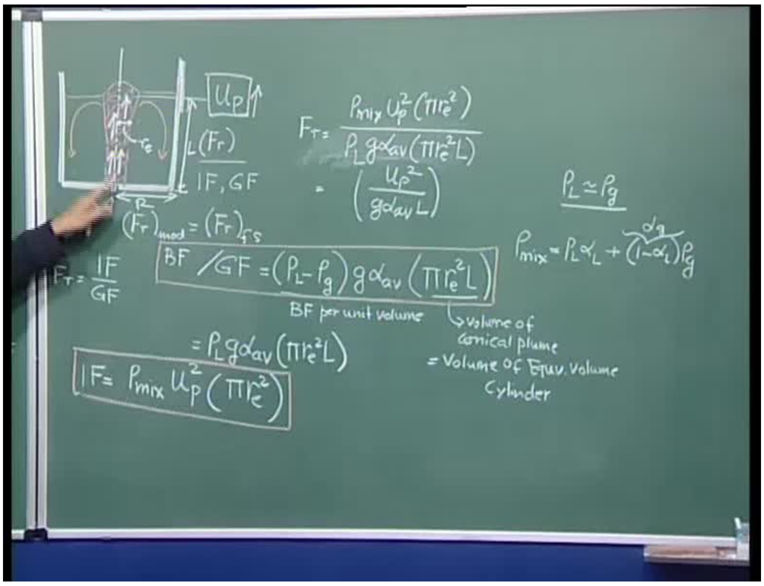

In this analysis, we do not see radius of the vessel coming into the picture,therefore we conclude that the analysis tells us that in the system, the depth of the liquid is the more representative length, because you remember the potential energy supplied by the gas depends on the depth of the liquid, like there is a depth of the liquid more is going to be the liquid recirculation in the system. So, it is this particular direction which is important in quantifying the forces which will be acting over the liquidand not this particular dimension.

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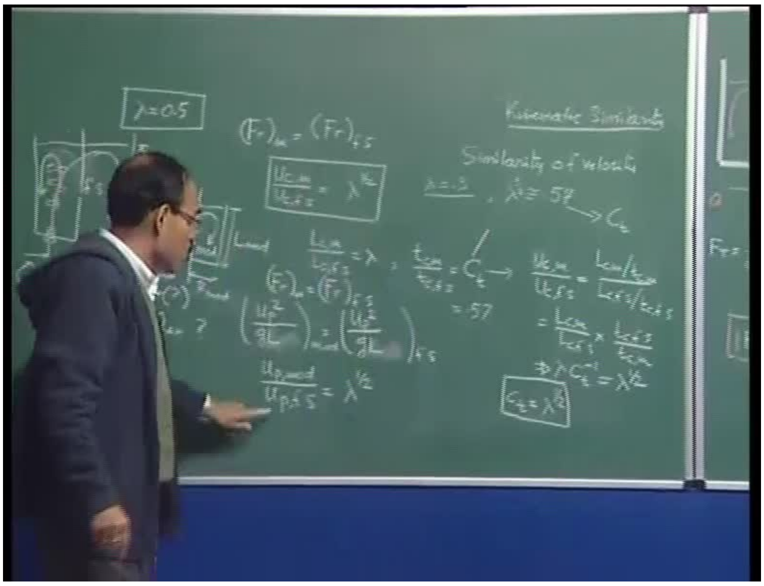

So,therefore, the analysis shows their indeed the depth of the liquid is meaningful length scale as far as you know ascribing one of the two characteristic, length parametersto a characteristic length itself.

So,therefore we conclude that in the case ofladle systems gestured, ladle systems we can consider the plume raise velocity to be a characteristic velocity and the length scale, the depth of the liquid as the characteristic length scale.

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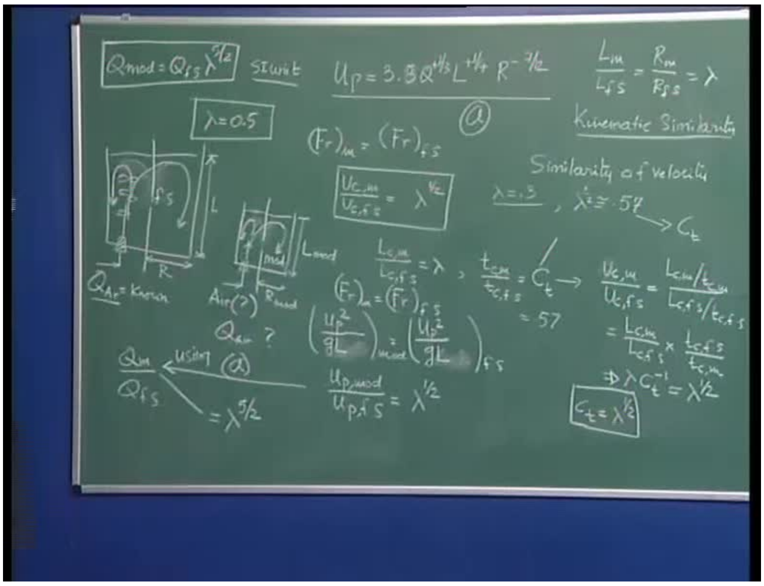

Now, we have an expression for plume rise velocity as I have shown you that what is the plume rise velocity in s i unit, this comes out to be something like 3 point 5, it is 3 point 3 into q raise to the power minus 1 by 3 plus 1 by 3, l raise to the power plus 1 by 4 and r raise to the powerminus 7 by 12.

The plume velocity increases as I have given some other expressions, when I was talking, because there are many expressions which are available in the literature and these expressions are more or less similar and produce equivalent results.

So, S I unit meters per second, this is the gas flow rate raise to the power 1 by 3 which tells us increase the gas velocity, plume rise velocity increases same is to with l, but it obeys an inverse relationship with r which tells us that larger is the radius of the vessel.

Now, we also note that l model by l full scale is equal to r model by r, full scaleis equal to lambda, this relationship always holds good, because the systems are geometrically similar. So, if I now substitute this equation into this particular expression in that case I will be able to replace l and r in terms of lambda, and the entireequation will boil down to at expression relating q, in model to q in full scale in terms of lambda, because r l and r can be replaced in terms of lambda.

And if we do that, we will find out that q model this equation and this expression, so you use this expression using a,I can say using then you can say q model by q full scale come out to be.

So therefore, the flow rate scaling equation boils down to q model is equal to q full scale raise to the power (( )), therefore this is the final expression and I would say that if you knowthis scale factor, and since you know thefull scale flow rate, you should be able tofind out that what is the model flow rate.

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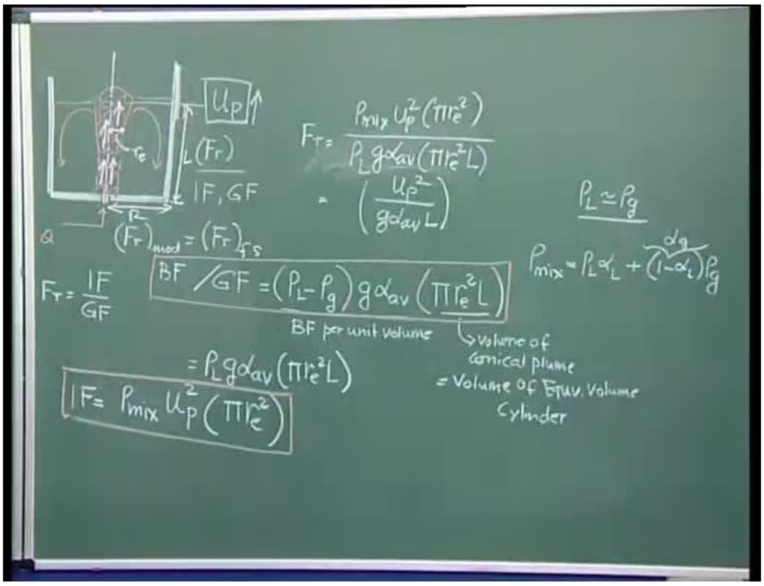

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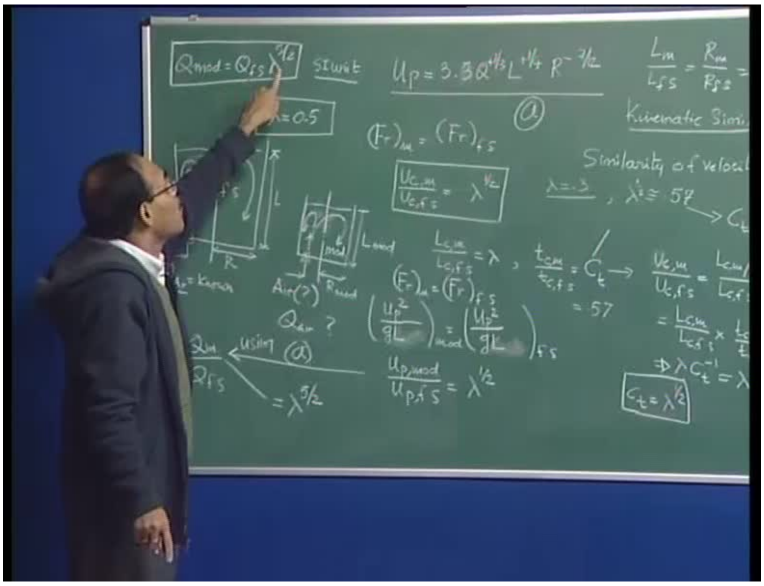

So,I have been found out the model flow rate that you have geometrical similarity, already that nozzle or tuyere or porous plug is relevant, no issue ofsettling similarity at this particular point, we can say by using this flow rate in accordance with the equation here,I can find out .So,for example I, can say that if you know this lambda to the power 5 by 2 comes out to be point 1, and if you are using a model flow rate, which is hundred meter cube per second in that case hundred multiplied by point 1.

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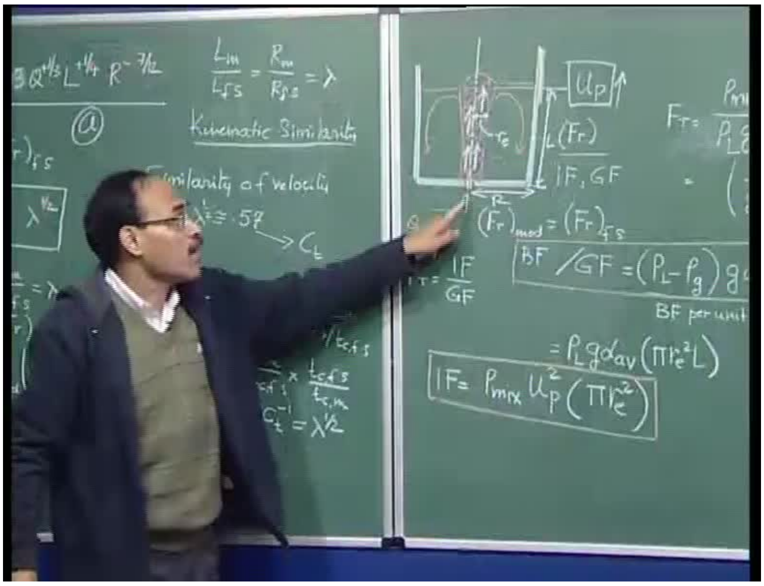

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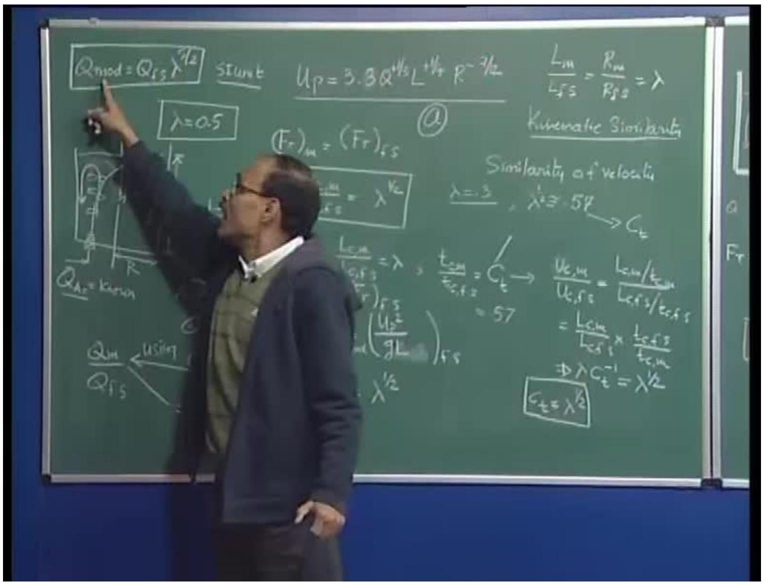

So, ten meter cube per second of flow rate, we have to use here in order to get. So, corresponding to the full scale flow rate and chosen scale factor, you are going to have model flow rate, and therefore with that model flow rate you will ensure dynamic similarity between the two.

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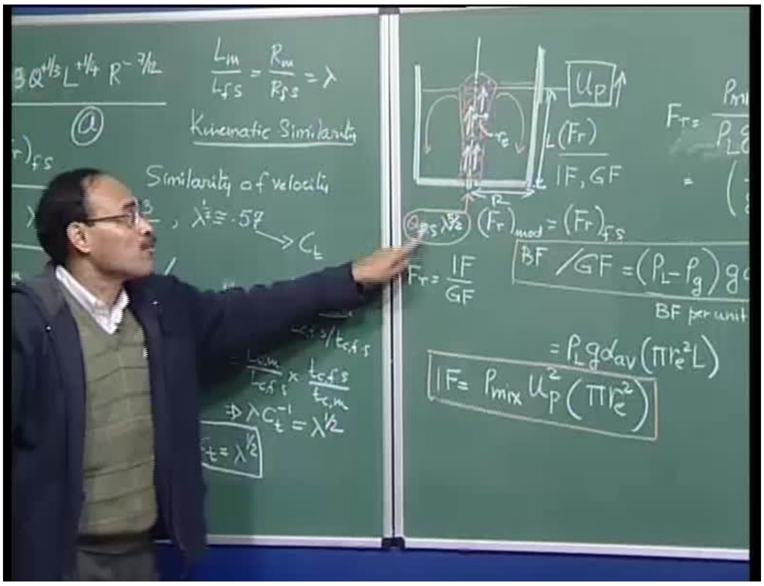

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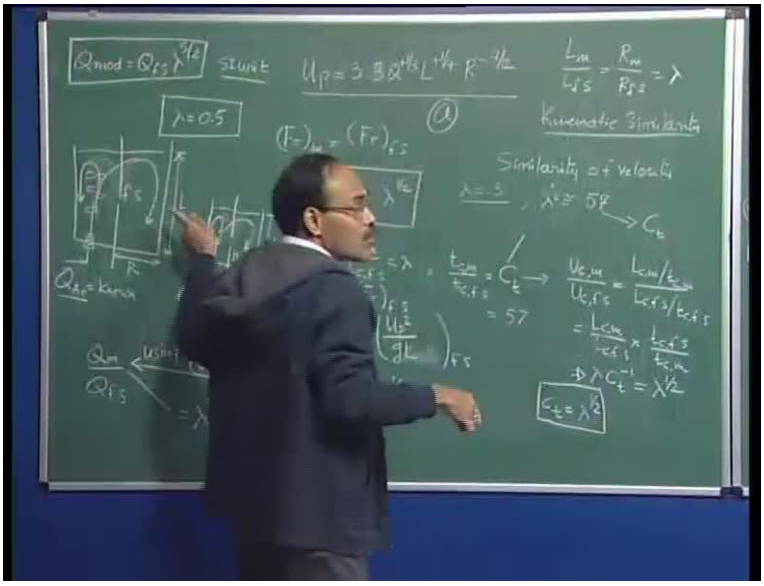

So, once the flow rate, right flow rate is use and this flow rate,therefore I am going to say it is going to be q full scale, what is the flow rate that you’re going to usinginto lambda to the power 5 by 2, and that is the flow rate actually we are going to introduce into, and this flow rate if you use then I am going to say that this system is going to be dynamically similar with the full scale system.