Materials and Energy Balance in Metallurgical Processes

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Module No. # 01

Lecture No. # 08

Errors In Measurements

I will talk on Errors in Measurements. Material and heat balance calculations are based on several assumptions like a stoichiometric requirement of the reactant and accordingly the production of the product. We also take several values on heat of formation, specific heat capacity and heat of reaction at 298 kelvin. So, based on all these assumptions, after doing material and heat balance, we make certain predictions. For example, mass flow rate is entering into the system and mass flow rate exiting the system. Another prediction that we make by doing heat balance calculation is that of the temperature. Another prediction, you will like to make is about the gas flow rates. Suppose in a process, air is required as an oxidizing medium. You predict the amount of air that you require and the prediction is based on stoichiometric amount or a stoichiometric requirement of the reaction.

You would like to see, whether these predictions are correct or not. Gas flow rates are also one of the important outputs. Then certain amounts of weights are put, 100 kg, 200 kg. You are measuring certain amount and you are putting it into the system for reaction. So, what I mean to say is that the objective of material and heat balance calculation is to predict these outputs and so one can control the process.

Now, the predictions are also required to check through measurements. It means, if you can measure these values, then on line, we can measure the predictions. Accordingly, we can correct our material and heat balance energy formulation because we are making certain assumptions while calculating the outputs, which I have mentioned earlier. Therefore, the errors in the measurement are a very important issue. You know how you are going to measure.

Suppose, you want to measure the mass flow rate of the gases, then you will use rotameter or you will use mass flow meter. If you want to measure the volume flow rate, you will be using say rotameter. If you want to measure pressure, you will be using some type of sensors. Similarly, you will be measuring temperature through thermocouple or through pyrometer. So, all these measuring devices have certain errors in measurements. So, let us discuss, what are the errors in measurements? How they interact with each other?

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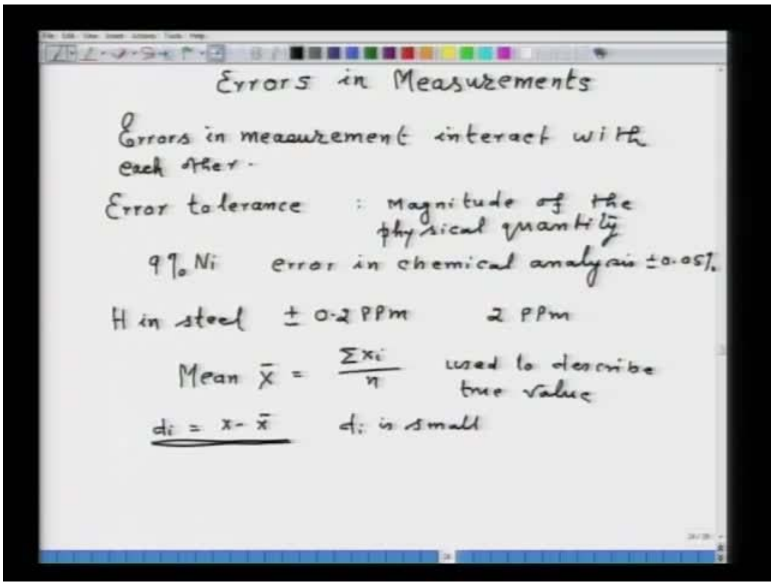

In fact, the errors in measurement interact with each other. Now, here, one should also be clear that something like error tolerance. Now, this error tolerance means, how much amount of error you can tolerate in the measurement of a physical quantity. Now, this error tolerance depends upon the magnitude of physical quantity.

Now, I mean, for example, stainless steel contains 9 percent nickel and you are making an error in chemical analysis. Say, error in chemical analysis consists of plus or minus 0.05 percent. Then, probably in that magnitude of 9 percent nickel, this 0.05 percent error in chemical analysis may not be that significant; it may be totally insignificant. Suppose, if you are measuring hydrogen in steel, you are making an error of plus or minus, 0.2 ppm. The magnitude involved is of the order 2 ppm, then 0.2 ppm looks to be very small in number, but it has a lot of significance. That means measurement is considered as highly inaccurate. So, what I mean that accurate or inaccurate measurement can be judged in relation to the magnitude of the physical quantity; it depends upon what is the magnitude of the physical quantity.

Now, actually the true value of any measured physical quantity is not known. We do not know what the true value is. What we do is that we make several measurement and we take a mean of several measurement. Well, this is approaching to the true value or we call this is a true value. So, we say mean or arithmetic mean, say, X bar is equal to sigma x i upon n, we can use to describe true value. It means that residual d i is equal to x minus x bar and that determines the accuracy of the measurement. Say, for example, if you say d i is small, then we say measurement is precise, but a precise measurement is not necessarily an accurate measurement. An accurate measurement can be precise or cannot be precise and that will depend upon the residual contained between x and x bar.

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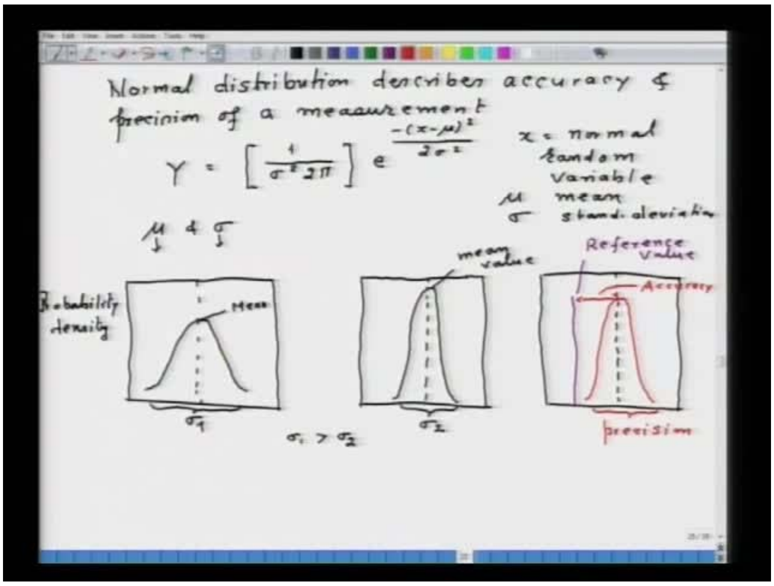

In general, the normal distribution of any measurement describes accuracy and precision of a measurement. It means, any random valuable Y in normal distribution is equal to 1 upon sigma square 2 pi into e to the power minus x minus mu whole square upon 2 sigma square. Here, x is a normal random variable, mu is the mean and sigma is the standard deviation. So, the graph of the probability function is described by this normal distribution function. It will depend upon the values of mu and sigma. Mu determines the center of the graph and sigma determines the height and width of the graph. So, if I want to plot. For example, (Refer Slide Time: 09:54) let us take some different situation, I plot probability density and this is the center of the graph. Then this is how normal distribution appears and this value is called mean and this one is the sigma and I say as sigma 1 because I want to compare the other one. So, let us take another type of distribution with same probability density on the Y- axis and this is a center of the graph. Let me take sigma 2 and this is again the mean value.

Now, as I said, sigma determines the height and width of the graph. So, here, you see that sigma 1 is greater than sigma 2. So, now in terms of precision and accuracy, let us again consider this type of function and this is again the center of the graph. Let us take this is somewhere, here is our reference value and this we call as a reference value. Our distribution is something like this. So, this particular difference is called accuracy and this is called precision. (Refer Slide Time: 11:26)

It means you can also understand that sigma is the standard deviation. It is also called spread of the distribution function. You can also consider that it decides the precision of a measurement. Number 2, a normal distribution is symmetric in nature and it is a bell shaped curve. So that is typical about the normal distribution. Now, there are different type of errors that one can do. One of the errors is the random error.

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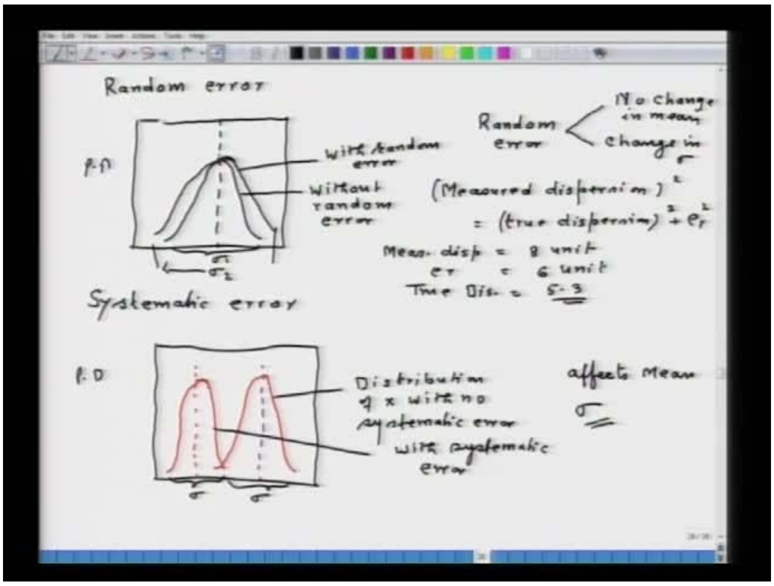

Now, random error normally depends on the mood of the person and that means while measuring, what mood he has and how he is reacting to a particular measurement. So, in fact, the random error and its effect can be shown on normal distribution like this and this is again the center of the graph. So, in this plot, if I take the distribution without random error, the same person is making this distribution with random error because sometimes his mood is good he may read more values and when his mood is bad he may read less values. So, you are saying the error or the random error in measurement does not affect the average value. So, we can say the random error makes no change in the mean and you have seen the mean is same in both the measurements. So, no change in mean, but there is a change in sigma. change in sigma means increases the dispersion.

So, if you want to determine the true value, then measure dispersion measure square is equal to true dispersion square plus e square r. Where, e r is the magnitude of the error due to random error. Now, let us measure dispersion to 8 units and e r corresponding to 6 units, then true dispersion will be equal to 5.3. So that is how one can determine, what is the true dispersion.

Now, another type of error is the systematic error. Similarly, systematic error can also be shown on normal distribution plot. So, this is the… (Refer Slide Time: 16:08) Now, the term systematic error means, you are making error systematically and you can very well visualize. It may not affect the standard deviation, but it may affect the mean of the measurement because they are the systematic error. So, accordingly, if I want to represent, this is one situation and this is another situation. So, (Refer Slide Time: 16:56) what do you see now from this particular curve? It is the distribution of x distribution of x with no systematic error. This is with systematic error and here of course, it is the probability density.

So, what you note from here? There is a change in the mean. So, systematic error affects mean value and you see the mean as shifted towards the left, when your measurement has systematic error. It did not change the standard deviations. We have seen that the value is more or less same; whereas, here you see this was the standard deviation in one case. Now, (Refer Slide Time: 18:07) this is one and this is two. So, the systematic error does not affect the standard deviation of the distribution and that means the dispersion may remain same, but it will affect the mean value. So, one has to see, what you are measuring and what type of error you are doing.

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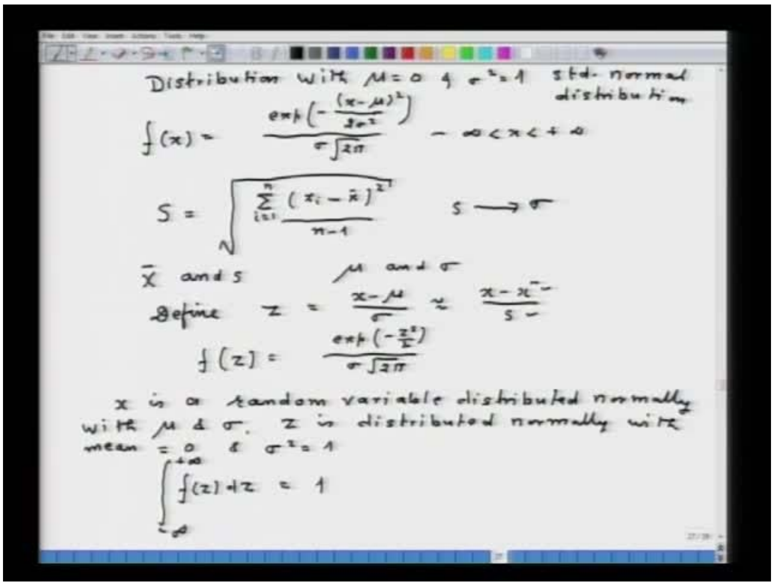

Now, distribution with mu is equal to 0 and sigma square is equal to 1. It is called as standard normal distribution. It can be represented by f x equal to exponential minus x minus mu upon whole square upon 2 sigma square upon sigma into square root of 2 pi and this is valid for minus infinity less than x less than plus infinity.

Now, in most of the situation, the mu, which is the mean of the normal distribution is the true mean and it is not known. So, what is being done. We determine the standard by making a large number of measurement by using the formula S. I think all of you know the standard deviation of any measurement. It can be determined through this formula; i is equal to 1 to n x i minus x bar whole square upon n minus 1. We say S approaches to sigma in the limit of large number of measurements. So, what we can say is that x bar and S can be used as an estimate for mu and sigma of the normal distribution.

Now, we define Z, if we define equal to x minus mu upon sigma, it is approximately equal x minus x bar upon S. You know mu and sigma of the normal distribution are not known. So, we estimate and make large number of measurements to find out x bar and S. We again define a function f z and that will be equal to exponential minus z square upon 2 by sigma into square root 2 pi. So, we can say, x is a random variable distributed normally with mu and sigma. Once we specify mu and sigma, we can trace out the graph and then z is distributed normally. I mean that am using the normally not in the literal sense. Normally means normal distribution with mean equal to 0 and sigma square is equal to 1. So, in this particular form, the total area under the curve say f z dz minus infinity to plus infinity is equal to 1 under the condition, when mean is equal to 0 and sigma square to 1, the total area under the curve become is equal to 1.

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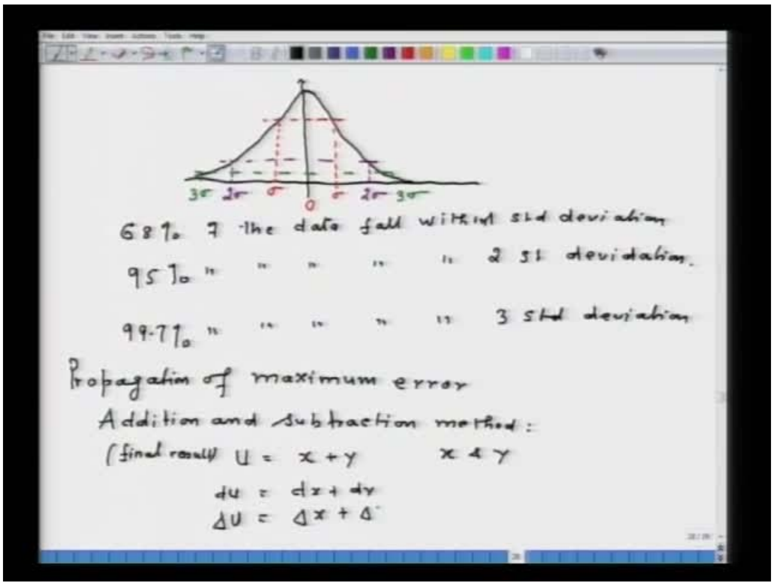

The normal distribution looks something like this. Now, this is symmetrical (Refer Slide Time: 23:36) and the value is 0. This is sigma and this is also sigma; this is 2 sigma and this is 2 sigma, this is 3 sigma and this is 3 sigma. So that is a transform normal distribution by putting value of z and I have said earlier. So, in this form, what does it mean? It means 68 percent of the data will fall within 1 standard deviation. 95 percent of the data will fall within 2 standard deviation. 99.7percent of the data will fall within 3 standard deviation. So, that is what the accuracy and precision means. It means if you want precise value, then the standard deviation should be small. So, all these things I have explained here.

Now, let us see the propagation of maximum error. Now, this one is the addition and subtraction method. Now, when errors are small, then this method can be used. Suppose, you have a final result, which is u and it comprises the measurement on x and measurement of y. So that your final result you report is the value of u and it consist of independent measurement of x and y. So that u is equal to x plus y and I call it as a final result, which is based on x and y. They are independent measurement they are the measurement and you some total and report the value of u. So, now, I can just put du is equal to dx plus dy or in terms of finite increment, I can say delta u is equal to delta x plus delta y.

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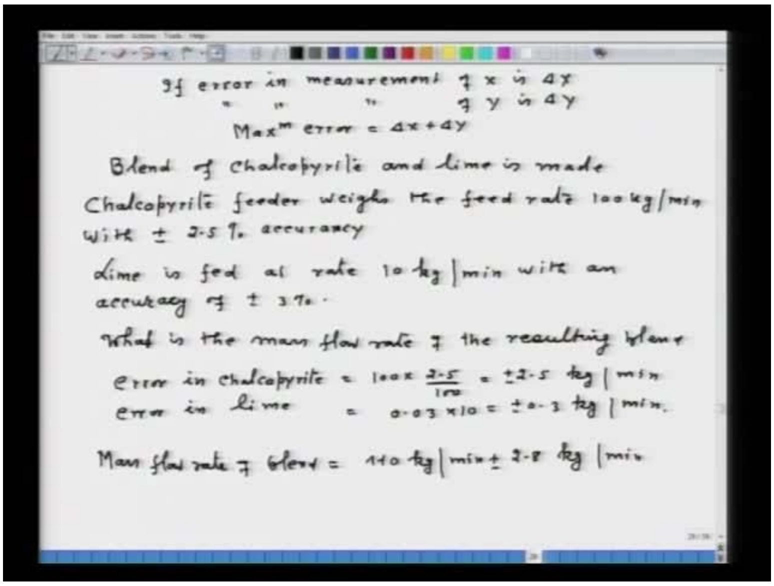

If error in measurement of x is delta x and error in measurement of y error is delta y, then maximum error will be equal to delta x plus delta y. Now, let me illustrate by taking a particular problem. For example, in copper extraction, you roast the chalcopyrite concentrate and you prepare a blend of chalcopyrite and lime. So, a blend of chalcopyrite and lime is made by weighing chalcopyrite and lime independently and then they are mixed into a container and fed into the plant.

Now, chalcopyrite has an independent measurement of the weight and lime has an independent measure of weight, like x and y is the two quantities. Now, let us say chalcopyrite feeder weighs the feed rate as 100 kg per minute. There are mass flow rate sensors, which can measure this feed rate and all the sensors are to be put in the feeder and you can accurately get the weight, but it has certain error. So, let us say, chalcopyrite feeder weighs the feed rate 100 kg per minute with some accuracy plus or minus 2.5 percent accuracy. Now, let us take lime because you have to make a mix of both and lime is fed at the rate of 10 kg per minute. It means, you are having a big container and from the container, certain mass flow rate of the lime is sent out and at the outlet you have put a sensor. It is measuring the rate of output of the lime and that rate is 10 kg per minute. It is with an accuracy of plus or minus 3 percent. What is the mass flow rate of the resultant feed? What is the mass flow rate of the resultant blend? We have to see that. Now, what we have done? Error in chalcopyrite error in chalcopyrite and error in lime are the two errors. So, error in chalcopyrite will be 100 into 2.5 upon 100 and that will be equal to 2.5 kg per minute. Of course, there will be plus and minus error in lime, it will be equal to 0.03 into 10 and you have plus or minus 0.3 kg per minute. So that mass flow rate of blend will be equal to 110 kg per minute plus or minus 2.8 kg per minute.

Now, this is an illustration and all the measuring instrument have certain error in that. So, these errors have to be accounted, while correcting the predictions made by material and energy balance; that is the importance of these measurements. Here, the mass flow rate of blend will be 110 kg per minute plus or minus 2.8 kg per minute. So, another way to find out this is by multiplication and division method.

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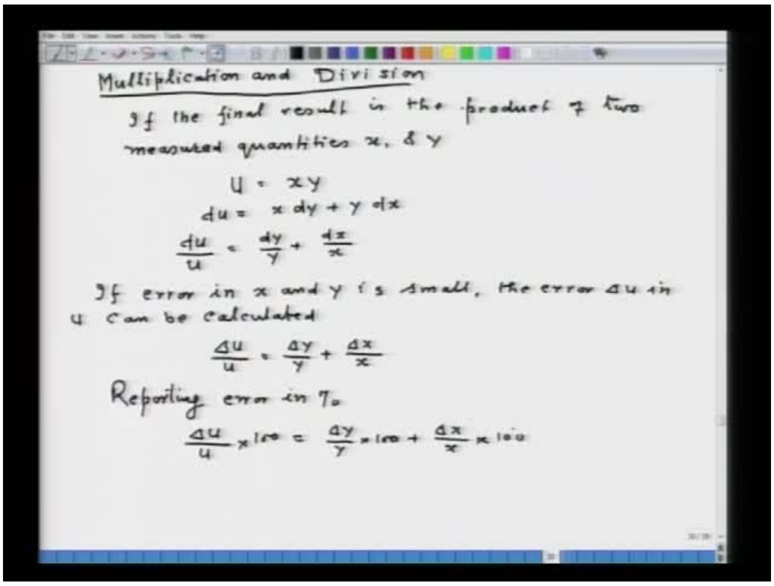

For example, in an earlier case, to the final result we are adding the 2 values; 2 independent measurements were added to get the blend and that is why we use addition method. The error in these measurement is added up and we have said the final blend consist of this plus or minus that error.

Now, here, if the final result is the product of two measured quantities, two measured quantities are x and y product. I mean u is a final result and that we want to get the measurement in the form of a product. u is equal to x into y and you can say du will be x into dy plus y into dx or we can also say du upon u will be equal to dy upon y plus dx upon x.

Now, if errors in x and y is small, the error delta u in u can be calculated from delta upon u and that will be equal to delta y upon y plus delta x upon delta x. Now, we can also report these errors in terms of percentage. So, It will be equal to delta u upon u into 100 and that will be equal to delta y upon y into 100 plus delta x upon x into 100.

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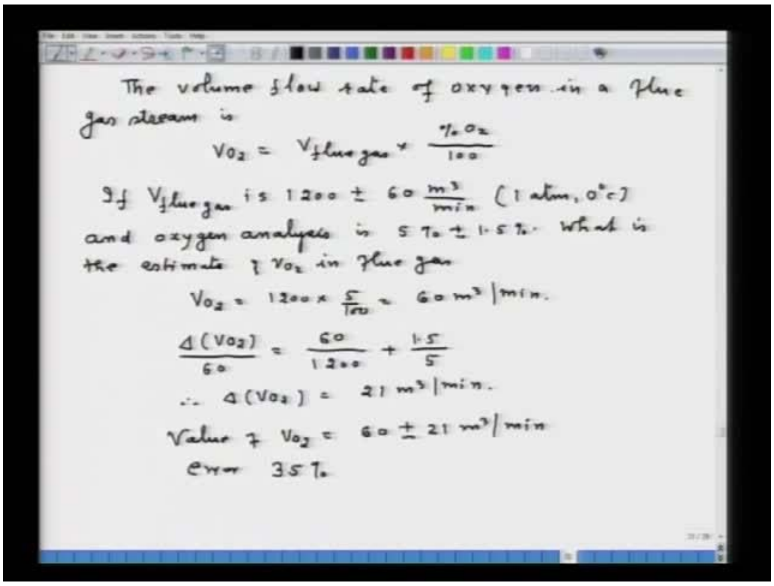

So, let me illustrate this by an example. Now, let the volume flow rate of oxygen in a flue gas stream is equal to VO2 and that is equal to V flue gas into percent O2 upon 100. It is a volume flow rate in oxygen in flew gas and this one gives it (Refer Slide Time: 37:16).

Now, if V flew gas is 1200 plus or minus 60 meter cube per minute, which is measured at one atm and 0 degree Celsius. Oxygen analysis is 5 percent plus or minus 1.5 percent, then what is the estimate of VO2 in flew gas? We can say VO2 will be equal to 1200 into 5 upon 100 will be 60 meter cube per minute. Delta VO2 upon 60 is equal to 60 upon 1200 plus 1.5 upon 5. Therefore, we get delta VO2 equal to 21 meter cube per minute. So, value of VO2 will be 60 plus or minus 21 meter cube per minute or you can say the error would be 35 percent. You can see how this error in measurement helps for the estimation purposes. Now, this result can also be obtained by adding percent error in each measurement.

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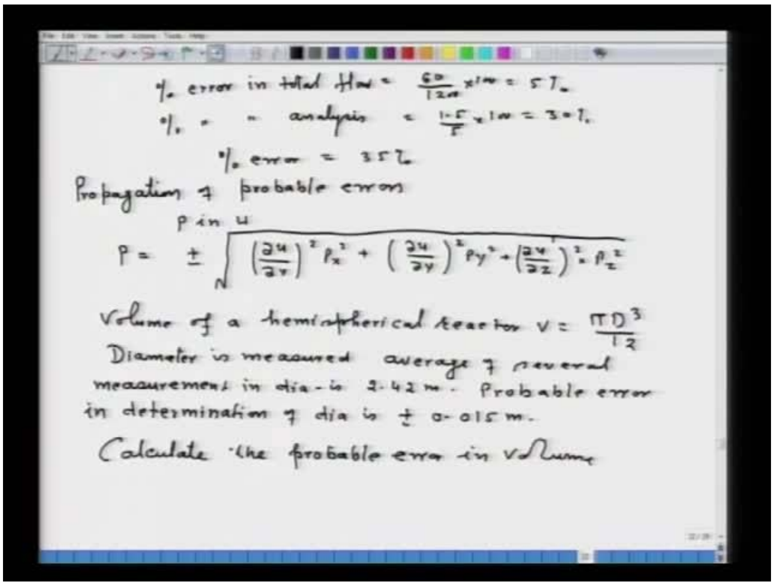

For example, percent error in total flow is equal to 60, then 1200 into 100 is equal to 5 percent. Percent error in analysis is 1.5 upon 5 into 100 and that is 30 percent. So, total error is become 30 plus 5. So, percent error in oxygen flow is again 35 percent, so, in this way also you can do it.

Now, I have to illustrate a propagation of probable errors. Now, the probable error P, for example, in measured quantity; u, it can be given by P and it is equal to plus or minus, if the independent variables are x, y and z, then the probable error P x minus u upon delta x whole square x square plus delta u upon delta y square P y square plus delta u upon delta z square plus into P z square. Here, we have 3 independent variable z y and z where P x is the probable error in x P y is the probable error in y and P z is the probable error in z.

Now, let us take an example, volume of a hemispherical reactor is give by v and it is equal to pi D cube upon 12, where D is the diameter of the hemisphere. Now, diameter is measured several times in several locations and you determine an average of several measurements; that means, diameter is measured. Diameter is measured for several times you made several measurement and the average of several measurement in diameter. Average of several measurement in diameter of the hemisphere is 2.42 meter. There is a Probable error in determination of diameter. Let us say, it is plus or minus 0.015 meter. Then calculate the probable error in volume. Now, you see this is a typical example of how the error propagates. You have done some errors in diameter and so it is propagating through and ultimately that error will be reflected in volume measurement. Similarly, when you do the heat balance, if you do some mistake in material balance that error will be propagated throughout the heat balance calculation. So that is a very serious error about the propagation and that is an important thing.

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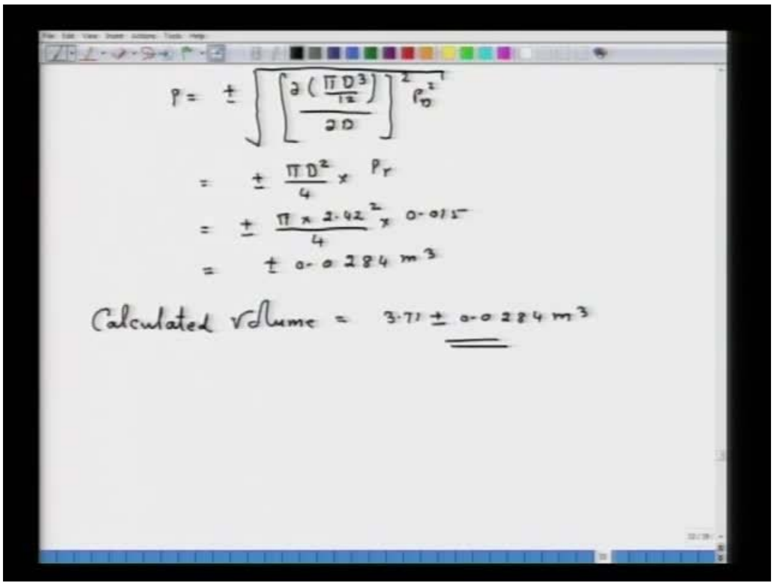

So, we can find out that P is equal to plus or minus, we have only one variable and that will be delta pi D cube upon 12 delta D and our formula into P r square or we can write P D square. We are measuring the diameter, only one variable and so the equation reduces. So, we differentiate and take the root and then we get plus or minus pi D square upon 4 into P r. I substitute the value plus or minus pi into 2.42 square upon 4 into 0.015. Ultimately, I will be getting plus or minus 0.0284 meter cube. So, the calculated volume would be by substituting D is equal to 2.42 meter. It will be equal to 3.71 plus minus 0.0284 meter cube and so that is the propagation of error.

Sir, I have doubt. Can you give me an example of maximum error using subtraction method?

In the lecture, I have mentioned addition and subtraction method, I can give you the example that uses subtraction method to find out the error.

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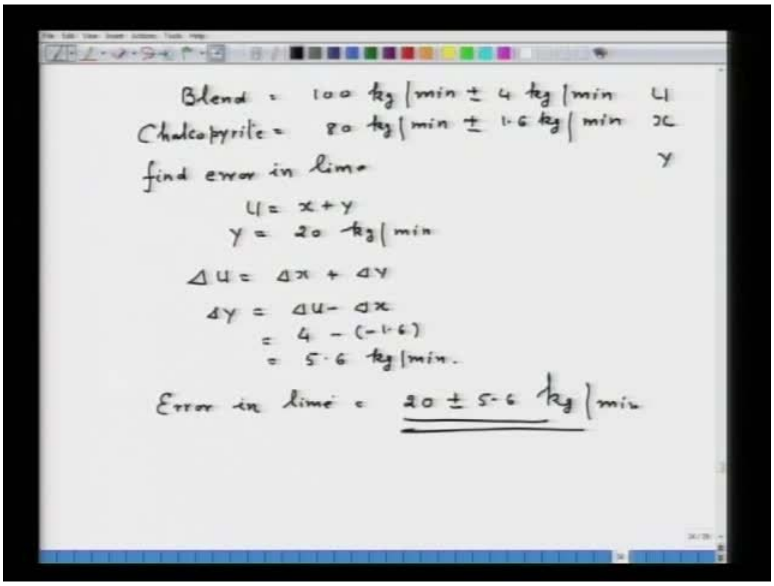

Now, let us take the example of blend and lime stone in chalcopyrite. So, it consist of 100 kg per minute with an error plus or minus 4 kg per minute In our equation, let us call u and say chalcopyrite is 80 kg per minute and with plus or minus error of 1.6 kg per minute. Let us say this as x, find error in lime and call this as y. Then we have u equal to x plus y. We get y is equal to 20 kg per minute. Of course, it is 100 minus 80 kg per minute.

Now, you know that error was delta u and that is equal to delta x plus delta y. You have to find out delta y and it will be equal to delta u minus delta x. Now, in delta u, we have 4 kg per minute. Now, one has to think because if you want the maximum error, then we have to use the minus sign of chalcopyrite and so, it is minus 1.6. So that is equal to 5.6 kg per minute. So, error in lime will be 20 plus or minus 5.6. Now, in fact, error in line is 5.6 kg per minute. So, the weight, which you are reporting with total and error will be 20 plus or minus 5.6 kg per minute.

Is that your question?

Yes sir, thank you sir.