

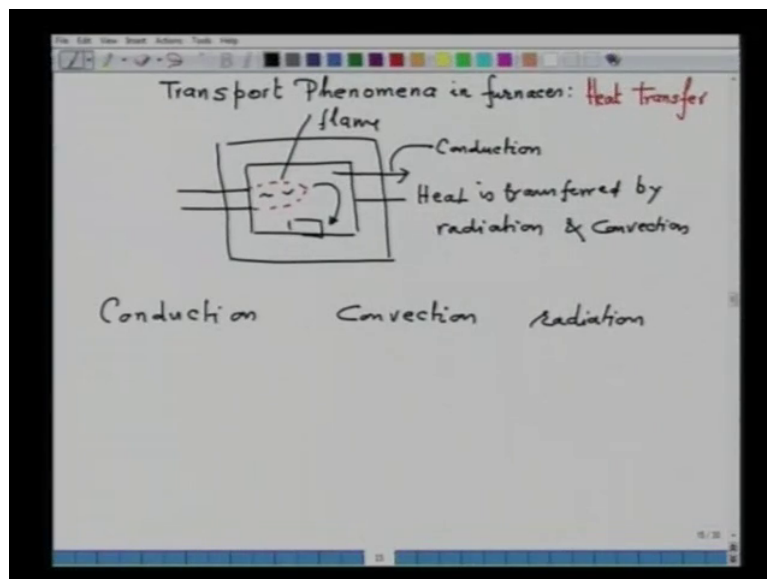
**Fuel, Refractory and Furnaces**  
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**Lecture No. # 28**

**Transport Phenomena in Furnaces: Heat Transfer and Refractory Design**

Today, we will start with the transport phenomena in furnaces, the second part that is the heat transfer. As I have repeatedly said a furnace is a thermal-encloser and is employed to heat the raw material through the process temperature and to carry out the function like roasting, smelting, centering, refining, converting, heating and so on. In all these function it is important that the transfer of heat from the flame to the walls of the furnace is takes place at a uniform rate. So that the furnace in the charge they are heated to desire temperature.

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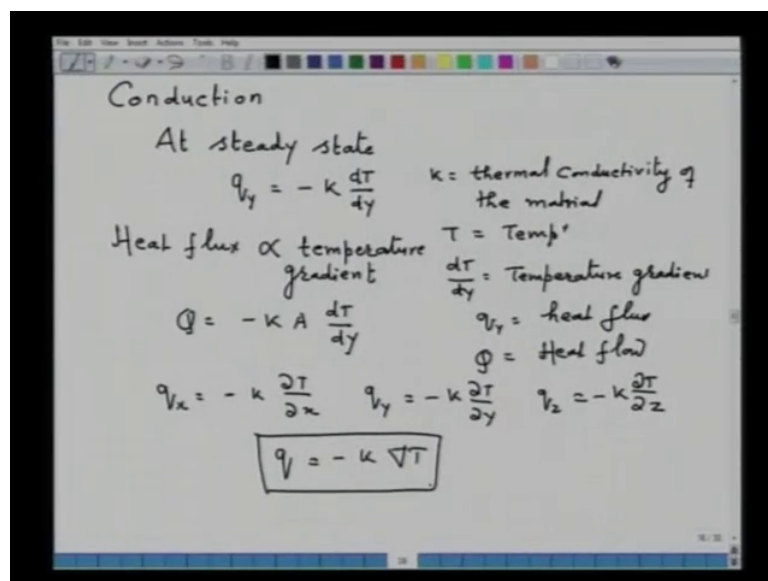
So, for example, if we consider a combustion chamber of a furnace which is lined with the refractive material and here we have a burner and this burner produces a flame. And here we have the charge so, this is the flame and the product of combustion transfers it is heat to the charge to the combustion chamber and hence heated up. Part of the heat is also transferred to the surrounding and hence heat losses are important. Therefore, for an efficient furnace

design it is important to consider the various mechanisms of heat transfer. If we see this particular combustion chamber of furnace then this is a flame and here heat is being transferred, heat is transferred by radiation and convection. Similarly, part of the heat it is also transferred through the refractory lining and through the surrounding. So, this particular transfer of heat from combustion chamber to the surrounding that is by the conduction.

So, in efficient design of furnace it also requires a fairly good knowledge of the heat transfer mechanism. And there is a loop the heat transfer mechanism of operative in furnace they are conduction, (No audio from 02:58 to 03:04) convection and radiation. In fact, conduction heat transfer occurs by transfer of vibration energy from one molecule to the other one. In case of convection the fluid flow is important, it is the flow of the fluid or it is the velocity of the fluid. That carries hot packets of fluid to the colder part and colder to the hotter part for the efficient heat transfer to occur. In case of radiation, there are electromagnet that is heat is transfer by electromagnet radiation. When a surface is irradiated, part of the heat is reflected, part of the heat is transmitted and part of the heat is observed.

So, all these three mechanisms are operative in the design of the furnaces and from the efficient point of view, from the energy conservation point of view it is important that we understand fairly well the role of the transfer.

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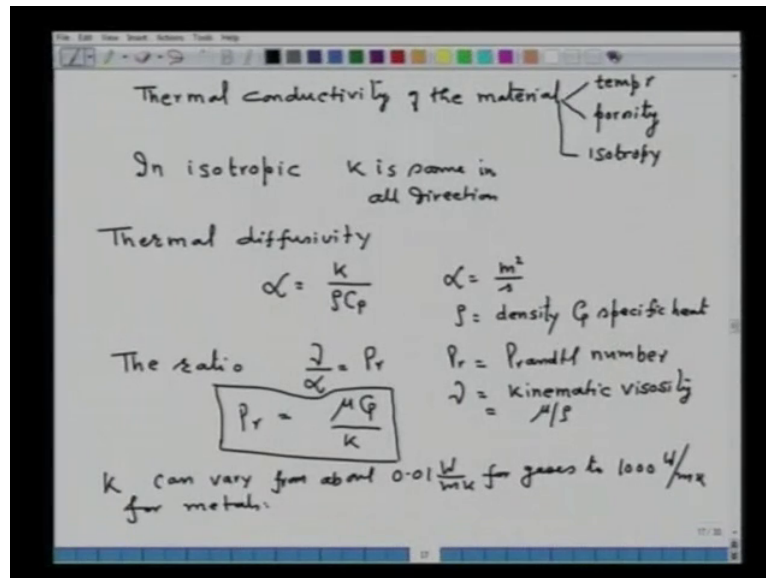


So, let first start with the conduction. Now, as I said that conduction involves flow of heat by transfer of vibration energy from one molecule to other. As the heat transfer occurs in the beginning, there will be unsteady state and at steady state, the flow of heat from high temperature to lower temperature occurs. And the flow of heat is proportional to the temperature gradient. All of us we are aware with the so called Fourier law of conduction. So, I am straight away writing at steady state by this I mean when temperature is not a function of time however, temperature is a function of the co-ordinates, that is x y and z.

So, at steady state for example, if we consider the y direction, then it flux that is equal to minus K into d T upon d y where, K is a thermal conductivity of the material and T is a temperature. And in fact d t upon d y is temperature gradient and q y is the heat flux, if we say q that is equal to heat flow. So, and the so, this is what is the Fourier law of heat conduction. That is in short, it says that heat flux is proportional to temperature gradient where as you want to write down this expression in the terms q which is the heat flow. Then q will be equal to K into area that is Q that will be equal to minus K A d T upon d y. That Q is now the heat flow that is in Watts and A is area and remember this area is perpendicular to the direction of flow.

So, now similar expression we can write down in all the three rectangle co-ordinates. So, if you write down that we can say the q x that is equal to minus K delta T upon delta x q y that is equal minus K upon delta t upon delta y and q Z that is equal to minus K delta T upon delta Z. So that we can write down q that is equal to minus K into del T. So, this is Fourier law of convection in three dimensions. Now, one thing you should know that q is equal to minus K and the minus sign is resulting because the heats flow from the high temperature to lower temperature.

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So, now what we are telling here is that this is the we are considering thermal conductivity. Thermal conductivity of the material, it depends upon temperature and it also depends upon porosity and it also depends upon isotropy, where as the material is isotropic called an isotropy. So, in isotropic media k is same in all direction. As regards porosity is concerned, lower is the porosity, higher is the thermal conductivity and higher is the porosity, lower is the thermal conductivity.

We can define another term from here is that thermal diffusivity. And thermal diffusivity is define as alpha that is equal to K upon rho into C p where, the dimensions of alpha that is equal to meter per second. So, the thermal diffusivity signifies the diffusion of heat into the system. Where K is again thermal conductivity, rho is the density and C p that is equal to specific heat of the material. Also, one number is important so the ratio of kinematic viscosity upon thermal diffusivity that is called P r where, P r is equal to prandtal number and this is the kinematic viscosity and that is equal to mu upon rho. So, if you substitute then we get P r that is equal mu upon mu into C p upon K. So, this prandtal number is also a very important dimension less number in case of studies on the heat transfer.

Now, the value of K which is the thermal conductivity, it can vary from about zero 0.01 Watt per meter Kelvin for gases to 1000 Watt per meter Kelvin for metals. So, in the Fourier's law of heat conduction we note, a heat flux is equal to K into d T by d y. This is a very important equation and this interconnects the heat loss and temperature gradient that depend upon the

property of the material that is the thermal conductivity of material. Based on this calculation one can design the different material by different conductivity as required by a particular equation. So, this particular expression that is a Fourier law of heat conduction is very very important.

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K for ceramic material  $\ll$  than metals.  
Summary of Units

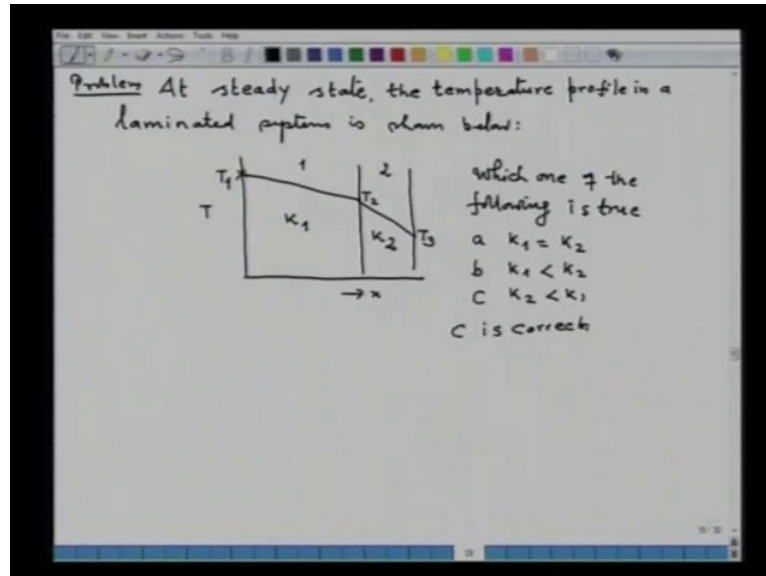
	SI	CGS	FPS
$q_y$	$W/m^2$	$\frac{cal}{cm^2 \cdot s}$	$\frac{Btu}{hr \cdot ft^2}$
T	K	$^{\circ}C$	$^{\circ}F$
y	m	cm	ft
k	$\frac{W}{m \cdot K}$	$\frac{cal}{cm \cdot s \cdot ^{\circ}C}$	$\frac{Btu}{hr \cdot ft \cdot ^{\circ}F}$
$C_p$	$\frac{J}{K \cdot kg}$	$\frac{cal}{C \cdot g}$	$\frac{ft \cdot lb}{ft^2 \cdot lb}$
$\alpha$	$m^2/s$	$\frac{cm^2}{s}$	$\frac{ft^2}{s}$
$\mu$	$Pa \cdot s$	$\frac{g}{cm \cdot s}$	$\frac{lbm}{ft \cdot hr}$

Now, as I already said K depends on temperature porosity. And also K for ceramic material is much much lower than metals than metals. Now, let us say the units that is important so I write down the summary of units here S I units, here C G S units and here F P S units. That is the delicious system foot pound and second. So,  $q_y$  S I unit is Watt per meter square, calorie per centimeter square second and in the F P S system that is B t u per hour feet square. Temperature that is expressed in Kelvin, here it is degree centigrade and here it is degree Fahrenheit, distance y, meter it is centimeter and it is feet. K Watt per meter Kelvin, here calorie per meter second into degree Celsius, here B t u over hour feet degree Fahrenheit. C p Joule per Kelvin into k g, here it is calorie per degree Celsius gram, here it is B t u feet per pound.

Then alpha is the thermal diffusivity is expressed meter square per second, centimeter square per second and feet square per second. Then mu is viscosity Pascal into second, here it is gram per centimeter second, here upon m upon feet into hour. So what I thought that these units are sometimes very important in order to understand the heat transfer from various

books. Because the various books are written from books that express in B t s or in B t u units or in F P S system, some in C G S and some in S I units.

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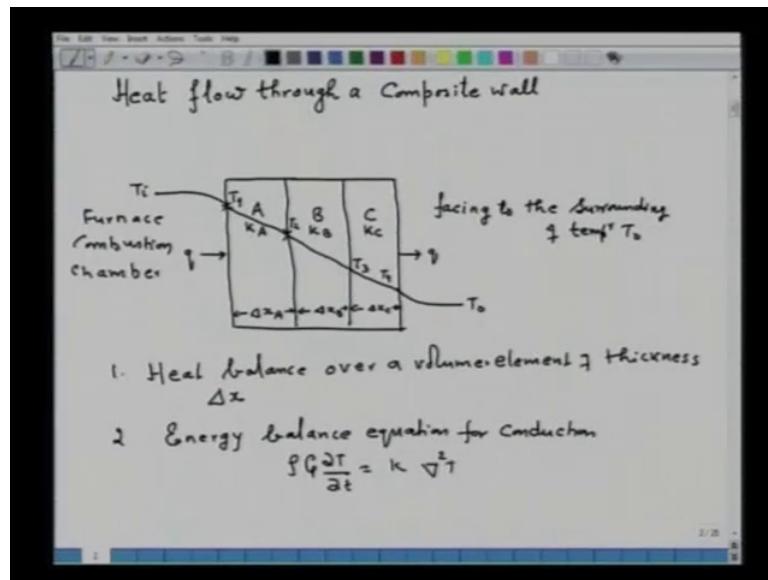
So, now let us take one very simple problem. For example, at steady state the temperature profile in a laminated system is shown below. This is just a problem for thinking. So, the profile is as follows now here this is the temperature and here is the distance in the x direction, here you have the temperature  $T_1$ , this is material 1 and this is material 2. Now, the temperature gradient is this one, this is  $T_2$  and here it is  $T_3$ . Now, suppose thermal conductivity of material 1 is  $K_1$  and thermal conductivity of material 2 is  $K_2$ .

Now, which one of the following statement is correct? a  $K_1$  equal to  $K_2$  b  $K_1$  less than  $K_2$ , c  $K_2$  less than  $K_1$ . So, you have to answer this question and you note that there is steep temperature gradient in the material two therefore, statement C is correct. Now, you can see in terms of Fourier's law of conduction because  $K$  is low temperature gradient is  $T$ . So that is why the answers for this is this one. Having studied Fourier law of heat conduction and having seeing the expression heat flux and temperature gradient which is related by thermal conductivity of the material. We are in the position now to study heat flow through a composite wall.

Composite wall means a wall which is constructed by refractory material by different conductivity and different thicknesses. Now, such a composite wall construction is very very

important in case of furnace design. In the furnace design, it is often required to construct the walls of the furnaces of the different type of the refractory depending upon the neat. As I have already pointed out in my earlier lecture from refractory material that refractory must be chosen as per the metallurgical objective.

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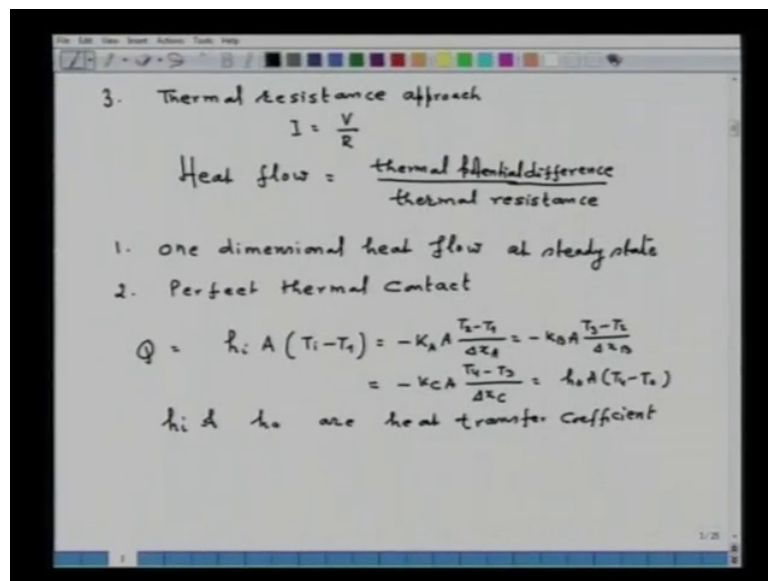
Now, let us see the heat flow through a composite wall this I mean. So, let us consider this is a wall I am just drawing sketch for you. So, this composite wall is constructed having three different types of materials the materials A, material B and material C. Material A have the thickness  $\Delta x_A$  this is thickness  $\Delta x_B$  and this is there is thickness  $\Delta x_C$ . Material A has thermal conductivity as  $K_A$  this is as  $K_B$  this is as  $K_C$ . Now, this particular composite wall is expose to the fluid at different temperature. So, the material which is having A is exposed to the combustion chamber of the furnace. And the such let us have this the temperature of furnace for example, here  $T_i$  this is the furnace combustion chamber. (No audio from 21:38 to 21:44)

And the other side is facing to the surrounding of temperature  $T_0$ . So, now if we wish to represent the temperature profile then from here say this will be here so, this is the temperature let us say it is  $T_1$ , there is a gradient let us take this temperature as  $T_2$ , this temperature as  $T_3$  and this temperature as  $T_4$  and from here it again goes to the surrounding which is  $T_0$ . So, this is the temperature profile. And the flux  $q$  is passing and the flux  $q$  will

be equal to out so that is what the composite wall. Now, we are require to find out an expression through which we can calculate the heat through this composite wall.

Now, there are several approaches to find out the heat flow. Now, one approach is, approach number one is doing heat balance over a volume element of thickness say delta x. now, this heat balance we have to perform for each material and then we have to substitute the Fourier's law of heat conduction and get the temperature gradient confirmed. So, this is one approach. You do heat input is equal to heat output in this material and then simultaneously solve the equation and you get the heat flow through a composite wall. Another approach is that utilize the generalize energy balance for conduction, utilize energy balance equation for conduction and generalized equation for conduction is say rho C p delta T upon delta t that is equal to K into del square T. So, this is also another approach through which you can proceed to solve the this particular problem.

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Now, the third approach is the so called thermal resistance approach, third approach is thermal resistance approach. Now, in the thermal resistance approach electrical analogy is used to find out the heat flux. Now, we all know that the flow of current that is equal to potential difference upon resistance. So, in an analogous fashion we can also say heat flow that is equal to thermal potential difference divide by thermal resistance. So, all that now the thermal potential difference is the temperature between the fluid which is the temperature of facing the walls of the composite wall, the walls of the composite construction towards the

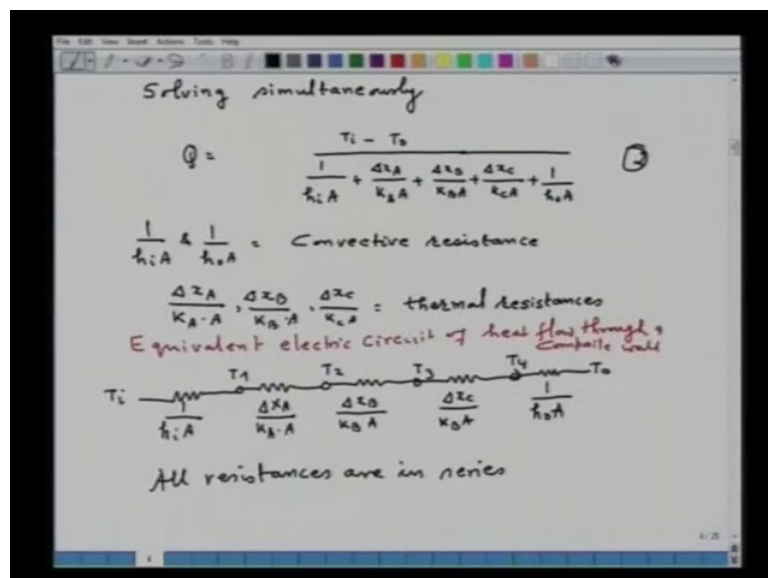


furnace and towards the surrounding and by calculating the thermal resistance we can find out the heat flow.

Now, since now let us some of the assumptions are one assumption is that we are considering one dimensional heat flow at steady state, this is the one dimension, a one assumption. And second assumption is that we are assuming perfect thermal contact between the various materials of construction with perfect thermal contacts means there is no air gap between the two materials. So, now since that this is a steady state, it cannot be accumulated so therefore, heat flow at each wall should be same and because heat flow across each wall should be same.

The temperature should drop depending upon the thermal conductivity of the material. So, in straight way we can write down heat flow that will be equal to  $h_i A (T_i - T_1)$  that should be equal to  $\frac{k_A A (T_2 - T_1)}{\Delta x_A}$  that should be equal to  $\frac{k_B A (T_3 - T_2)}{\Delta x_B}$  that should also be equal to  $\frac{k_C A (T_4 - T_3)}{\Delta x_C}$  and that must be equal to  $h_o A (T_4 - T_o)$ . Now, note here  $h_i$  and  $h_o$  are heat transfer coefficient. Now, this  $h_i$  is the heat transfer coefficient from the combustion chamber to the wall of the composite material facing the combustion chamber. So, it is important here to note that this temperature  $T_1, T_2, T_3, T_4$ , they are the temperature of the respective walls. Similarly,  $h_o$  is the heat transfer coefficient from the outer wall of composite to the surrounding.

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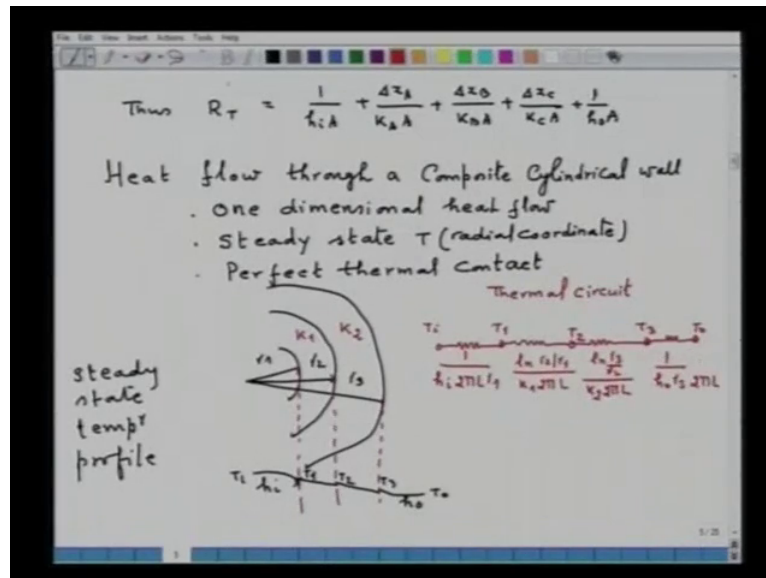


Now, solving this equation simultaneously, solving simultaneously we arrive that  $Q$  that is equal to  $T_i - T_o$  upon  $1/h_i + \frac{\Delta x_A}{K_A} + \frac{\Delta x_B}{K_B} + \frac{\Delta x_C}{K_C} + 1/h_o$  into  $A$ . So, let us say this equation is number two. Now, remember this  $1/h_i$  into  $A$  and  $1/h_o$  into  $A$ , they are the convective resistance to the heat flow. Whereas,  $\frac{\Delta x_A}{K_A}$  into  $A$ ,  $\frac{\Delta x_B}{K_B}$  into  $A$ ,  $\frac{\Delta x_C}{K_C}$  into  $A$ , they are the thermal resistances offered by the walls of different thickness and different thermal conductivities.

Now, from this we can consider that heat transfer rate is as a flow and the combination of  $K$ ,  $\Delta x$  and  $A$  as resisters to heat transfer. So, if you want to represent the whole thing by electrical analogy then what you can say? This is the one resistance, here the temperature  $T_i$  and the resistance over here is  $1/h_i$  into  $A$  so, this temperature is then  $T_1$ , this temperature is  $T_2$  because of the resistance  $\frac{\Delta x_A}{K_A}$  into  $A$ . Similarly, here third and this temperature  $T_3$  resistance is  $\frac{\Delta x_B}{K_B}$  into  $A$ . Now, fourth wall this is the temperature four  $T_4$  and resistance is  $\frac{\Delta x_C}{K_C}$  into  $A$  and ultimately we have temperature  $T_o$  and this is the resistance represented by  $1/h_o$  upon  $A$ .

So all the resistances they are in series, all resistances are in series and they can be at that to get the equation number two as we represent. So, this particular representation it can be considered as this is an equivalent electrical circuit of heat flow through a composite wall. So, now all the resistance can be added up and the one can obtain the so called  $Q$  that is the heat flow.

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So, the thermal resistance now thus total thermal resistance that is equal to  $\frac{1}{h_i} \ln \frac{r_2}{r_1} + \frac{\Delta x_A}{K_A} \ln \frac{r_2}{r_1} + \frac{\Delta x_B}{K_B} \ln \frac{r_3}{r_2} + \frac{\Delta x_C}{K_C} \ln \frac{r_4}{r_3} + \frac{1}{h_o} \ln \frac{r_o}{r_i}$ . So, this is how the heat flow through a composite wall is now with this expression one can design the composite wall of different thermal conductivity and different thickness, one can optimize them that well. If this is the thickness I want what should be the  $K$ , I should use or if I know the value of  $A$  what should be the value of  $\Delta x$ . So, this is a very important equation for the design of the walls of the for example, rectangular furnaces.

So, what we have considered is a rectangular wall construction of different material of different thermal conductivity and different thickness, but there are also cube furnaces. They are cylindrical in nature, there also dot, they also cylindrical in nature, and there also required to be insulated to as such next thing we will consider the heat flow through a composite cylindrical wall. Here, we consider one dimensional heat flow that means the gradient is along the thickness of the wall, this is the one second. We will assume steady state that means here temperature is a function of say function of radial coordinate. Since, we are considering one dimensional so temperature is a function of are only other dimension we are considering as a uniform one.

Third assumption is that assuming a perfect thermal contact over it is easy to assume over here, but it is very difficult to maintain a perfect thermal contact. Because the suspension's that

the refractory will always be rough and that such perfect thermal contact can be achieved nearly, but cannot be achieved in an absolute sense. Now, the reason why I mentioning over here because if there is not perfect contact there will be air gap, in the calculation we will consider thermal perfect thermal contact and hence no air gap.

So when we will apply this calculation or the result into the actual practice then this may not fairly hundred percent because in the actual practice the assumption of perfect thermal contact may not valid, there is always an air gap. An air gap as very very small thermal conductivity as in the later I will show you through a problem that what this thermal conductivity means so that part is to be consider very carefully. What I mean to say here that what assumption we are making that should also be consider when applying the result in the actual practice.

So now let us consider the composite cylindrical wall say this is for example one, this is the second and this one is the third. So, let us see this is the radius  $r_1$ , this is the radius  $r_2$  and this the radius  $r_3$ . Now, if I want to draw the temperature profile somewhere here this is the one temperature at this interface, this is the temperature here at the interface so, the temperature profile somewhere here, this is  $r_1$  is facing to the furnace. So, here if I have the temperature  $T_i$  and the temperature will drop here and this temperature let us say is  $T_1$ , this interface temperature is  $T_2$  and this interface temperature is  $T_3$  and from here it goes to the  $T_0$  so here you have heat transfer coefficient  $h_i$ , here, here you have heat transfer coefficient  $h_0$ . So, this is in fact a steady state temperature profile through a composite wall of thickness or different thicknesses and different thermal conductivities.

So, let us take the conductivity of this as  $K_1$  and this conductivity of this is  $K_2$ . So, we can also draw the thermal circuit analog to electric circuit. All that we are assuming that the walls of different thickness and different thermal conductivities offered resistance to heat flow, same as resistance offered the flow of current. In the same way the resistance offered by the break, it rather hampers the heat flow through the wall. So, if you want to represent the thermal circuit then for example, here somewhere we have temperature  $T_i$  and we have one say resistance which is  $\frac{1}{h_i} + \frac{2\pi L}{r_1}$ . Note that the wall which is facing the furnace, heat transfer coefficient correspond to the diameter of the radius of that particular wall.

Now, here because of the resistance temperature as come to value T 1 and now it meets the walls of different thickness. So, here I have brought temperature T 2 and the resistance is  $\ln r_2$  upon  $r_1$  divide by  $K_1 2 \pi L$ . Now, because of the say third wall I got here the temperature T 3 so this resistance is  $\ln r_3$  upon  $r_2$  divide by  $K_2 2 \pi L$  and ultimately it is through surrounding, this is the T 0 and here if these  $1$  upon  $h_0$  into  $r_3$ . Note, this side is refers to the diameter of the or the radius of the wall that is facing to the surrounding.

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Applying series concept

$$\text{Heat flow } Q = \frac{T_i - T_o}{\frac{1}{2\pi L r_1 h_i} + \frac{\ln(r_2/r_1)}{2\pi L K_1} + \frac{\ln(r_3/r_2)}{2\pi L K_2} + \frac{1}{h_o r_3 2\pi L}}$$

Thermal resistance  $R_t = \frac{\Delta x}{kA}$

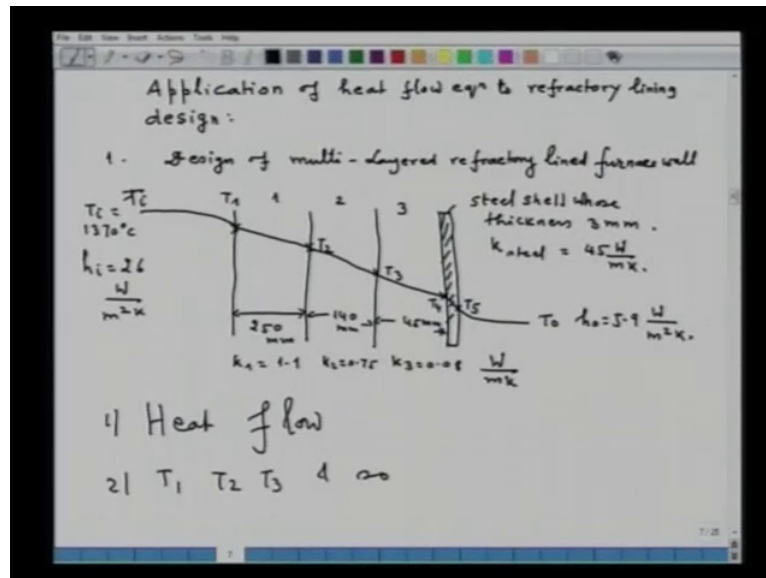
Cylinder  $R_t = \frac{\ln(r_2/r_1)}{2\pi L K}$

Surface to fluid  $R_t = \frac{1}{hA}$

So, now it is very simple. All the resistance are edit you because this is in series so, we can represent now. Applying series concept, we get the heat flow Q that is equal to T i minus T 0 upon  $\frac{1}{2 \pi L r_1 h_i} + \ln r_2$  upon  $r_1$  divide by  $2 \pi L K_1$  plus  $\ln r_3$  upon  $r_2$  divide by  $2 \pi L k_2$  plus  $1$  upon  $h_0$  into  $r_3$ . So, this is the heat flow Q through a composite wall of thermal conductivity K 1 and K 2. If we have three or four wall accordingly we can add resistance.

So, from here we can say that the thermal resistance for a rectangular wall and one dimension that thermal resistance is R t that is equal to  $\Delta x$  upon  $K$  into  $A$ . If we have cylinder in one dimension or we can say for infinite cylinder R t that is equal to  $\ln r_2$  upon  $r_1$  divide by  $2 \pi L$  into  $A$ . And thermal resistance from surface to fluid that is equal to R t that is equal to  $1$  upon  $A$  into  $h$ . So, this is how the concept of the thermal resistance.

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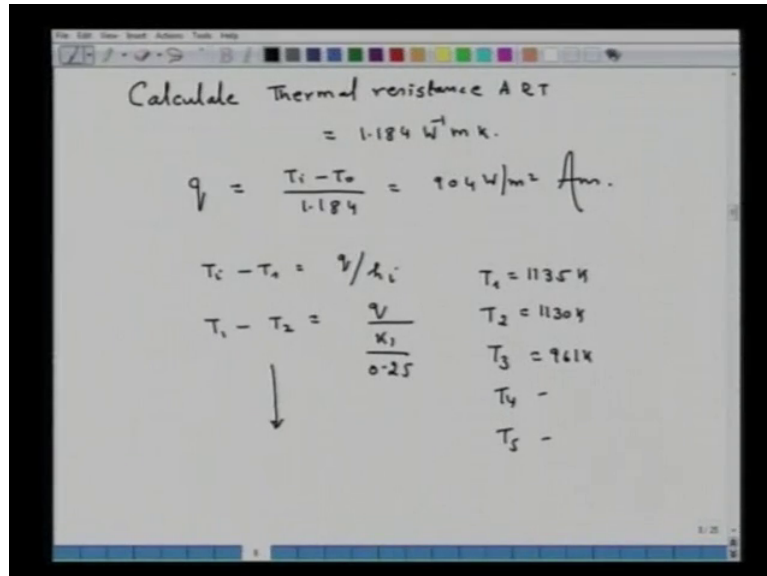


Now, let us apply this equation so as I will see say application of heat flow equation to refractory lining design. Now, let us first of all first let us consider design of multi layered refractory line furnace wall. Now, we are considering one dimensional heat flow so let us consider a furnace wall having three different type of material, just trying the sketch. (No audio from 46:43 to 46:52) So, this is temperature given as T 1, this is temperature T 2, this is temperature T 3, here is temperature T 4 and here is temperature T 5. Now, the wall say this is the one wall second third and this is the steel shell. And the whose thickness is 3 millimeter. So, this is wall one, wall two ,wall three, the thickness of this wall is 250 millimeter, thickness of this wall is 140 millimeter, thickness of this wall is 45 millimeter. K 1 that is equal to 1.1, K 2 0.75 and K 3 that is equal to 0.08, all in Watt meter Kelvin. K of the steel shell K steel shell is given to you 45 Watt meter Kelvin.

Now, if I represent the temperature profile then inside the material it would be linear and the defaces to the temperature. Let us consider this is the temperature of the furnace, this is temperature for example, of the furnace T i that is equal to 1370 degree Celsius and h i that is equal to 26 Watt per meter Kelvin. So as such there will be temperature drop this will be the T i and here accordingly we have T 0 and here h 0 is given to you 5.9 Watt per meter square Kelvin so, this is how a multi layered lining of different thicknesses. What you have to calculate? You have to calculate first of all the amount of heat flow so calculate heat flow and also calculate temperature say T 1 T 2 T 3 and so on at each interfaces so, this is what the

problem is. How will you calculate now? So, what you have to do first of all we have to calculate heat flow.

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So, we will calculate thermal resistance that is A into R T. I already given the formula so if you calculate thermal resistance that will come up to be equal to 1.18 meter Kelvin. So, from here I can calculate heat flux  $q$  that will be equal to  $T_i$  minus  $T_o$  upon 1.184 that will be equal to 904 Watt per meter square, this you can calculate and you get the answer. Now, you have to calculate the temperature at different interfaces, they are thermal resistance offered by each layer. You have to equal it with the heat flux because the heat flux cannot change.

So, for example, the  $T_i$  minus  $T_1$  that is equal to  $q$  upon  $h_i$  similarly,  $t_i$  minus  $T_2$  that is equal to  $q$  upon  $K_1$  upon 0.25 and similarly, you can proceed so you will be getting the temperature  $T_1$  that is equal to 1135 Kelvin temperature  $T_2$  that is equal to 1130 Kelvin and temperature  $T_3$  361 Kelvin similarly, you can calculate  $T_4$  and  $T_5$  and all these temperature can be easily calculated because heat flux to each material will be same. So, all that you have to equate the thermal resistance offered by each wall with the heat flux so only unknown is  $T_1$  so,  $T_i$  is known one can calculate  $T_1$ . Once you calculate  $T_1$  and  $T_1$  minus  $T_2$  where  $T_1$  is the unknown  $T_1$  is known and  $T_2$  is known, so that way one calculate the different temperature. Further example I will take in the next lecture.