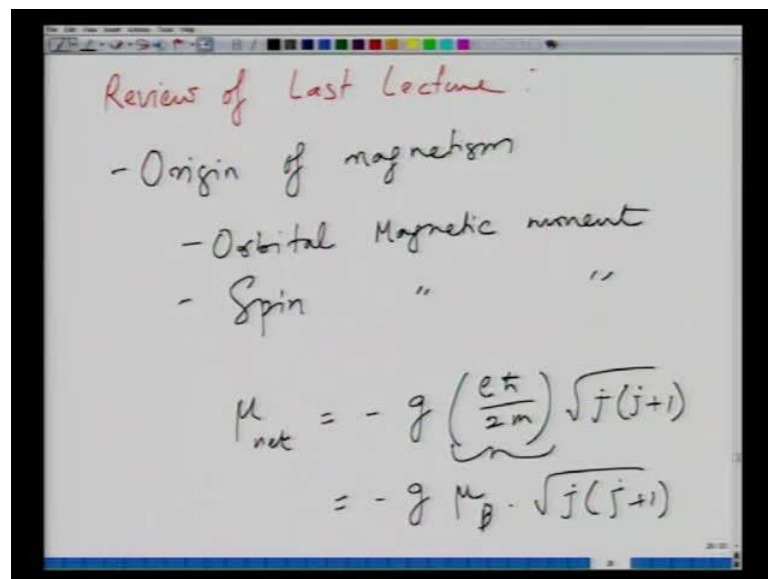


Electro ceramics
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Indian Institute of Technology, Kanpur

Lecture - 34

So, again, we start again new new lecture and in this lecture we will first review what we did last time and then we will go through the new contents of this lecture.

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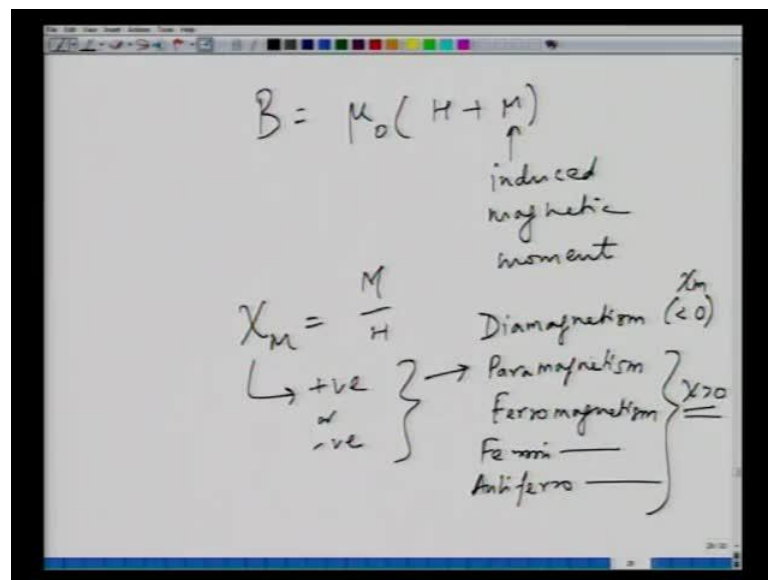
So, in the last lecture... Just let me just review of last lecture. So, in this previous lecture we discussed the origin of magnetism and we saw that origin of magnetic magnetism is essentially from three important contributions; one is the orbital magnetic moment, second is a spin magnetic moment, and third is the nuclear magnetic moment.

Now, nuclear magnetic moment contribution is often very small, so hence it is neglected in most of the quantitative exercises. Now, and we looked at essentially two contributions; one was orbital magnetic moment and then another was spin magnetic moment. And sum of these two in any material gives you the total magnetic moment and this total magnetic moment, which is μ_{net} is minus of g into $e h$ cross divided by $2 m$ into root of small j into small j plus 1. This $e h$ cross divided by $2 m$ is the smallest quantity of magnetic moment and this is called as $g \mu_B$, and this is called as Bohr

magneton into root j into $j + 1$. This small j is nothing but a characteristic quantum number, it depends upon which contribution is stronger.

So, this becomes s when you are talking about spin quantum number, it becomes l when it become, when you are talking about orbital quantum number. Similarly, this lande g factor g is again a characteristic number and its its magnitude is 2, when you are talking about spin parametricism and spin sorry spin purely spin magnetic moments, and 1 when you are talking about purely orbital magnetic moment. So, this was the microscopic picture of magnetic moment.

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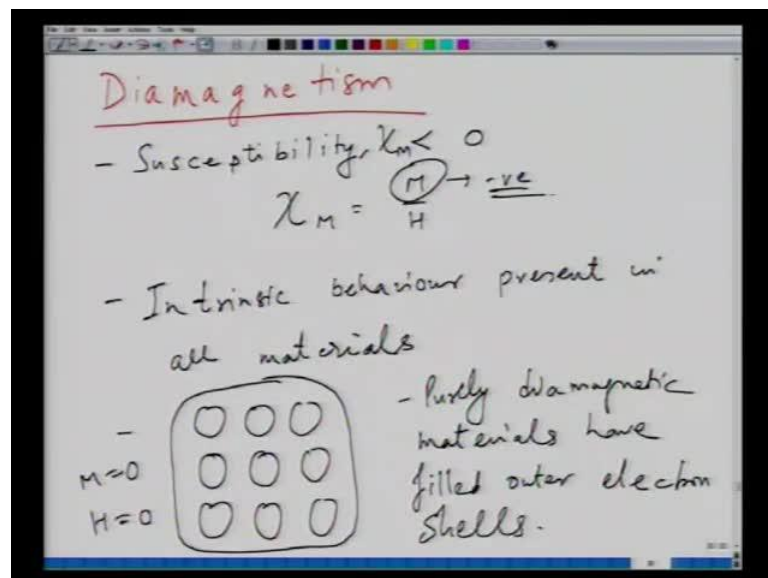
The macroscopic picture of magnetic moment was upon insertion of magnet inside a coil and this gives rise to $\mu_0 H + M$. This M was the induced magnetic moment by inserting a material and this M can be positive or negative both. The quantity which characterises the sign of M is susceptibility and which is nothing but M by H . This susceptibility is a similar quantity in the sense that it is to dielectric susceptibility, in the sense that it quantifies the... It gives a measure of the response of magnetic response of the material.

However, unlike dielectric materials this can be both positive as well as negative. Depending upon what sign it has the magnetism has various categories and based on that we we we defined magnetism into; diamagnetism, which has negative susceptibility and

then paramagnetism, ferromagnetism, ferrimagnetism, and then anti ferromagnetism. All of these will have chi greater than 0.

Now, depending upon all these four effects of different magnitudes of chi, but they are all greater than 0. So, we classified the various kinds of magnetism on this macroscopic picture of magnetism, which is based on magnetic susceptibility and its sign. So, what we will do in this lecture is, we will start with the quantitative picture of each of these kinds of magnetism and then finally move towards the some of the ceramic magnetic materials.

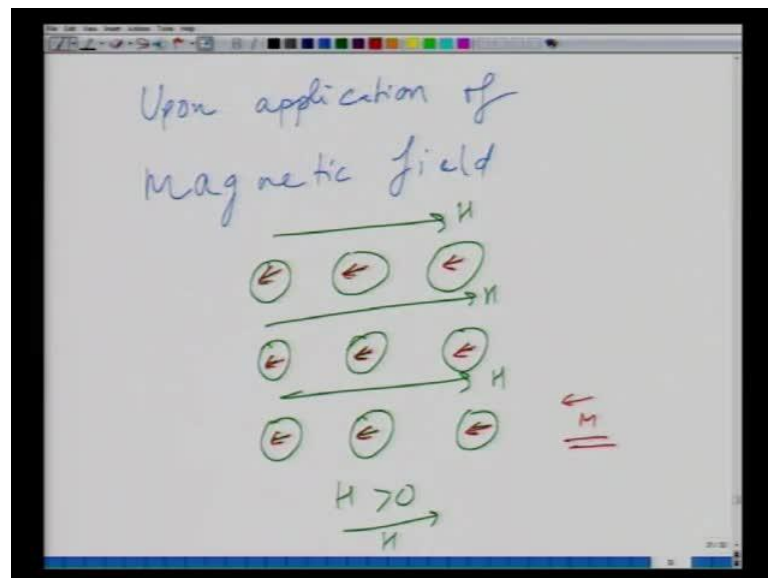
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So, we start with a concept of diamagnetism. Now, diamagnetism as we said earlier it happens in materials which have susceptibility, susceptibility less than 0. Now, what it means is that essentially, this chi M is equal to M by H. Now, since H is positive, which means that M has to be negative. So, M this is the, which is the negative quantity which means, you can see from this sign of the susceptibility that the magnetisation which is induced in the materials is negative, which means it happens in the direction opposite to the applied field. Typically such a phenomena, this phenomena happens in all sorts of materials, but this diamagnetic effect is a is a intrinsic effect for any material. It is just that some materials have only diamagnetic character and other materials have overwhelmingly paramagnetic and ferromagnetic etcetera character.

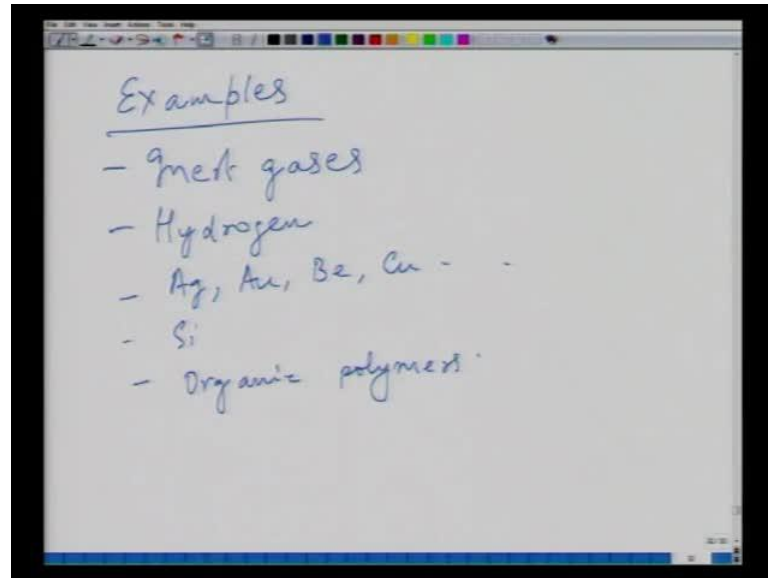
So, this is inherent or intrinsic behaviour present in all materials. It is just that its magnitude is different in different materials and the explicit observation of this effect also depends upon the fact whether, the material has any other type of magnetism present in that. Now, basically diamagnetic materials are the materials, which in the absence of any field they do not have any magnetism. So, purely diamagnetic materials. So, so you draw a atomistic picture, this is the atomistic picture. So, in the absence of... So, when H is equal to 0, then M is also equal to 0, which means there is no sort of picture, which gives you any clear picture of the spins. This typically happens in the materials with filled or closed or, filled outer electron shells. So, purely diamagnetic materials are... Materials have filled outer electron shells and now when you apply magnetic field...

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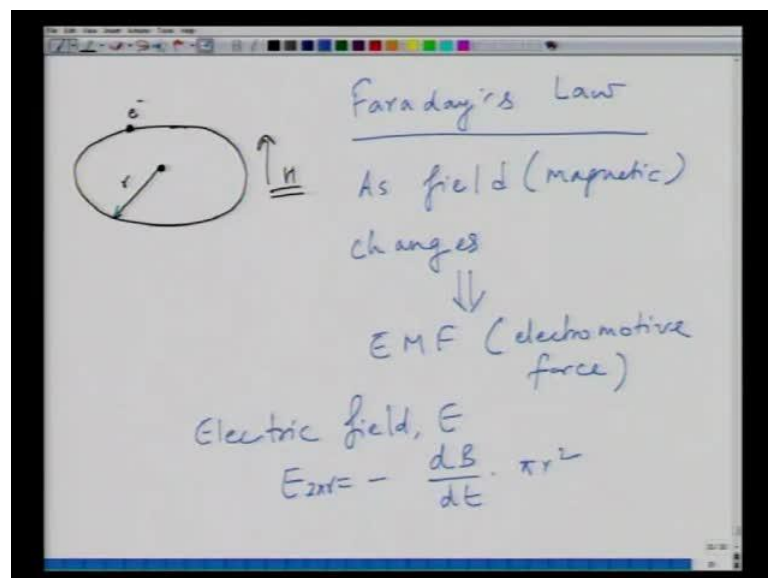
Upon a application of magnetic field, picture is slightly different. So, when we draw this picture again, you have these atoms and in these, in this case, suppose when you apply H greater than 0 and this H is in this direction, what happens is that; this is the field direction H , the magnetism is in this direction. So, this would be M , let me just put it different colour so that it is easy to recognise, this factor is M . So, M opposes H . Now, examples of such kind of effect or examples of materials which show such effect.

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For example, inert gases, purely this effect, I mean that purely diamagnetic materials. So, these are the examples; inert gases, hydrogen, variety of metals like silver, gold, beryllium, etcetera etcetera, copper, many non metals like silicon silicon. Silicon is a semiconductor with covalent bonding and no available electrons as such, and then organic polymers. So, variety of these materials are typically diamagnetic in nature or purely diamagnetic in nature. Now, this is the qualitative picture. Now, what is the quantitative picture?

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For the quantitative picture, you want to assume orbit of radius r , this is the orbit of radius r and you have, so this is the centre of the orbit. This centre so coincides with the centre of the atom. So, you have an electron somewhere here; let us say this is an electron. So, what you do is that, you turn the magnetic field in the vicinity of this atom, so you put on the magnetic field H or B . Now, when you put on a magnetic field, now there is something called in physics which is called as Faraday's law. Faraday's law essentially says that, as you change is applied, as the magnetic field changes, so this field is basically magnetic. As the magnetic field changes, this change in the magnetic field gives rise to what is called as EMF or electro motive force or, simply let us say electric field.

So, this electric field which is tangential to the motion of tangential to this orbit. So, let us say electric field E , is given as it is nothing but minus of dB by dt . Now, you need to get the units right, so here you multiply by the area. So, basically we are talking about the flux, magnetic flux and since this is tangential, you need to multiply it this by the circumference. So, E multiplied by $2\pi r$ is equal to minus of dB by dt into πr^2 , this is nothing but your Faraday's law

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The image shows a whiteboard with the following handwritten equations and text:

$$E = -\frac{r}{2} \cdot \frac{dB}{dt}$$

Electric field \rightarrow force \rightarrow Torque

$$\begin{aligned} \text{Torque, } \tau &= F \cdot r \\ &= -eE \cdot r \\ &= -eEr \end{aligned}$$

$$\tau = \frac{\partial J}{\partial t}$$

$$\frac{dJ}{dt} = -e \cdot \left(-\frac{r}{2} \frac{dB}{dt} \right) r$$

This gives you E to be equal to minus of r by 2 in into dB by dt , so this is the expression that you get for electric field. Now, when you have this electric field present there, what does this electric field do? This electric field gives rise to a force and this force gives rise

to a torque, and this torque is nothing but torque tau is nothing but $F \cdot r$. And F is what? Minus of $e E$ multiplied by r .

So, let us say both of them are collinear. So, as a result it becomes minus of $e E r$ and this tau, this torque is nothing but is time derivative of angular momentum. So, tau is nothing but $\frac{dJ}{dt}$, this is again from your classical physics. So, this minus of $e E r$, you can say $\frac{dJ}{dt}$ is equal to minus of e multiplied by capital E , which is minus of r by $2 \frac{dB}{dt}$ multiplied by r . Let us say... So, let us place this $\frac{dJ}{dt}$ because we need to take similar terminology on the both sides. Now, here you can see that negative, negative cancel each other.

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$$\frac{dJ}{dt} = \frac{er^2}{2} \frac{dB}{dt}$$

Integration of above eqn leads to

$$\int_{J_1}^{J_2} dJ = \frac{er^2}{2} \int_0^B dB$$

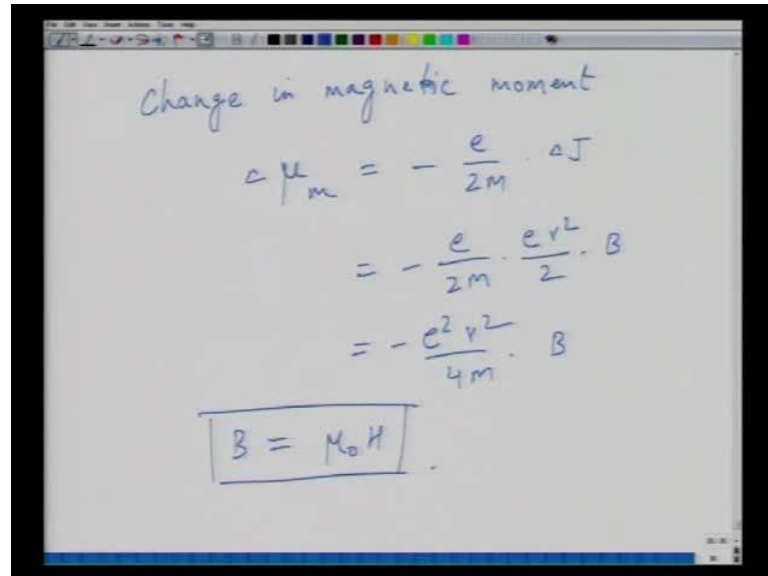
$$\Delta J = \frac{er^2 B}{2}$$

And what you get here, in the end is $\frac{dJ}{dt}$ to be equal to $e r^2$ by 2 into $\frac{dB}{dt}$. So, this is the expression that you get for... So, time derivative of angular momentum is equal to time derivative of magnetic flux multiplied by some constant because all of these parameters here are constant.

Now, you need to integrate this equation. So, integrating this equation from, this J of course, will change from J_1 to J_2 and B will change from 0 to B . So, when you integrate of above equation leads to... So, essentially you have dJ , J_1 to J_2 $e r^2$ by 2 is constant, 0 to B , dB and this gives rise to ΔJ , to be equal to $e r^2 B$ by 2 . This is what is the extra angular momentum, which provided to electrons when you apply this

magnetic field. So, let us box this quantity because this is an important expression. So, this extra momentum leads to change in the change in the magnetic moment.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, it says "Change in magnetic moment". Below that, three equations are written:

$$\Delta \mu_m = -\frac{e}{2m} \cdot \Delta J$$
$$= -\frac{e}{2m} \cdot \frac{e r^2}{2} \cdot B$$
$$= -\frac{e^2 r^2}{4m} \cdot B$$

At the bottom, the equation $B = \mu_0 H$ is boxed.

So, this corresponding change in magnetic moment because we know that angular momentum is related to magnetic moments. So, as a result this change in magnetic moment $\Delta \mu_m$ and this you know was equal to minus of e by $2m$ because it is purely orbital, so g is equal to 1 multiplied by ΔJ . And this becomes minus of e by $2m$ into $e r^2$ divided by 2 into B , and this becomes equal to minus $e^2 r^2$ divided by $4m$ into B . So, this is the change in the magnetic moment that you get. Now, replace B to be equal to $\mu_0 H$. So, this is this is from macroscopic picture that you know, so replace this in our equation. So, what you get?

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$$\Delta \mu_m = - \left(\frac{e^2 r^2 \mu_0}{4m} \right) \cdot H$$

Assuming atoms to be
spherically symmetric,

$$\langle r^2 \rangle = \frac{2}{3} \langle \bar{r}^2 \rangle$$
$$\Delta \mu_m = - \frac{e^2 \bar{r}^2 \mu_0}{6m} \cdot H$$

Delta mu m to be equal to minus of e square r square mu naught divided by 4 m multiplied by H. So, this for this two-dimensional picture that we have taken, now typically if you take for if you take atoms to be spheres. So, let us say assuming atoms to be spherically symmetric, this r square essentially changes to 2 by 3 r bar square. So, as a result our r bar becomes a mean radius and this delta mu m becomes minus of e square r bar square mu naught divided by 6 m into H.

So, this is the change in the magnetic moment. Now, here you see, it is very important observation that here you see that H is a positive quantity, mu naught is a positive quantity, r bar is a positive quantity, m is a positive quantity, e is a positive quantity, which means this delta mu m is negative in nature. And this negative sign essentially illustrates that the change in magnetic moment or magnetisation, which was induced as a result of applied field was opposing the applied field. So, this is very important conclusion from this. So, if you have now now we want to calculate what is the magnetisation?

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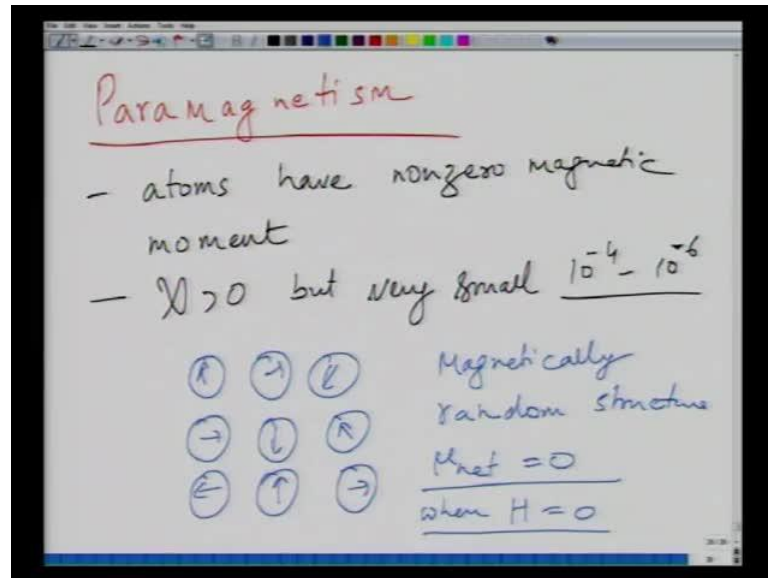
The image shows a whiteboard with handwritten text and equations. At the top, it says 'N: atoms per unit volume'. Below that, 'Magnetization, M'. The main equation is $M = -N \cdot \frac{e^2 \bar{r}^2 \mu_0}{6m} \cdot H$. Below this, a boxed equation shows $\frac{M}{H} = \chi_{dia} = -\frac{N e^2 \bar{r}^2 \mu_0}{6m}$. An arrow points from the boxed equation to the text '(Temperature independent)'.

Assume you have N atoms per unit volume and so as a result the magnetisation M is given as, M is equal to minus of N multiplied by e square \bar{r} bar square μ_0 divided by $6m$ into H . And if you take H on the other side, so M by H and this becomes nothing but χ_{dia} , and this is equal to minus of $N e$ square \bar{r} bar square μ_0 divided by $6m$. So, this is in again important expression, the expression for diamagnetic susceptibility, which is negative because you can see all the quantities in these expressions are in this expression are positive. As a result you get negative susceptibility for the diamagnetic contribution.

Now, remember this effect is inherent in all the materials. It is just that some materials such as with those with complete electron shells they have only diamagnetic behaviour, they do not have any other magnetic effect. So, as a result they are called as diamagnetic behaviour, but since you can see that, since there are electrons in all the materials this effect is bound to be present in all the materials. It is just that those materials which have other effects dominating such as, which are strongly paramagnetic or strongly ferromagnetic, they do not have this effect, get suppressed or over shadowed by a large extent, so that is why you do not see this effect. Otherwise, it is a very intrinsic effect which is present in all the materials. So, this is the analysis of diamagnetic susceptibility and you can also see from this expression that this susceptibility is temperature independent.

So, you can write here, temperature independent, this is an important conclusion. Now, what we will do is that, we will take an analysis for another kind of magnetism that we discussed last time and that is nothing but paramagnetism.

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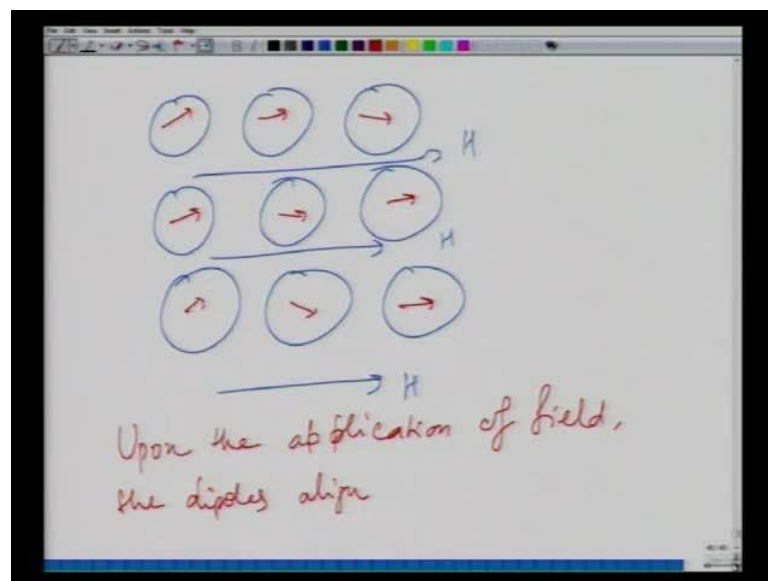
These paramagnetic materials are rather different, fundamentally what they have is, they have atoms in these materials, which have a non-zero magnetic moment. So, some essential features of these materials are; atoms have non-zero magnetic moment. Now, these and this non zero magnetic moment can be sum of again, as I said orbital and a spin magnetic moment, but still susceptibilities of these materials are very small. So, chi is a very small number, it is a positive number, so chi is greater than 0, but very small. It is of the order of 10 to power minus 4 to 10 to power minus 6, in fact 10 to power minus 4 is on the higher side, mostly it is closer to 10 to the power minus 5, minus 6.

Now, what is the reason? The reason is although each atom tend to have finite magnetic moment, which means it is magnetically polar in that sense, but all these magnetic moments just like a paraelectric material or just like normal dielectric material, all these magnetic just like a normal polar dielectric material, all these magnetic moments are randomly distributed, with respect to each other. So, what you have a picture, so you have a picture in which you have these atoms and these atoms will have magnetic moments in all the directions. So, this is basically magnetically random structure, as a result μ_{net} is equal to 0 when H is equal to 0. So, when you apply magnetic, when you

when you do not have any magnetic field then this magnetic moment is 0. So, that is why in the absence of any magnetic field these paramagnetic materials do not show any magnetisation, despite having atoms which are magnetically polar.

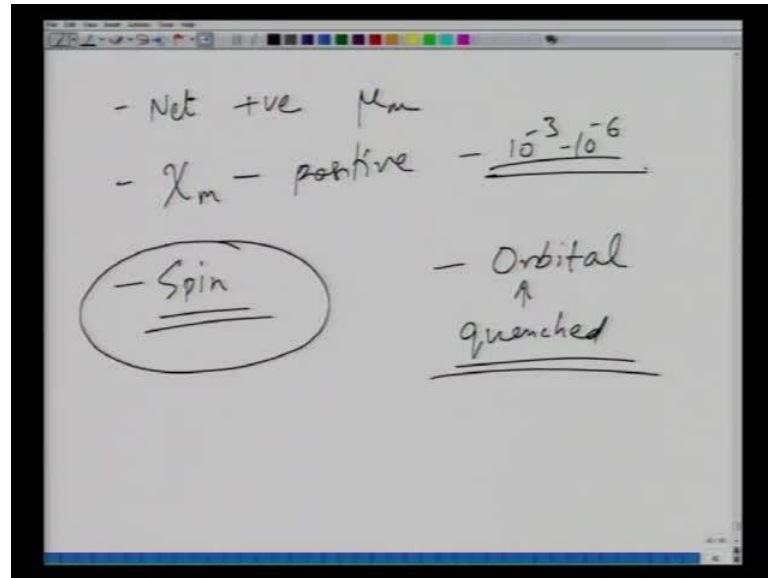
However, when you apply magnetic field and this is and this is essentially this random distribution is essentially, because of thermal energy dominating. So, thermal energy prevents these magnetic moments aligning with respect to each other. So, large thermal effects dominate this kind of effect. Now what happens when you apply field?

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Upon application of field, if you draw the picture again. So, upon a application of field, suppose you apply the field in this direction, this is H , what happens here is... Now, here there is alignment depending upon the strength of field, there is depending upon the strength of field there is alignment of magnetic moment, along with the direction of applied field. So, what you have essentially is upon the application of field, the dipoles align. Now, when you have this kind of picture, what happens is that then the material will have net positive magnetic moment.

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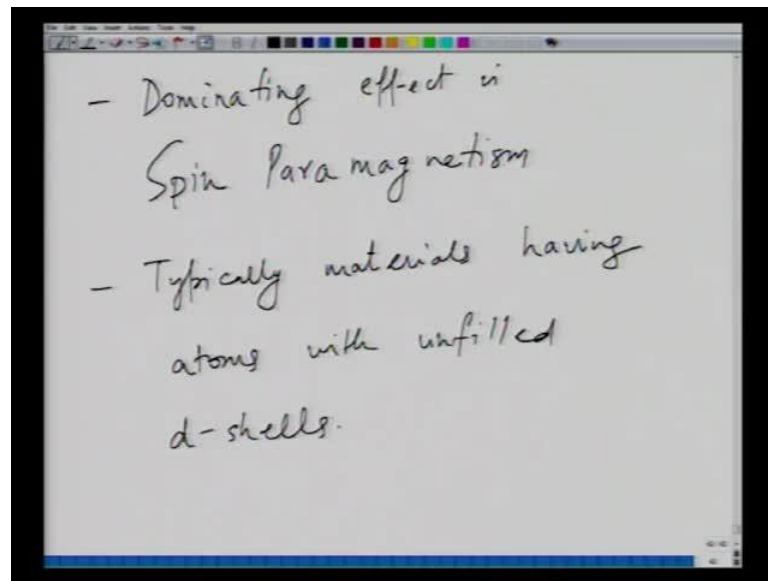


So, essentially what you have is a net positive magnetic moment and this gives rise to susceptibility, which is positive, but the overall effect remains small because this overall value is only 10 to power minus 3 to minus 6 , with respect to applied field.

So, M by H is essentially of the order 10 to power minus 3 to minus 6 . Now, you have two kinds of magnetism as we said earlier; you have a spin and you have orbit. Typically in most paramagnets orbital contribution is called as quenched. Now, what his quenching means is that, in these materials typically the atoms have surrounding field and this surrounding field sought of couples these orbits with respect to the lattice. As a result these orbits, the moments or let us say the orbital magnetic moments do not switch when you switch the field. So, as a result they essentially remain ineffective.

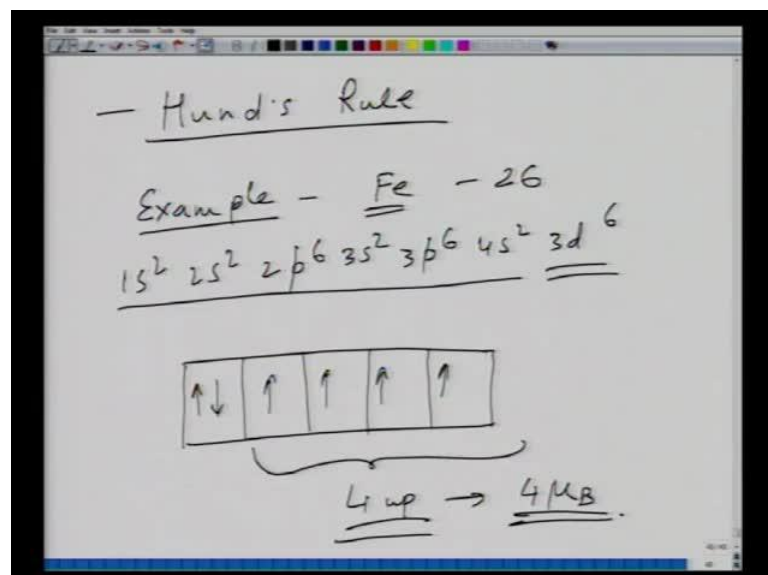
So, what you have essentially coming into picture as a major force of magnetism is essentially a spin paramagnetism, which is a major force in most of the paramagnetic materials. For for for a paramagnetic materials essentially in this discussion we will only consider the spin paramagnetism, because orbital contribution is essentially quenched in these materials, for the for the reasons which are beyond this course. But essentially it is because of the local filed, which is which is which is or the the field which is created by the surrounding ions in the lattice. As a result these orbits are essentially coupled to the lattice and the magnetic moments are unable to flip back and forth, they cannot reorient or orient themselves along with the field, as a result they are supposed to be frozen.

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So, a spin paramagnetism is essentially the dominating factor, and spin magnetism is found in materials which have unfilled d-shells, so typically materials having atoms with unfilled d-shells. If you have materials having atoms with unfilled d-shells they also tend to follow what is called as Hund's rule.

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So, this Hund's rule is essentially a basic physics law, which says that the the moment for this atom is a maximized by alignment of spins in the in the in the in one direction by filling up in one direction first and then going for another direction. So, without violating

the laws of quantum mechanics, especially the Pauli's exclusion principle you can maximize the moment. For example, you take the effect of you take the you take the example of iron, Fe. Now, Fe is 26, so 26 will make 1s 2, 2s 2, 2p 6, 3s 2, 3p 6 and 4s 2 and 3d 6, so this is the configuration for Fe, so 10 10 plus 10 and 6, 26. So, if all these shells are filled, these are not going to contribute to magnetism at all. So, only this shell which is unfilled is going to contribute to the magnetism. So, the picture for d-shell is, you have d-shell and this has 5 sub shells, so these are the 5 sub shells.

Now, one way of doing, arranging these 6 electrons is, you put one up one down, one up one down, one up one down, but this would not give you any magnetism because all the spins cancel each other, as a result you do not have any magnetic moment. Now, what Hund's rule does is, it allows the 5 shells to be filled first with one spin direction. So, 5 up go in the five unfilled shells and then one remaining goes in one of these, so as a result what you have is 4 up and this 4 up gives rise to four μ_B of magnetic moment in iron. So, this is typically, this is this is a typical picture with atoms having unfilled d-shells and this helps in materials having a magnetic moment, as a result. So, this iron has 4 μ_B of magnetic moment, it is a magnetic polar material. Similarly, you can have nickel, cobalt etcetera, all of these have unfilled partially occupied d orbitals and as a result you have a finite magnetic moment.

So, as a result each iron atom will have a magnetic moment of 4 μ_B . The picture may be slightly different for 4 f materials, because in 4 f materials they have f electrons. So, as a result as a result you have both orbital and spin contributions coming into picture for those kind of materials. Now, you must now you must come to what is called...

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The image shows a whiteboard with three equations for paramagnetic susceptibility. The first equation is $\chi_{para} = \frac{M}{H}$. The second equation is $\chi_{para} = \frac{N \mu_m^2 \mu_0}{3RT}$, where μ_m is circled and an arrow points from it to the text "Temperature dependent". The third equation is $\chi_{para} = \frac{C}{T}$, where C is underlined and an arrow points from it to the text "Curie's Constant".

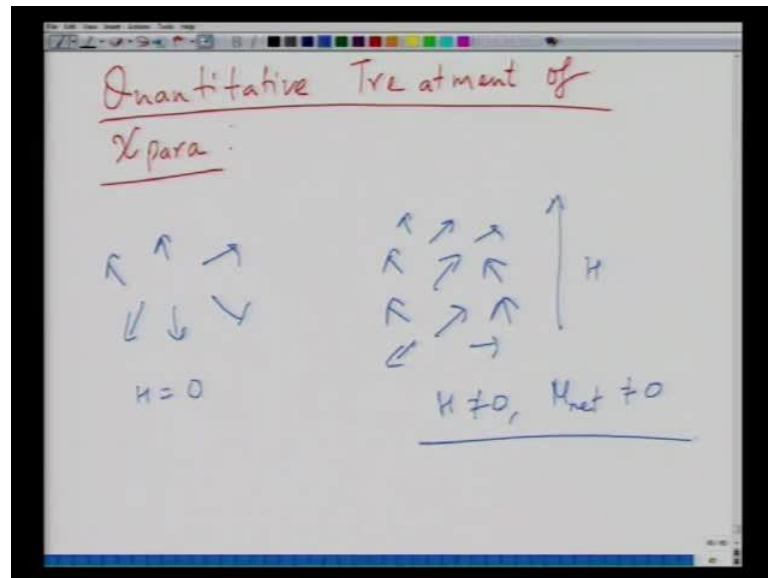
So, a finite magnetic moment and this which is positive, so as a result this susceptibility of these materials χ_{para} is also a positive quantity. It is given as M by H and this is equal to $N \mu_m^2 \mu_0$ divided by $3kT$, and this is the same expression that if you remember you got it for polar dielectrics, and when we are and when we are talking about the orientation polarization.

So, what this paramagnetic susceptibility shows you, it depends upon μ_m , which is the magnetic moment for the atoms and T , which is the temperature. So, it is a temperature dependent susceptibility and this typically this susceptibility is shown to follow what is called as Curie's law. So, as a result this χ_{para} is often written as C by T . So, this C becomes Curie's constant and this C is nothing but $N \mu_m^2 \mu_0$ divided by $3k$, so it becomes a Curie's constant. So, N is nothing but number of atoms, μ_m is the magnetic moment number of atoms per unit volume, μ_m is the magnetic moment, μ_0 is the permeability of free space, k is the Boltzmann constant and T is the temperature.

So, susceptibility is inversely proportional to the temperature, which is the same conclusion that we drew when we were doing orientation polarization. And this is sort of this sort of make sense because as a temperature increases, the susceptibility goes down. What it means is that as the temperature increases, the thermal energy which causes the randomization of magnetic moments that increases, so as a result the magnetic response

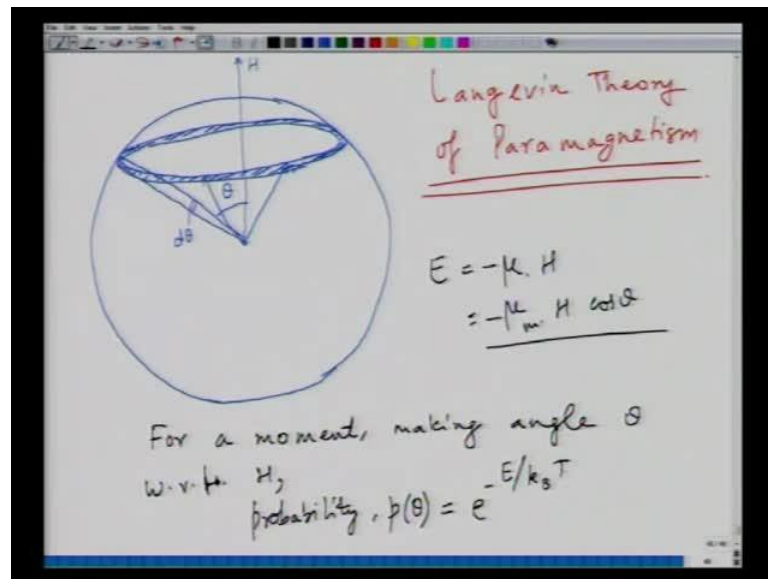
of the material goes down. So, essentially you need to apply a larger field overcome the last thermal energy at higher temperatures.

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Now, we can do a quantitative treatment of this. So, the quantitative treatment of paramagnetic susceptibility is similar to what we did for orientation polarization. So, essentially here we assume that each of these atoms have a finite magnetic moment μ_m and this μ_m is finite, but its direction can be random. So, within a material all these μ_m can be in various directions. So what you will essentially have is, in the direction of, in the in the in the absence of... So, H is equal to 0 will be completely randomized picture and when you have finite H then these will be little random, but with the propensity to align in the direction of applied fields. So, H is not equal to 0 and μ_{net} is not equal to 0.

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So, what essentially you have a picture like this, if you draw a unit sphere and this unit sphere will have, let us say magnetic field direction H like this. In this you can have these, let us say this is a segment of sphere, and this basically circumference. This circle essentially denotes the magnetic moment, which are all pointed in this direction. So, this is the centre of the sphere and so centre of the... So, you have centre of the sphere, so this is the magnetic field direction and you have all the magnetic moments going round like this. And you can have these magnetic moments at any angle, so you have this H perpendicular right up and then you can have all these magnetic moments, which are like this.

So, they can be at any angle θ , so this is one θ , this is another θ , this is another θ and they all could be any side. So, basically they make a cone around the applied magnetic field. Essentially what you have is this kind of picture. So, let us say they just take for this small unit area and let us say this is angle is $d\theta$, and this angle is θ . So, what we are going to do is that, we are going to just analyze the way we did for orientation polarization and this analysis is also called as Langevin theory of Paramagnetism.

This is an important concept in the understanding of magnetism. So, what essentially you have is, host of these whole array of these magnetic moments aligned at various angles with respect to H and these magnetic moments as I said, make a conical structure around

this magnetic field. So, what we will do is that, using the classical mechanics we will derive an expression for the susceptibility of these materials. So, let us say for a moment which makes an angle, making an angle making angle theta with respect to H. So, what is for a moment which makes an angle theta with respect to H, what is the probability that it occupies a certain state E, certain energy state E.

Now, you know that the energy of these dipoles is or energy of these magnetic dipoles in this, is the analogous way as of dielectric materials is given as $\mu \cdot H$. So, this is nothing but $\mu_m H \cos \theta$, minus of $\mu \cdot H$. So, naturally theta is, as the theta is 0 the energy becomes minimum, so the natural propensity of course is to having an angle closer to H rather than having farther from H, when you apply H. This is this is the whole reason why the magnetic moment align themselves with the applied field rather than pointing another direction, although the case for antiferromagnetism is entirely different.

So for a for a moment μ_m making an angle theta with respect to H, you can write what is the probability that it will be in energy state E and this is given by Boltzmann statistic. This probability is given as p, let us say and this is given as $e^{-E/k_B T}$ to the power minus, this energy E divided by $k_B T$.

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$$p(\theta) = \frac{e^{-E/k_B T}}{\mu_m H \cos \theta / k_B T}$$

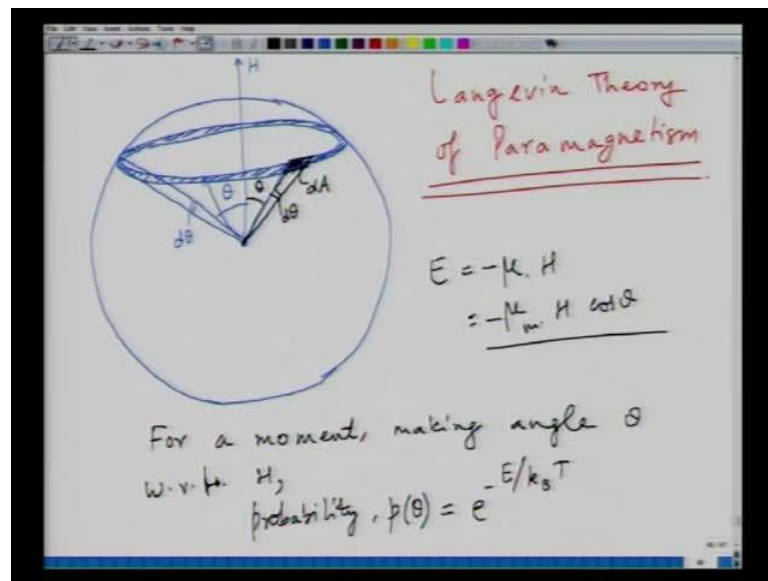
$$= e$$

No of magnetic moments, that lie between θ and $\theta + d\theta$ w.r.t H,

So, $p(\theta) \propto e^{-\frac{E}{k_B T}}$ and this is equal to $e^{-\frac{\mu_m H \cos \theta}{k_B T}}$. So, this is the probability with which this will align, so naturally align in the direction of field ($\theta = 0$). Naturally $\theta = 0$, this probability will maximize.

So, number of magnetic moments which lie between θ and $\theta + d\theta$, with respect to H . So, what we do is that, so we take this fractional surface area of the surrounding sphere. So, if you go the previous picture, so you basically take this fractional surface, you can choose any fractional surface area. So, let us say this is the fractional surface area, these are proportional to the fractional surface area then of course, if you want to take over the whole ring, you need to integrate it. So, this fractional surface area dA , as shown in this picture, this is dA .

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So, essentially taking over this fractional surface area, so again you can say that this θ and this is $d\theta$, this makes it more clearer.

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Handwritten notes on a whiteboard:

$$p(\theta) = \frac{e^{-E/k_B T}}{H_m H \cos \theta / k_B T} = e$$

No. of magnetic moments, that lie between θ and $\theta + d\theta$ w.r.t. H , \propto fractional area dA .

$$dA = 2\pi r^2 \sin \theta d\theta$$

So, again... So, this number of... So, they are proportional to this fractional area dA and this dA is given as, you can find it out from the solid angle, is $2\pi r^2 \sin \theta d\theta$. So, number of magnetic moments that are lying between this angle θ into θ plus $d\theta$ are proportional to basically this fractional area dA . This dA can be calculated using the solid angle as $2\pi r^2 \sin \theta d\theta$, just in the similar way as we did in dielectric material.

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Handwritten notes on a whiteboard:

Overall probability, $P(\theta)$.

$$P(\theta) = \frac{e^{-H_m H \cos \theta / k_B T} \cdot \sin \theta d\theta}{\int_0^\pi e^{-H_m H \cos \theta / k_B T} \sin \theta d\theta}$$

total no. of magnetic moments

So, as a result the overall probability $p(\theta)$ for atoms making basically angle between θ and $\theta + d\theta$. So, this $p(\theta)$ is given as $e^{-\frac{\mu_m H \cos \theta}{k_B T}} \sin \theta d\theta$ divided by $\int_0^\pi e^{-\frac{\mu_m H \cos \theta}{k_B T}} \sin \theta d\theta$. So, this is the overall probability and this denominator is essentially the total number of magnetic moments.

So, now what we have worked out is, what is the overall probability for atomic moment making an angle between θ to $\theta + d\theta$. So, now for each atom you have a contribution, which is $\mu_m \cos \theta$.

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The image shows a whiteboard with the following handwritten equations:

$$\text{Each atom} \rightarrow \mu_m \cos \theta$$

$$M = N \mu_m \langle \cos \theta \rangle$$

$$= N \mu_m \int_0^\pi \cos \theta \cdot p(\theta) d\theta$$

$$M = \frac{N \mu_m \int_0^\pi e^{\frac{\mu_m H \cos \theta}{k_B T}} \cos \theta \sin \theta d\theta}{\int_0^\pi e^{\frac{\mu_m H \cos \theta}{k_B T}} \sin \theta d\theta}$$

So, each atom has $\mu_m \cos \theta$ contribution, so if you have N number of atoms, you can find out the total magnetisation. So, magnetization M , which is parallel to the applied field for the whole system will be equal to N ; number of atoms per unit volume multiplied by $m \cos \theta$ and this $\cos \theta$ will be for all the $\cos \theta$ s, that you for all the $\cos \theta$ s that you have for all... The, this should be small m this should be small μ_m rather.

So, this would be equal to $N \mu_m \int_0^\pi \cos \theta p(\theta) d\theta$ and this should be equal to, so M would be equal to $N \mu_m$ multiplied by $\int_0^\pi e^{-\frac{\mu_m H \cos \theta}{k_B T}} \cos \theta \sin \theta d\theta$, divided by $\int_0^\pi e^{-\frac{\mu_m H \cos \theta}{k_B T}} \sin \theta d\theta$. So, you just go

back to the previous picture. We have these 0 to π to the power $\mu_m H \cos \theta$ $k_B T \sin \theta$, this is the number of total number of magnetic moment, and this is essentially... So, this expression gives you the probability essentially, that you have moments between θ and $\theta + d\theta$. Then when you multiply this by essentially $\mu_m \cos \theta$, you get the total magnetic moment and so this works out. Now, this is worked out in a similar fashion as we did in orientation polarization.

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The image shows a whiteboard with the following handwritten derivation:

$$M = N \mu_m \left[\coth \left(\frac{\mu_m H}{k_B T} \right) - \frac{k_B T}{\mu_m H} \right]$$

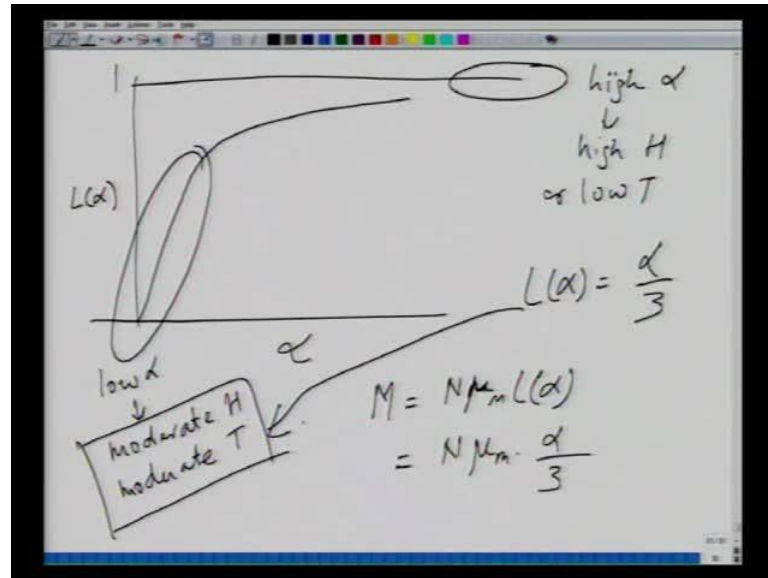
$$= N \mu_m \left[\coth \left(\alpha \right) - \frac{1}{\alpha} \right]$$

Langevin Function.

$$= \underline{N \mu_m L(\alpha)}$$

So, this M , the magnetisation works out to be N into μ_m into \coth of hyperbolic $\mu_m H$ divided by $k_B T$ minus of $k_B T$ divided by $\mu_m H$. If you remember this $\mu_m H$ by $k_B T$, we assume it α . So, this becomes $N \mu_m \coth$ of hyperbolic α minus 1 divided by α and this is nothing but your Langevin function, this is given as $N \mu_m L(\alpha)$. So, essentially it goes in, the solution goes in the similar manner as in the as in the dielectric materials.

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So when you plot this L alpha for... Alpha something like that before approaching a value 1 at the very high values of alpha or at very high values of field essentially or, at very low temperatures, when the thermal fluctuations are lower. So, this 1 would essentially be high alpha and that would mean high H or low T. On the lower side would we would have is, in this side what we will have is, low alpha and that would mean moderate H, moderate T and this is what is more realistic picture. So, as a result this L alpha, so this m which is given as $N \mu_m L(\alpha)$, it takes up a value of $N \mu_m \alpha$ by 3, at these lower values for all practical fields essentially and at low enough temperature. So, this L alpha becomes equal to alpha by 3 at these conditions.

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The image shows a handwritten derivation on a whiteboard. At the top, the magnetization M is given as $M = \frac{N\mu_m^2 H}{3k_B T}$. Below this, the paramagnetic susceptibility χ_{para} is defined as $\chi_{para} = \frac{M}{H} = \frac{N\mu_m^2}{3k_B T}$. A note indicates that in SI units, the formula becomes $\chi_{para} = \frac{N\mu_m^2 \mu_0}{3k_B T}$. At the bottom, a note states "as $T \uparrow$, $\chi_{para} \downarrow$ ".

So, as a result the M becomes equal to $N \mu_m^2 H$ divided by $3 k_B T$ or rather M by H is equal to $N \mu_m^2$ divided by $3 k_B T$. So, this is the expression for χ_{para} , which is nothing but your paramagnetic susceptibility. If you put that in SI units, in the SI units it will become, χ_{para} will be equal to $N \mu_m^2 \mu_0$ divided by $3 k_B T$. Again, so you can see that this is the origin for C by T nature of this paramagnetic susceptibility.

So, this is a very important expression that you have, that we have derived for the paramagnetic materials having temperature inverse temperature dependence of susceptibility. Essentially as temperature increases χ_{para} decreases and this essentially because of increase thermal randomisation and as a result you have say a kind of picture. Now, you can you can make some modifications to this expression. The modifications can be made in the form of in the form of adding both explain or orbital contributions or taking the quantum mechanical picture.

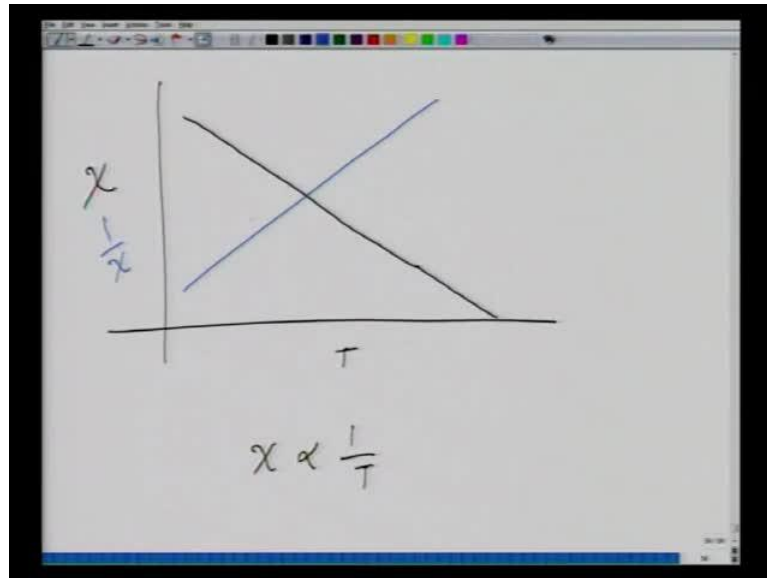
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The image shows a whiteboard with handwritten text and equations. At the top, it says "Q-M picture" with a horizontal line underneath. Below this, the equation for paramagnetic susceptibility is written as $\chi_{para} = \frac{N g^2 j(j+1) \mu_B^2}{k_B T}$. The numerator is circled, and the denominator $k_B T$ is also circled. Below this equation, the simplified form $\chi_{para} = \frac{C}{T}$ is written, with a wavy line underneath it. At the bottom, the text "Curie-Weiss Law" is written and underlined.

If you take a quantum mechanical picture then this... I am not going to details of this quantum mechanical picture, but quantum mechanically this chi Para works out to be $N g^2 j(j+1) \mu_B^2$ divided by $k_B T$ because the point here is... Unless you quantise μ , you do not have a quantised number of such a moments.

So, in order to have quantisation you need to have quantum mechanics come into picture, so as a result you have characteristic quantum number. So, this again become nothing but equal to C by T , if you take everything else as everything else as a constant. But this is quantum mechanical picture, which you have present for these materials. Now, and this behaviour is called as Curie-Weiss law. So, just like ferroelectric, dielectric materials or dielectric materials you have this Curie-Weiss behaviour, which you have for these paramagnetic materials.

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Now, so essentially you can see that if you plot the susceptibility versus temperature, so this is temperature and susceptibility χ . So, you have a transition temperature T_c and up to which these materials, as you see that χ is proportional to $1/T$. So, χ will decrease as a function of temperature, before sort of becomes completely 0.

So, this is how the expression is or if you plot, another better way of plotting this is plotting $1/\chi$, so $1/\chi$ naturally it increases. So, this will be behaviour for $1/\chi$ and this is more appropriate picture. So we have done this derivation, we will take this little more forward with respect to some more analysis.

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Theoretical values				
Ion	Elec Config.	Calculated		μ_m (Measured)
		$g \sqrt{l(l+1)}$	$g \sqrt{s(s+1)}$	
<u>Transition Elements</u>				
Mn ³⁺	3d ⁴	0.0	4.9	4.9
Fe ³⁺	3d ⁵	0.0	5.92	5.9
Fe ²⁺	3d ⁶		4.9	5.4
Ni ²⁺	3d ⁸		2.83	3.2
<u>Rare Earths</u>				
Ce ³⁺	4f ¹ 5s ² 5p ⁶	2.54		2.4
Nd ³⁺	4f ³ 5s ² 5p ⁶	3.62		3.5

So, essentially this $1/\chi$ varies like this, you can see some of the theoretically derived values of, theoretical values of magnetic moment. As you go to the previous slide, here we wrote this χ_{Para} in terms of $N g^2 \mu_B^2 j(j+1) / 4kT$ and this j here again has put into here in this expression because it helps in not only giving you a quantised picture. But also it helps you in separating the contributions from completely orbital and completely spin contribution. Because this j will take value of spin quantum number when it is spin contribution, it will take orbital quantum number when it takes the orbital moment is dominating.

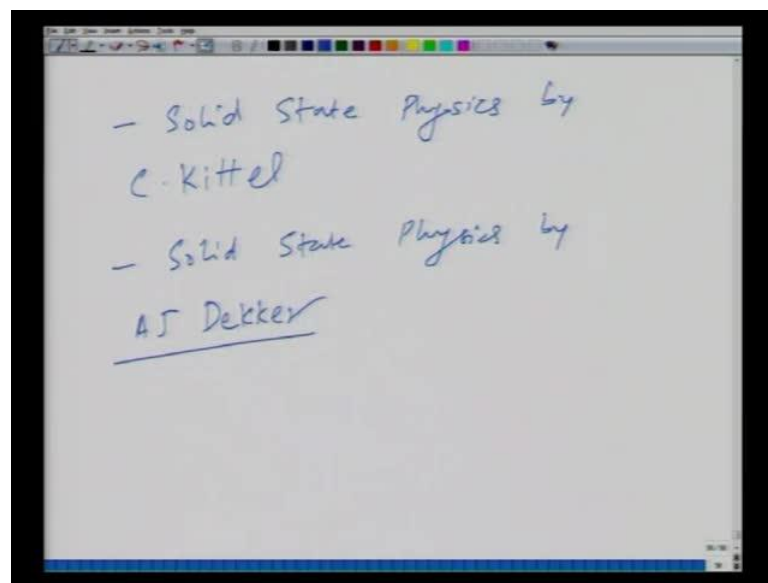
So, as a result if you if you look for some number, so for example, you have this is the ion and then configuration, electronic configuration and then calculated value of... So, you calculate it for both orbital as well as spin. So, orbital is root of $J, J+1$ and then spin is g into root of $s, s+1$ or rather make it $l+1$ and then measured value of, this is measured value. So, electronic configuration, I am going to take only for the last few shells. So, for d for transition elements, let us say in case of manganese $3+$. Manganese $3+$ has $3d^4$ configuration and this will be 0, and this would be 4.9 and the measured is 4.9, so fairly accurate the spin contribution.

Similarly, Fe $3+$, you have $3d^5$ for Fe $3+$ and as a result the measured value is for Fe $3+$ is 5.92, this is 5.9. So, actually make this... So, if you calculate for l , the values are absorbed, basically they do not make any sense for these materials. So Fe $2+$ $3d^6$

and this is 4.90 and the measured is 5.4, so fairly close. If you take for nickel 2 plus its 3d 8 and this is 2.8 and what you measure is 3.2. So, this is for transition elements, for rarer as I said for orbital for m is contributing. So for example, for cerium 3 plus you have a 4f 1, 5s 2, 5p 6 and this gives rise to 2.54 here and what you measure is 2.4.

Similarly, for neodymium 3 plus you have 4f 3, 5s 2, 5p 6 this gives rise to 3.62 orbital contribution and what you measure is 3.5. So, this quantum pictures gives you very good estimate of orbital and very good differentiation between orbital and spin contributions.

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I am not going to go into details of this, but if you want to go into details of this magnetic, quantification of magnetic susceptibility you can follow; Solid State Physics book by Kittel, Charles Kittel or Solid State Physics by Dekker, A J Dekker.

So, these are some books which you can which you can refer to for better understanding or for detail discussion. Now, we will close this lecture here. In the next lecture we will look at other two kinds of magnetism or other kinds of remaining kinds of magnetism, essentially ferromagnetism and a anti ferromagnetism and ferrimagnetism.

Thank you.