Electroceramics

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Lecture - 23

So, we start with the new lecture. And what we will do is that, we will first review the last lecture, because this lecture is nothing but continuation of the previous lecture. So, now, where did we start? We started with the analytical treatment of frequency dependence of various polarization mechanisms. So, if we go to previous lectures. Now, previous lectures, what did we discussed was, if we go back a few slides what we had was, we started treating this.

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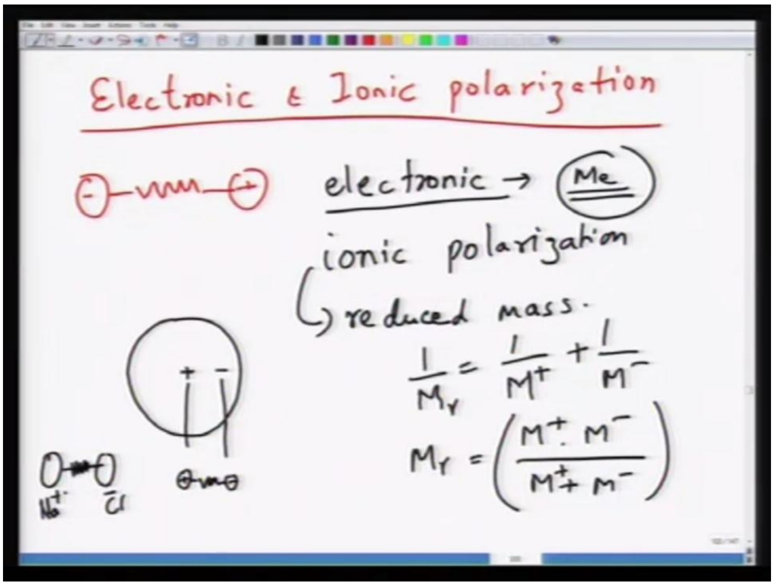

So, we started with this case of a electronic and ionic polarization, where we treated this charged dipole as a linearly oscillating mechanical dipole or linear mechanical oscillating oscillator. Now here, whether you have atom sitting here or a group or a, so in case of electronic system, so suppose this is the picture.

Now, this is the centre of let us say positive charge, this is the centre of negative charge, and the centre of positive and negative can be treated as a positive, negative and dipole. So, this is at an atomic scale. Now, you can have at lattice's scale, where you have a system like sodium chloride. So, you have a lattice like this, and here you have let us say sodium atom, here you have chlorine. So, positive, negative and this also makes a dipole. So, you can treat these two systems like a mechanical oscillating system.

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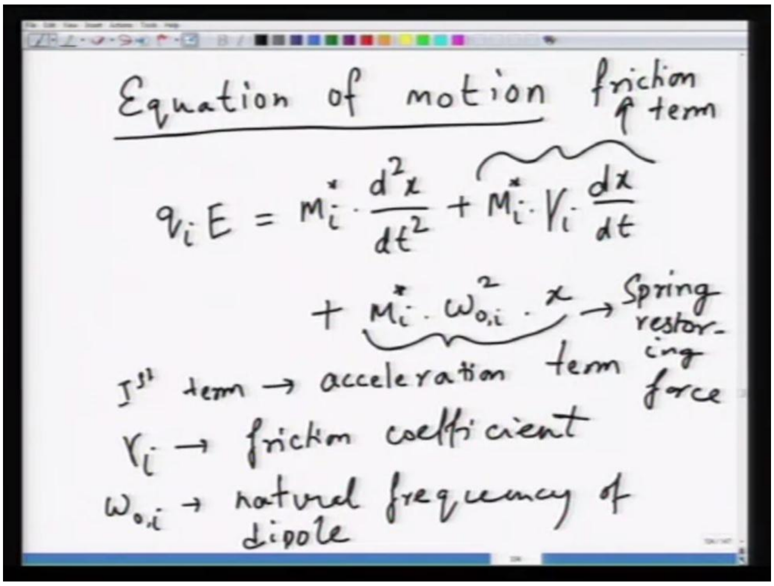

Equation of motion

$$q_i E = M_i^* \cdot \frac{d^2x}{dt^2} + \overbrace{M_i^* \cdot \gamma_i \cdot \frac{dx}{dt}}^{\text{friction term}}$$

$$+ \underbrace{M_i^* \cdot \omega_{0,i}^2 \cdot x} \rightarrow \text{Spring restoring force}$$

1st term → acceleration term

γ_i → friction coefficient

$\omega_{0,i}$ → natural frequency of dipole

And here, if you write you can write an equation of motion for both of these systems, where the force which is due to applied field is equal to various forces which are then generated as intern. Now, this applied field is a c in nature. So, you have an acceleration term, followed by, you have a friction term which would be true for any system if damping is present. And then you have a string constant related system, where which is called as spring restoring force. Now, this is because we have considered the dipole like a spring.

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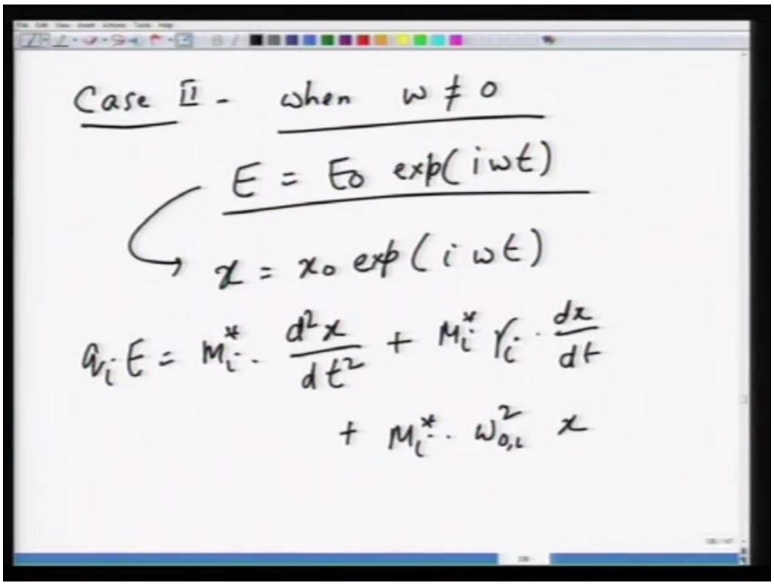

Case II - when $\omega \neq 0$

$$E = E_0 \exp(i\omega t)$$

$$x = x_0 \exp(i\omega t)$$

$$q_i E = M_i^* \cdot \frac{d^2x}{dt^2} + M_i^* \gamma_i \cdot \frac{dx}{dt} + M_i^* \cdot \omega_{0,i}^2 \, x$$

So, and when you do analysis when you, when you put in the value of E is equal to E naught exponential i omega t, which is a alternating electric field. So, correspondingly the displacement for this field should be x is equal to x naught exponential i omega t. And when you make the displacement, when you make the, when you replace the value of E in this expression, you get an expression for x.

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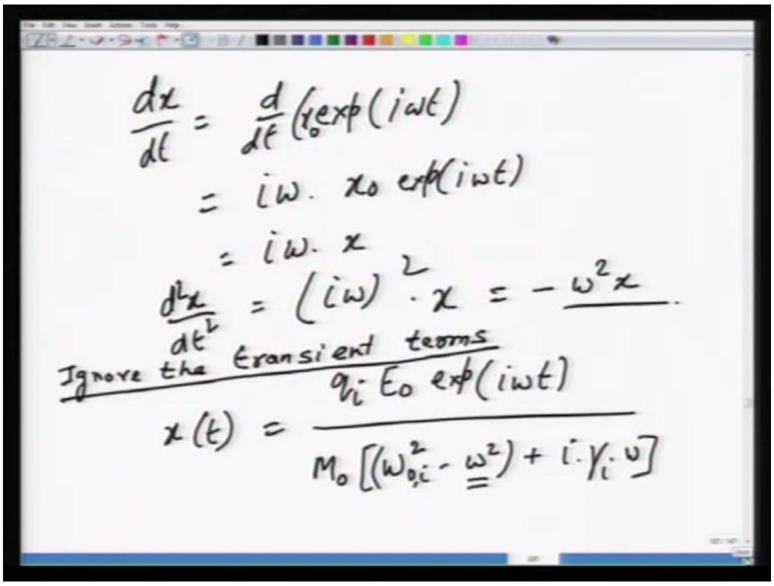

$$\frac{dx}{dt} = \frac{d}{dt}\left(x_0 \exp(i\omega t)\right) = i\omega \cdot x_0 \exp(i\omega t) = i\omega \cdot x$$

$$\frac{d^2x}{dt^2} = (i\omega)^2 \cdot x = \underline{-\omega^2 x}.$$

Ignore the transient terms

$$x(t) = \frac{q_i E_0 \exp(i\omega t)}{M_0\left[(\omega_{0,i}^2 - \underline{\underline{\omega^2}}) + i \cdot \gamma_i \cdot \omega\right]}$$

And the expression for x is, so, you calculate various things d x by d t d 2 x by d t square etcetera.

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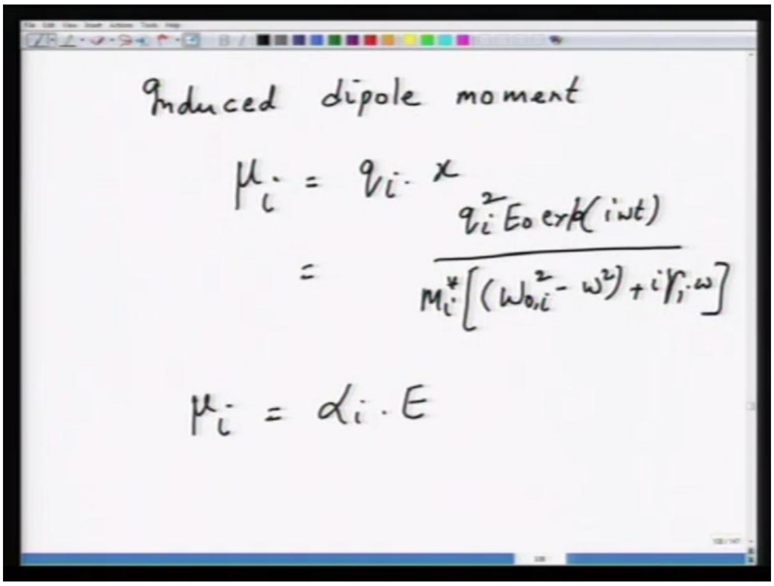

Induced dipole moment

$$\mu_i = q_i \cdot x$$

$$= \frac{q_i^2 E_0 \exp(i\omega t)}{M_i^*\left[(\omega_{0,i}^2 - \omega^2) + i\gamma_i \cdot \omega\right]}$$

$$\mu_i = \alpha_i \cdot E$$

And the expression for x has frequency dependence as you can see what we did from this x was to calculate the dipole moment and this dipole moment could be nothing but product of charge multiplied by the distance. And this gives you, this can give you polarizability which could be pertaining to electronic or ionic mechanisms, because that is dipole moment is equal to alpha into E.

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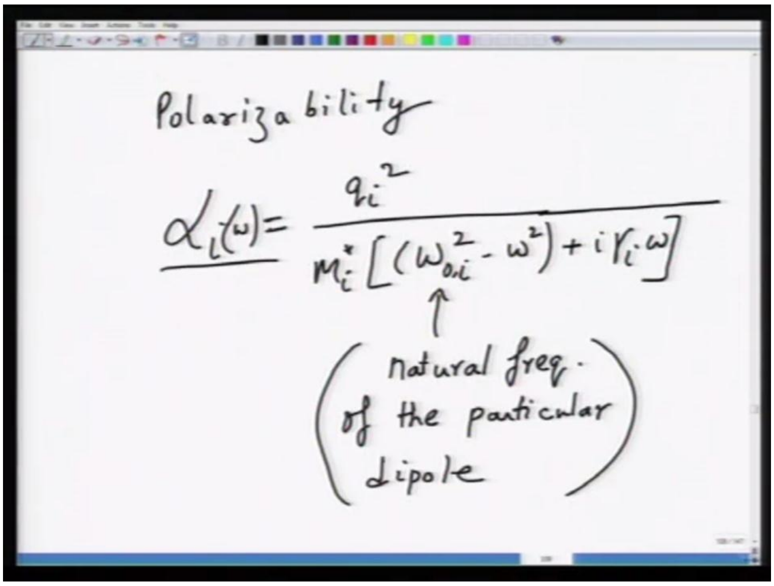

Polarizability

$$\underline{\alpha_i(\omega)} = \frac{q_i^2}{M_i^*\left[(\underset{\uparrow}{\omega_{0,i}^2} - \omega^2) + i\gamma_i \cdot \omega\right]}$$

(natural freq. of the particular dipole)

And from this, we work out what is polarizability? And here, we see that polarizability has frequency dependence, very clear frequency dependence, and it is also complex in nature.

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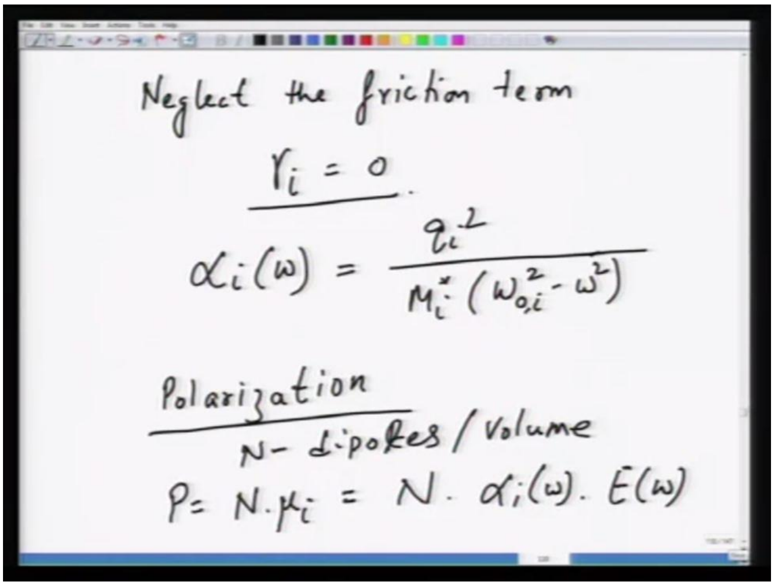

Neglect the friction term

$$\underline{\gamma_i = 0}$$

$$\alpha_i(\omega) = \frac{q_i^2}{M_i^*(\omega_{0,i}^2 - \omega^2)}$$

Polarization

N- dipoles / Volume

$$P = N.\mu_i = N.\alpha_i(\omega).E(\omega)$$

Now, given the fact that this is complex in nature your dielectric constant is going to be constant complex in nature.

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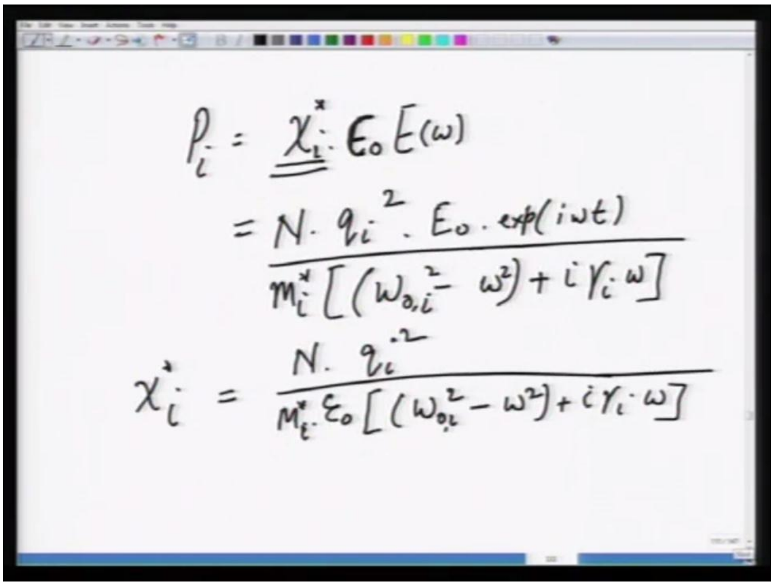

$$P_i = \underline{\underline{\chi_i^*}} \cdot \epsilon_0 E(\omega)$$

$$= \frac{N \cdot q_i^2 \cdot E_0 \cdot exp(i\omega t)}{m_i^*[(\omega_{0,i}^2 - \omega^2) + i\gamma_i \cdot \omega]}$$

$$\chi_i^* = \frac{N \cdot q_i^2}{M_i^* \cdot \varepsilon_0 [(\omega_{0i}^2 - \omega^2) + i\gamma_i \cdot \omega]}$$

As a result, we can explain, and also polarization, and susceptibility all of them are going to be complex in nature.

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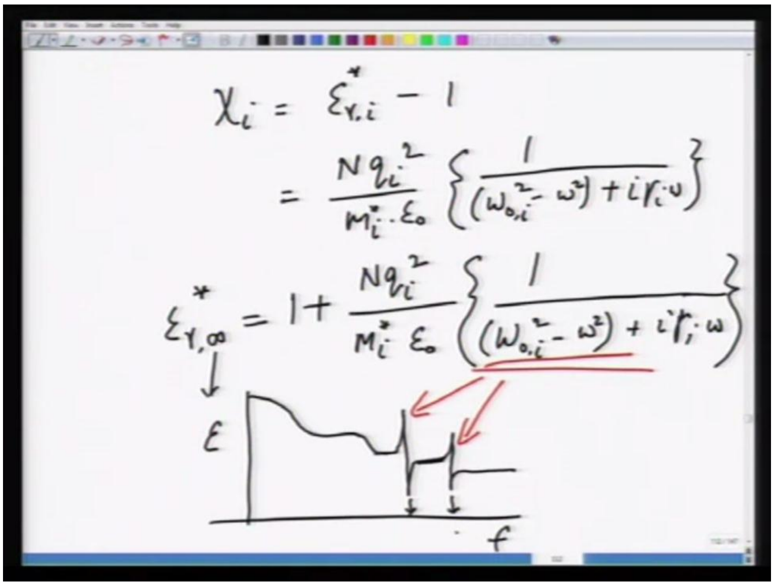

$$\chi_i = \varepsilon^*_{r,i} - 1$$

$$= \frac{N q_i^2}{M_i^* \cdot \varepsilon_0} \left\{ \frac{1}{(\omega_{0,i}^2 - \omega^2) + i\gamma_i \omega} \right\}$$

$$\varepsilon^*_{r,\infty} = 1 + \frac{N q_i^2}{M_i^* \varepsilon_0} \left\{ \frac{1}{(\omega_{0,i}^2 - \omega^2) + i\gamma_i \omega} \right\}$$

And if you define this susceptibility as epsilon r star minus 1, given that is a complex quantity, you can work out the static part of the dielectric constant as 1 plus N q i square divided by M i star epsilon naught and this frequency dependent term, and in the dielectric constant versus frequency plot. These two peaks which occur at high frequencies at, let us say 10 to the power 15 Hertz 10 to the power 13 Hertz.

These are the regions where resonance occurs which means the oscillation of dipole is in completely, is in resonance with the applied field. So, both of these things match with each other, and that is where loss becomes maximum, and this can be very nicely seen in the plot.

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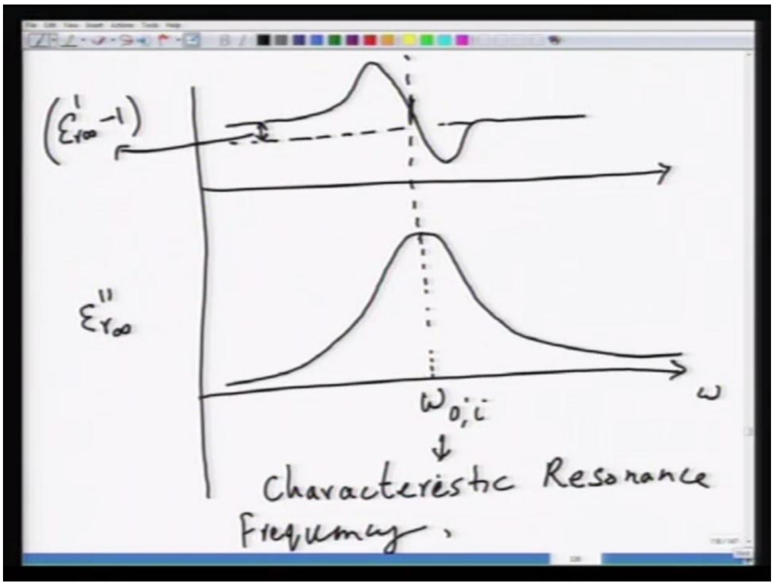

So, this epsilon star can be broken down again into epsilon prime, and epsilon double prime. And when you plot them versus frequency, you see a resonance term, resonance at an omega naught which is the characteristic frequency related to either ionic or electronic. And this omega naught is going to be higher for electronic than for ionic and you see a loss term the maxima in the loss lattice characteristic frequency, and this is true for both the mechanism. So, this was the analysis based on a c field.

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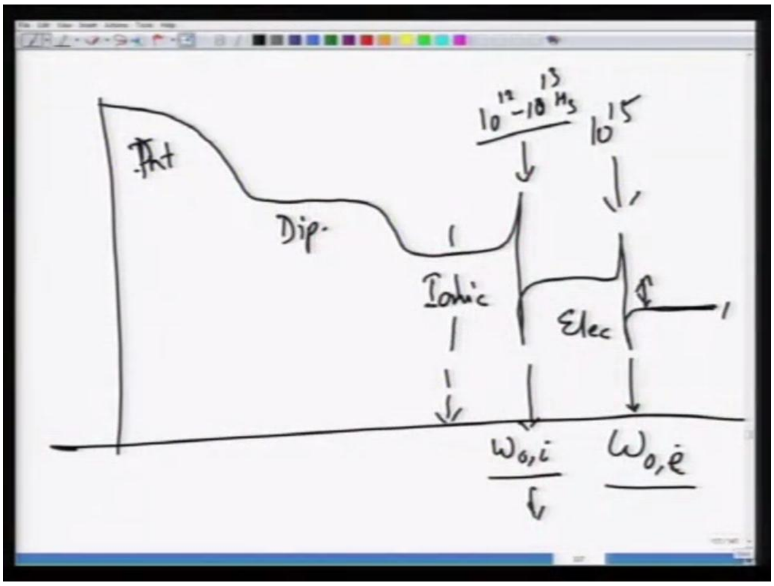

And from this then we worked out.

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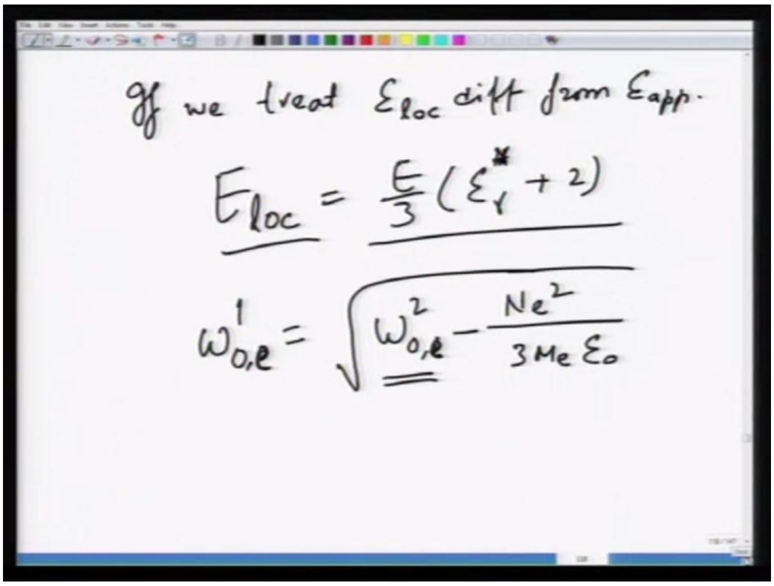

If we treat $\mathcal{E}_{loc}$ diff from $\mathcal{E}_{app}$.

$$E_{loc} = \frac{E}{3}(\varepsilon_r^* + 2)$$

$$\omega'_{0,e} = \sqrt{\omega_{0,e}^2 - \frac{Ne^2}{3M_e\varepsilon_0}}$$

And from this, you can also get the same term that you obtained in case of simple analysis, that we did earlier in our work out the polarizability.

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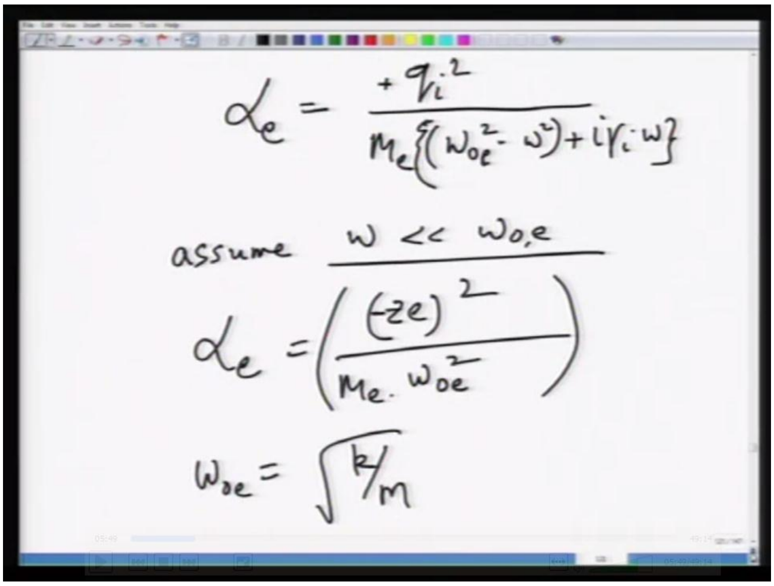

$$\alpha_e = \frac{+q_i^2}{M_e\{(\omega_{0e}^2 - \omega^2) + i\gamma_i\omega\}}$$

assume $\omega << \omega_{0,e}$

$$\alpha_e = \left(\frac{(-ze)^2}{M_e \cdot \omega_{0e}^2}\right)$$

$$\omega_{0e} = \sqrt{k/m}$$

And that if you do by treating omega as root k by m.

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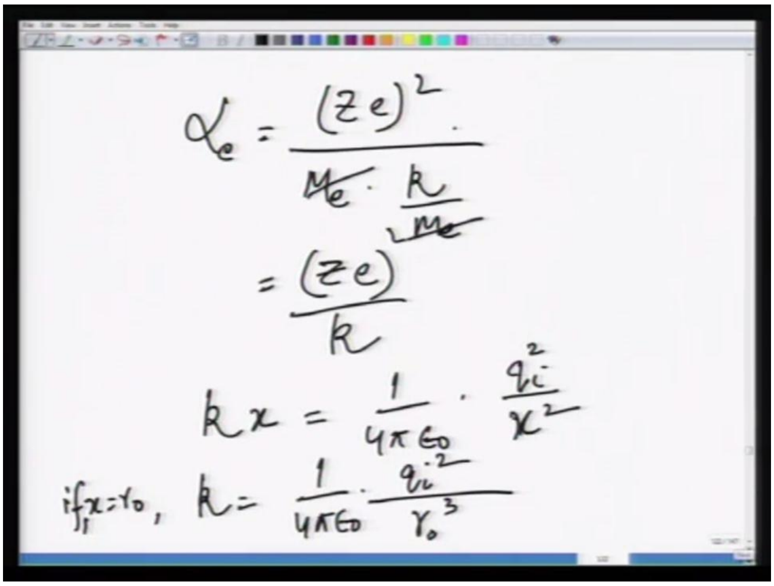

$$\alpha_e = \frac{(ze)^2}{\cancel{M_e}\cdot\frac{k}{\cancel{M_e}}}$$

$$= \frac{(ze)}{k}$$

$$kx = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_i^2}{x^2}$$

if $x = r_0$, $k = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_i^2}{r_0^3}$

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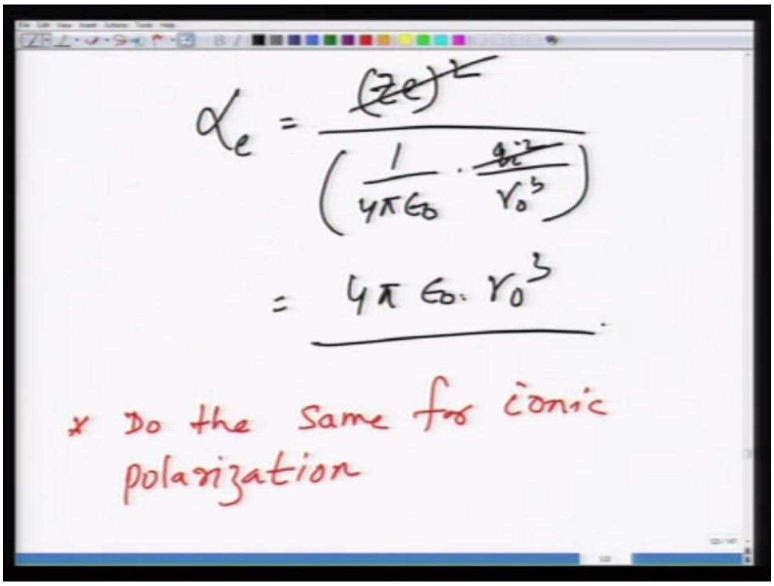

$$\alpha_e = \frac{\cancel{(ze)^2}}{\left(\frac{1}{4\pi\epsilon_0}\cdot\frac{\cancel{q_i^2}}{r_0^3}\right)}$$

$$= \underline{4\pi\epsilon_0\cdot r_0^3}.$$

* Do the same for ionic polarization

You can make it look equal to get the same expression 4 pi epsilon naught r naught cube. So, basically what we did has been correct so far, because you can make both of these, because you can make this a c equation converge into the simple equation that we got earlier.

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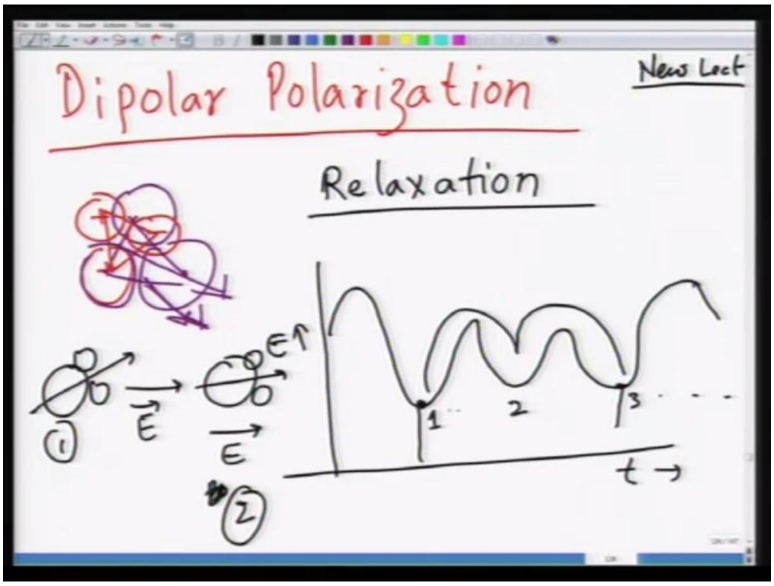

So, then in the last lecture, we started doing dipolar polarization, which is about the materials having permanent dipole moment. So, what happens in such a situation is you have a dipole like this. And this dipole, let us say has a dipole moment in this direction, and this is let us say position 1. And when you apply field, let say in this direction field is in this direction. And this rotates itself in the direction of applied field, and this is now its new position. So, this is position this is 2. So, in order to move from position 1 to position 2 and then back to some other position, which is the equivalent position? This requires this, this phenomenon is called as you know relaxation, and it is not just short range movement just like in case of electronic ionic dipoles. It is like hopping from 1 equilibrium position another equilibrium position.

So, as a result the energy versus time plot can be plotted in terms of various potential energy wells, where electron, where the molecule groups of molecules or molecule changes its position from 1 to 2 to let us say 3 and so on and so forth. And each time, when it does it, let us say from going from 1 to 2, it has to overcome an energy barrier. And this is the activation energy which is required for migration. So, this is like the diffusion process in nature, and this process called as relaxation molecule relaxes into new statistically equivalent position.

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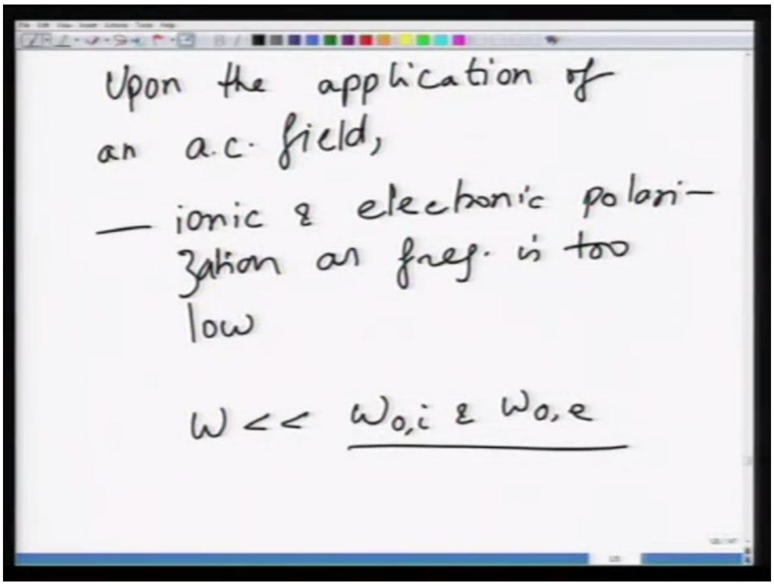

Upon the application of an a.c. field,

— ionic & electronic polarization as freq. is too low

$\omega << \underline{\omega_{0,i} \ \& \ \omega_{0,e}}$

So, and since, this is diffusion in nature, the masses of entities involved is much more higher, the frequency at which this process happens is significantly lower as compared to the characteristic frequency for ionic or electronic polarizations.

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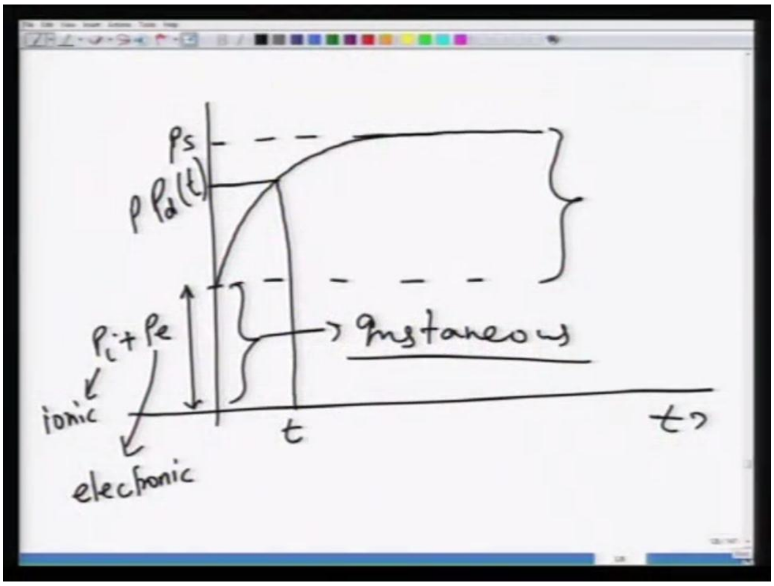

And this happens to be in the range of Mega Hertz or so. And as a result the way polarization develops is. So, when you plot the polarization as function of time, you have an instantaneous development of polarization component which is nothing but sum of ionic electronic. Because you know that electronic ionic mechanisms are extremely fast.

So, that is why this term becomes more or less instantaneous, followed by slow development of polarization to a value which is saturation polarization and that is the polarization when all the dipoles aligned themselves to the direction of applied field. Because when you apply a field, now field is time dependent. Not all the dipoles respond to the frequency of applied field at the same time. That is why dipoles take their own time to align in the direction of applied field, as a result there is a time dependence of dipolar polarization until it saturates into P s.

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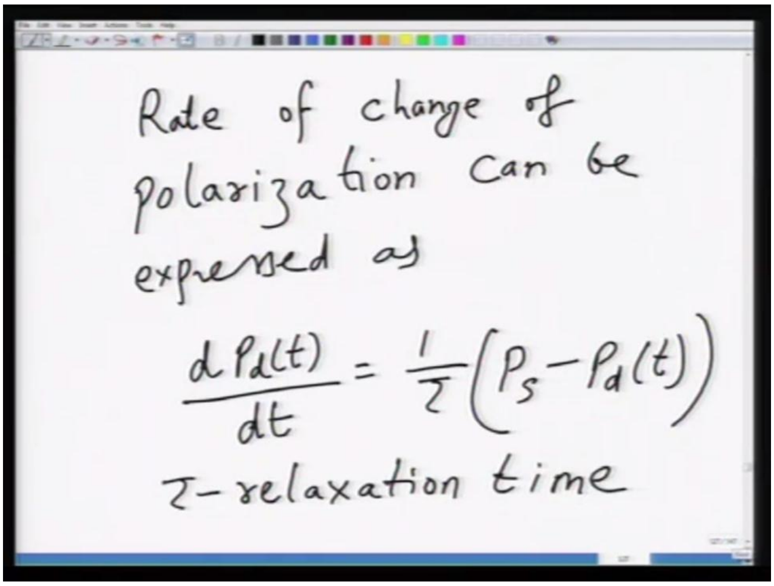

Rate of change of polarization can be expressed as

$$\frac{d P_d(t)}{dt} = \frac{1}{\tau}\left(P_s - P_d(t)\right)$$

τ - relaxation time

So, now, this rate of change of polarization can be expressed mathematically. So, you get an expression for rate of change of polarization d P by d t is equal to 1 over tau P s minus P d t. Now, this is the expression which I have given you without any derivation at this point of time, but in order we can derive this, that is what we did last time.

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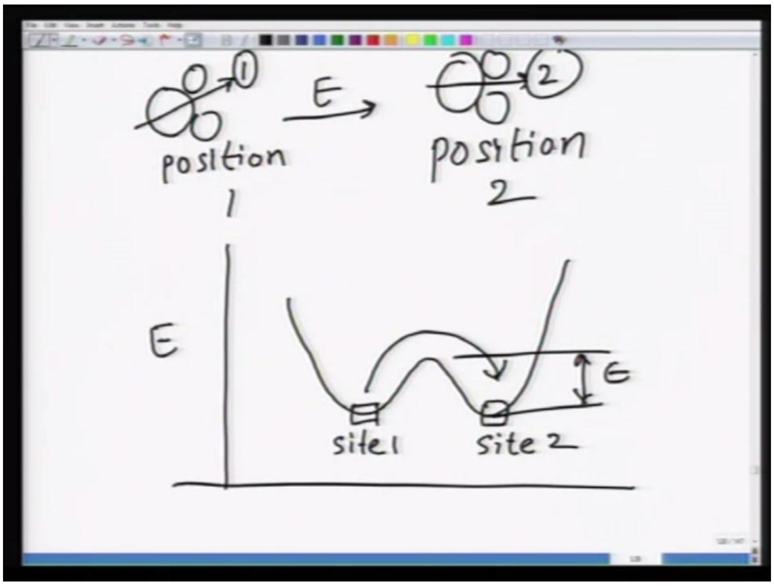

So, for that you choose a model which is called as a bi-stable model. So, what you do is that you consider this hopping model from site 1 to site 2. Let us say between 2 sides hopping is taking place. And this is characterised by this energy barrier. Let us say E or E a whatever, you call it. And so, what we have is, the dipole goes from position 1 to position 2 upon a application of electric field.

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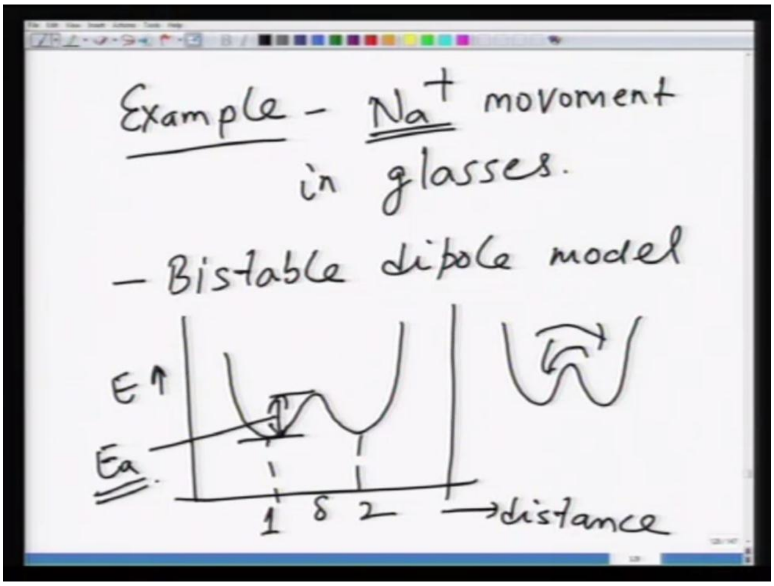
Example - Na^+ movement in glasses.

- Bistable dipole model

And so, what happens is, when you consider this model, when let us say there is no electric field, and if you have this potential well it can hop in this direction, it can hop in this direction. So, there is a finite probability of hopping in both the directions.

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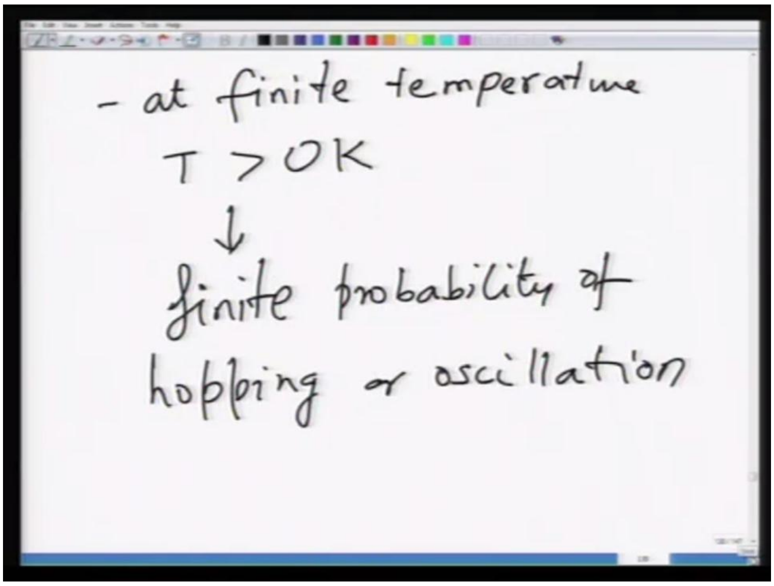

- at finite temperature

$T > 0K$

↓

finite probability of hopping or oscillation

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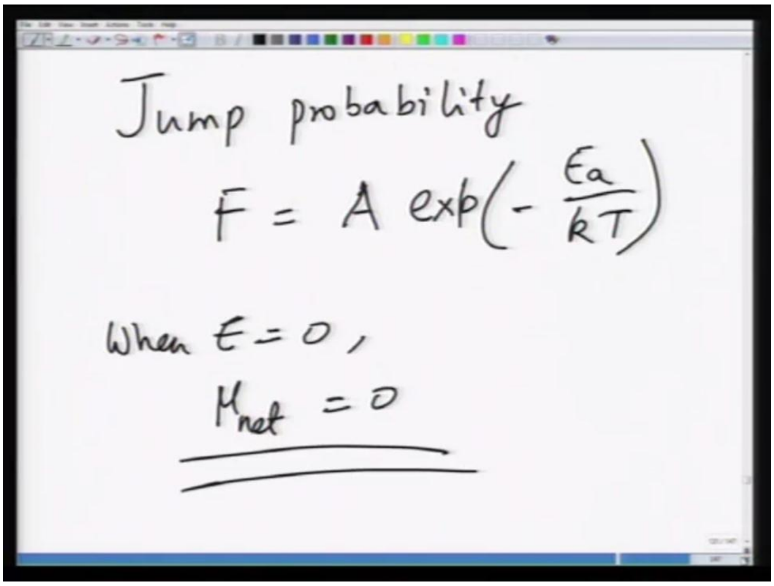

Jump probability

$$F = A \exp\left(-\frac{E_a}{kT}\right)$$

When $E = 0$,

$\mu_{net} = 0$

And this hopping probability or jump probability can be written as a exponential minus E a by k t following Maxwell and Boltzmann statistics. And when field is 0 then of course, the net probability of jump in either direction is equal to 0 as a result, you do not get any polarization and thermal effects dominate.

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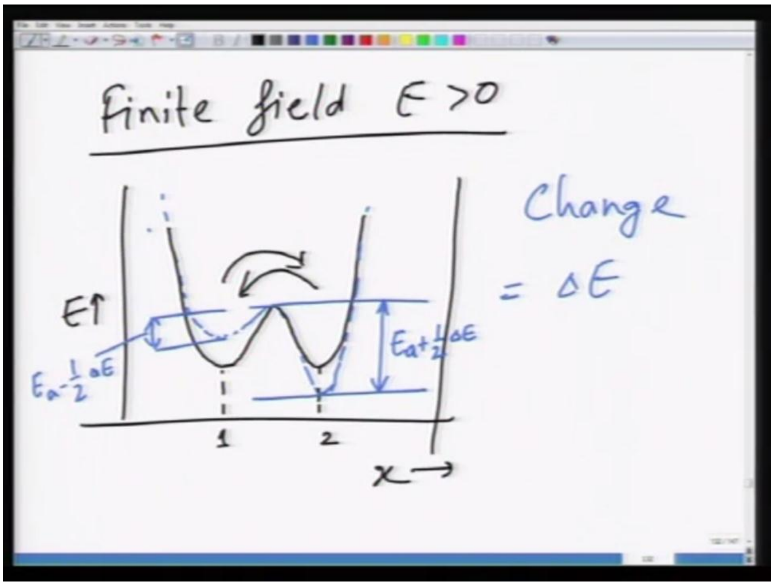

Now, when the field is large enough, then potential energy will tilts on one side, let us say. So, depending upon the direction of applied field. So, what happens is that the energy barrier on one side becomes smaller and the energy barrier on another side becomes larger. So, for anything to jump from this side to that side, it has to now overcome a larger energy barrier. And for energy anything to hop from this side to that side it has to overcome a small energy barrier.

Now, this energy now, this change in the potential energy well configuration results in what we call as change in the jump probability as a result you have net accumulation or depletion of charge at one particular site.

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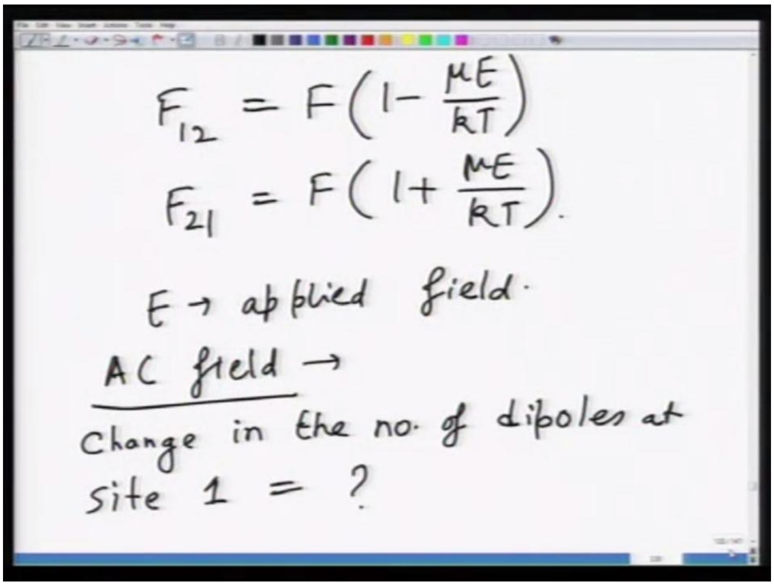

$$F_{12} = F\left(1 - \frac{\mu E}{kT}\right)$$

$$F_{21} = F\left(1 + \frac{\mu E}{kT}\right).$$

E → applied field.

AC field →

Change in the no. of dipoles at site 1 = ?

So, you have a change in the charge. So you can write the changed probabilities as F 1 2 can be written as F into 1 minus mu E by k T. And second can be and 2 1 can be written as F into 1 plus mu E by k T. So, you can see that, now this was, if you just wondering why it is like this, then what has happened is. So, this is nothing but the probability factor multiplied by another factor 1 minus mu E by k T depending upon the application of electric field. And this mu is nothing but the energy. And this and you can get the same. Now, this you can relate to, if you remember what we did in case of derivation of ionic conductivity. In ionic conductivity also, we took a model like this hopping model, and there also we have worked out the polarizabilities probabilities of jump.

So, if you wondering where these expression come from, just go back to that module the, which is module number 3, and look how we worked out the probabilities of jump upon the application of electric field in both the directions. And how we arrive at these expressions? So, as a result, you have change in the number of dipoles at site 1, and these since because of probabilities are unequal.

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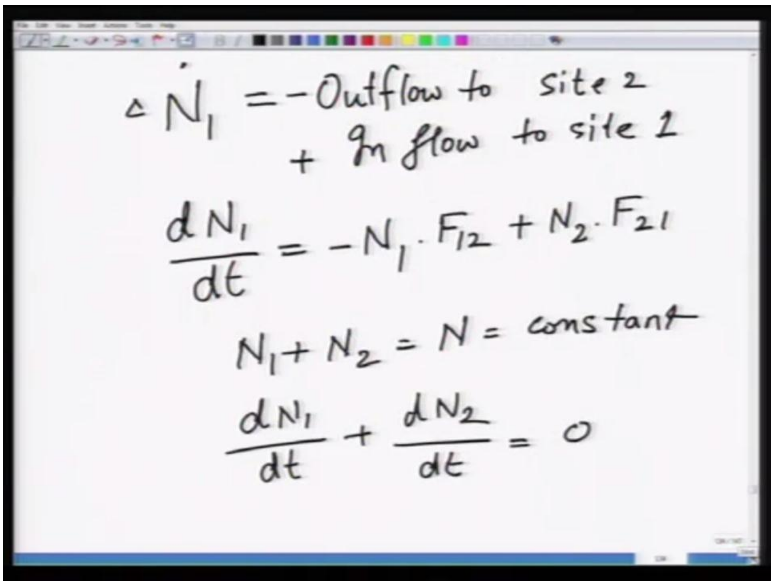

$\Delta \dot{N}_1 = -$ Outflow to site 2 + In flow to site 1

$$\frac{dN_1}{dt} = -N_1 \cdot F_{12} + N_2 \cdot F_{21}$$

$$N_1 + N_2 = N = \text{constant}$$

$$\frac{dN_1}{dt} + \frac{dN_2}{dt} = 0$$

If that is true then delta N 1 dot can be defined as outflow to site 2 plus inflow to site 1. So, outflow to site 2 can be given as minus, because you are losing charge at site 1. And what is coming into site 1 is inflow from site 2 to site 1. So, that adds as a result you have d N 1 by d t as minus N 1 F 1 2 plus N 2 F 2 1. And we know that N 1 plus N 2 is constant which is equal to capital N. And from this, you can also write d N 1 by d t plus d N 2 by d t net change is equal to 0.

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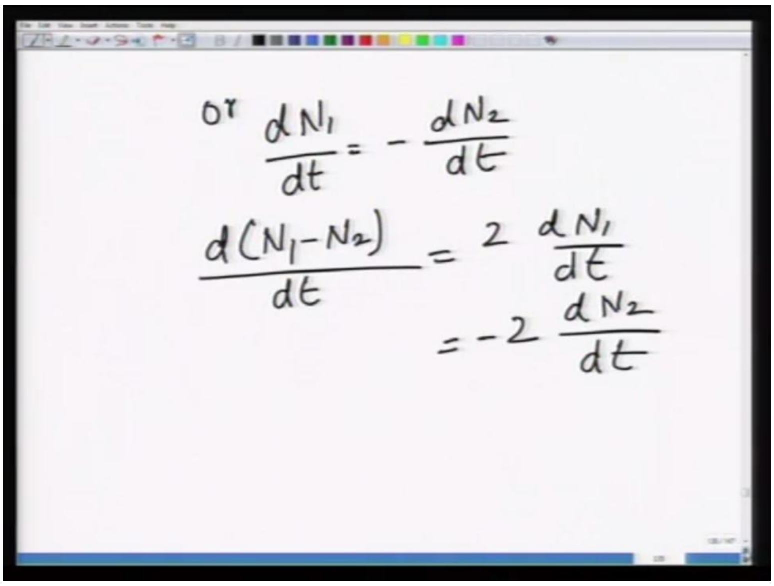

or $\frac{dN_1}{dt} = -\frac{dN_2}{dt}$

$$\frac{d(N_1 - N_2)}{dt} = 2\frac{dN_1}{dt}$$
$$= -2\frac{dN_2}{dt}$$

And then we proceeded on further.

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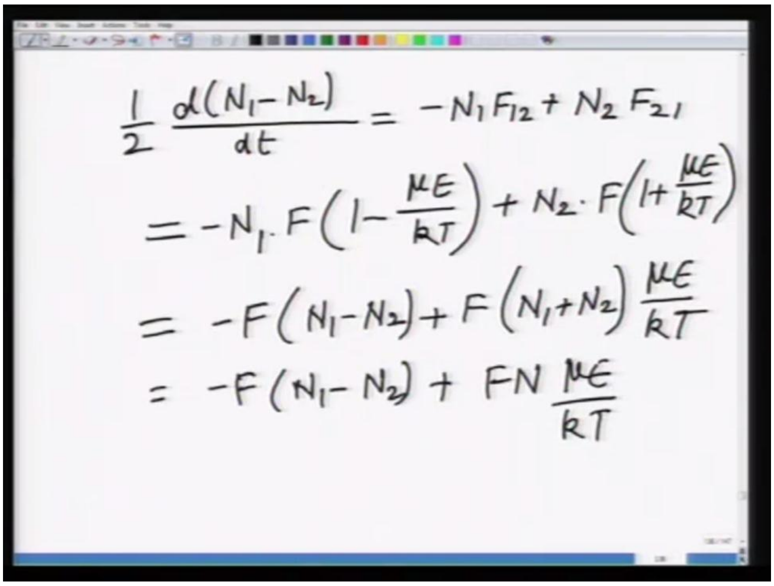

$$\frac{1}{2}\frac{d(N_1-N_2)}{dt} = -N_1F_{12}+N_2F_{21}$$

$$= -N_1\cdot F\left(1-\frac{\mu E}{kT}\right)+N_2\cdot F\left(1+\frac{\mu E}{kT}\right)$$

$$= -F(N_1-N_2)+F(N_1+N_2)\frac{\mu E}{kT}$$

$$= -F(N_1-N_2)+FN\frac{\mu E}{kT}$$

To work out what half of d N 1 minus N 2 by d t was and then...

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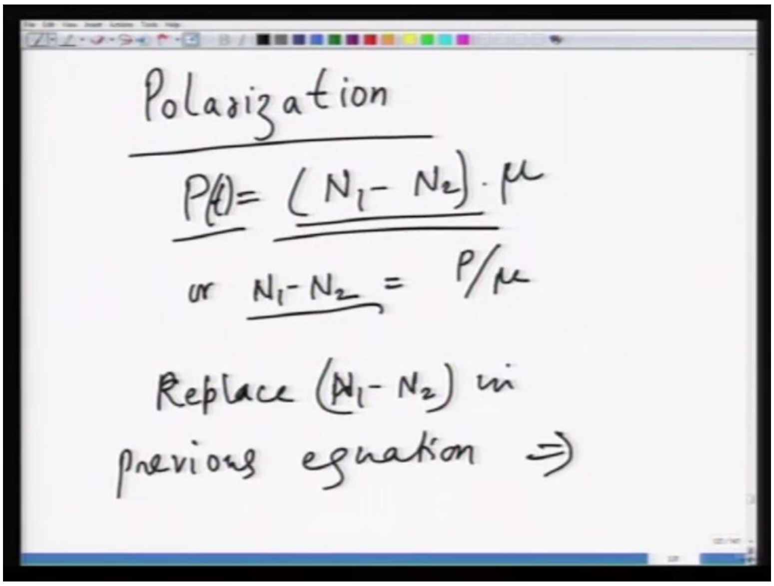

Polarization

$$\underline{P(t)} = \underline{(N_1-N_2)}\cdot\mu$$

or $\underline{N_1-N_2} = P/\mu$

Replace (N_1-N_2) in previous equation ⇒

Finally, we derived; we put it into a expression for polarization. Now, polarization can be written as P t polarization is a function of time is equal to N 1 minus N 2 which is the change in the number of charge or charge density, at that multiplied by the dipole moment. And then so N 1 minus N 2 becomes equal to p by mu, p is nothing but time dependent polarizations. So, you can write it either p of p d t or p t.

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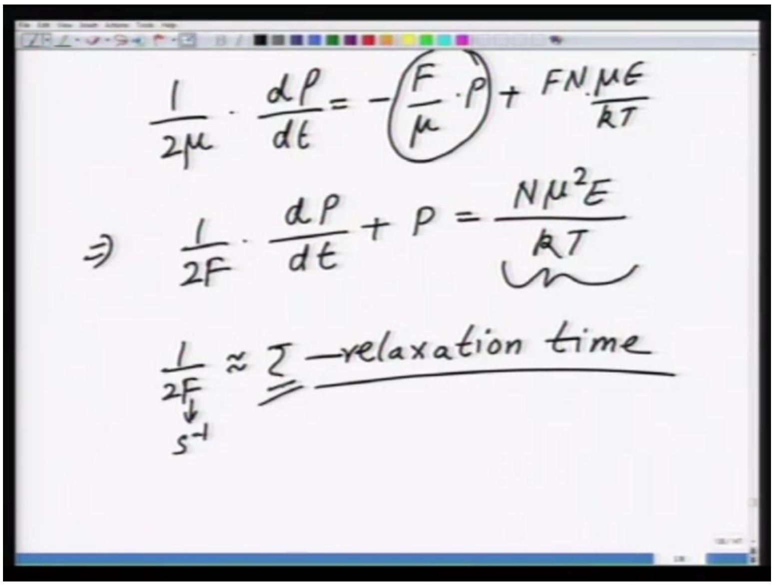

$$\frac{1}{2\mu} \cdot \frac{dp}{dt} = -\boxed{\frac{F}{\mu} \cdot p} + FN\frac{\mu E}{kT}$$

$$\Rightarrow \quad \frac{1}{2F} \cdot \frac{dp}{dt} + p = \frac{N\mu^2 E}{kT}$$

$$\underset{\downarrow \atop s^{-1}}{\frac{1}{2F}} \approx \tau \text{ — relaxation time}$$

So, this expression that we wrote earlier, if you make that replacement, it becomes 1 over 2 mu d p by d t to be equal to minus F divided by mu, p plus E N mu E by k T. Now, so now, if you make proper replace, if you make proper rearrangement here, what you get is 1 over 2 f into d p by d t plus p. If you take this on the left hand side, this term on the left hand side is equal to N mu square E by k T. And this, you know what it is? This is nothing but your dipolar polarizability, and 1 over 2 F is defined as, because F is probability in second minus 1 E. This is tau which is the relaxation time.

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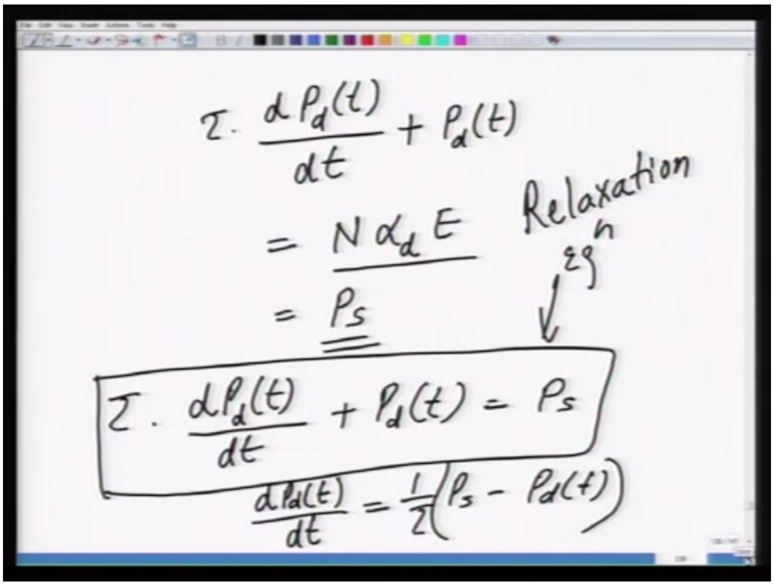

$$\tau \cdot \frac{dp_d(t)}{dt} + p_d(t) = \underline{N\alpha_d E} = \underline{\underline{p_s}}$$

Relaxation eq^n

$$\boxed{\tau \cdot \frac{dp_d(t)}{dt} + p_d(t) = p_s}$$

$$\frac{dp_d(t)}{dt} = \frac{1}{\tau}\left(p_s - p_d(t)\right)$$

So, tau d p d by d t plus p d by plus p d t is N alpha d E, so, this expression N into E. So, you know N mu square E by k T becomes N alpha d E. So, you can relate what is mu here, mu is alpha is mu square by k T. And this is nothing but your saturation polarization when all the dipoles are aligned in the direction of applied field. And so, you get an expression tau d p d by d t plus p d t is equal to p s. And this is what we proposed earlier that p d t was d p d t by d p t rate of change of polarization was equal to 1 over tau into p s minus p d t. So, we get back to same relaxation equation that we proposed earlier.

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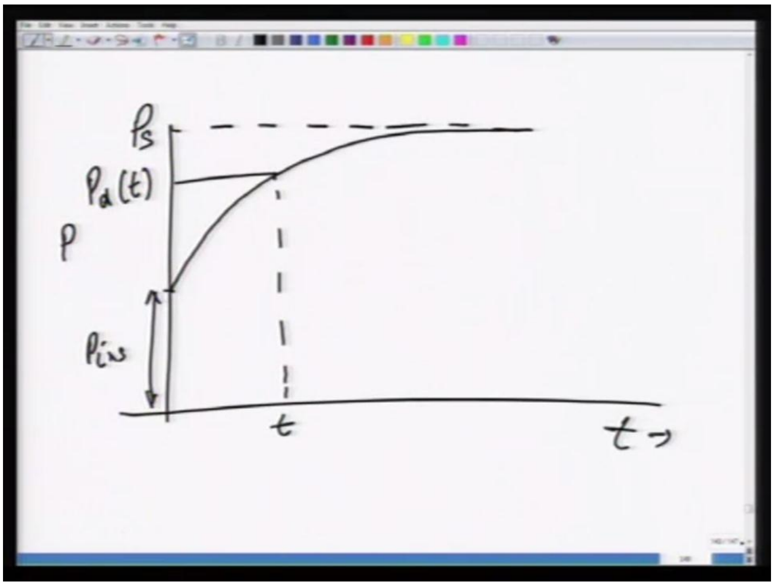

Again this explains what we did earlier.

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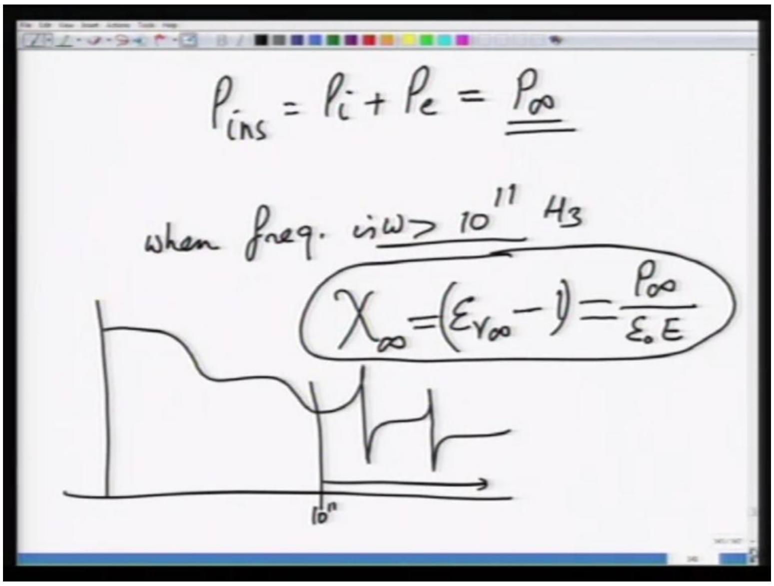

$$P_{ins} = P_i + P_e = \underline{\underline{P_\infty}}$$

when freq. i.e. $\underline{\omega > 10^{11}}$ Hz

$$\boxed{\chi_\infty = (\varepsilon_{r\infty} - 1) = \frac{P_\infty}{\varepsilon_0 E}}$$

And the sum of now, p instantaneous can be taken as sum of p i plus p e. And that can be defined as p infinity. And when, and this would be in the high frequency range typically higher than 10 to power 11 Hertz. And the susceptibility corresponding to this would be chi infinity is equal to epsilon r infinity minus 1 divided is equal to p infinity divided by epsilon naught E which is simple I mean because you know that chi is nothing but ratio of polarization to the displacement due to applied field in vacuum also.

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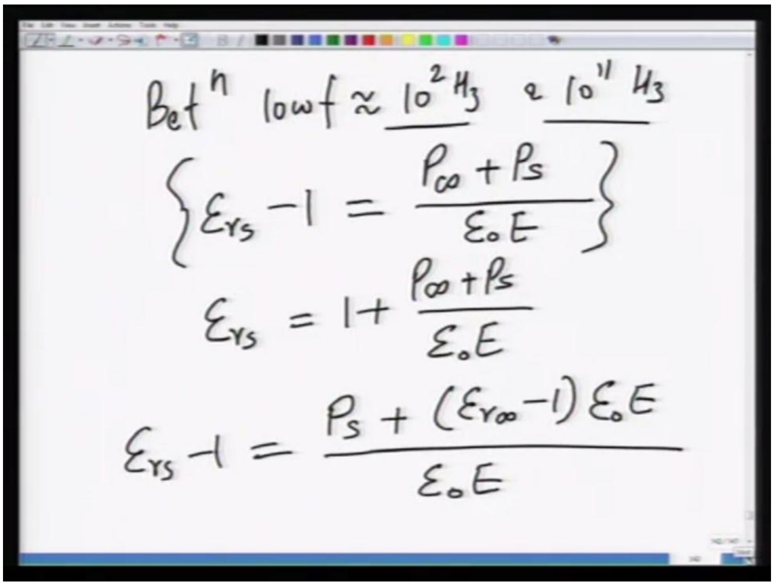

Bet^n low f ≈ $\underline{10^2 \text{ Hz}}$ & $\underline{10^{11} \text{ Hz}}$

$$\left\{ \varepsilon_{rs} - 1 = \frac{P_\infty + P_s}{\varepsilon_0 E} \right\}$$

$$\varepsilon_{rs} = 1 + \frac{P_\infty + P_s}{\varepsilon_0 E}$$

$$\varepsilon_{rs} - 1 = \frac{P_s + (\varepsilon_{r\infty} - 1)\varepsilon_0 E}{\varepsilon_0 E}$$

So, between the low, at the low frequencies, you can write similar equation as epsilon r s minus 1 which is the chi is equal to p infinity plus p s divided by epsilon naught E. So, epsilon r s becomes 1 plus p infinity plus p s divided by epsilon naught E. And then epsilon r s minus 1 becomes equal to p s plus epsilon r infinity minus 1 epsilon naught E divided by epsilon naught E.

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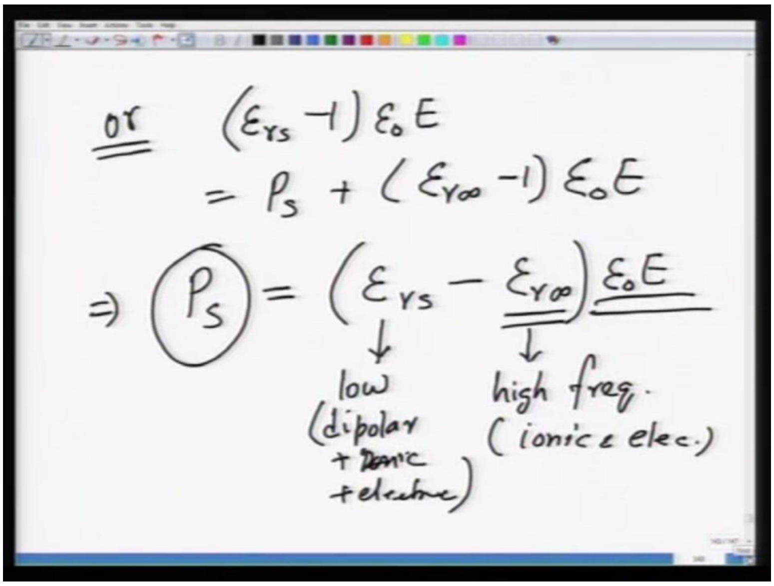

or

$$(\varepsilon_{rs} - 1)\varepsilon_0 E = P_s + (\varepsilon_{r\infty} - 1)\varepsilon_0 E$$

$$\Rightarrow \boxed{P_s} = \Big(\underset{\substack{\downarrow \\ \text{low} \\ \text{(dipolar} \\ \text{+ ionic} \\ \text{+ electronic)}}}{\varepsilon_{rs}} - \underset{\substack{\downarrow \\ \text{high freq.} \\ \text{(ionic \& elec.)}}}{\underline{\varepsilon_{r\infty}}}\Big)\underline{\varepsilon_0 E}$$

So, you can jumble this around and then what you get is p s is equal to epsilon r s minus epsilon r infinity. So, where epsilon r s is the low frequency static dielectric constant which corresponds to ionic and electronic. Yes so, this is your low frequency dielectric constant a static dielectric constant which combines dipolar ionic and electronic minus the epsilon r infinity which is the high frequency contribution which is due only, due to ionic and electronic. So, the difference of these two will correspond to the dipolar component that makes sense.

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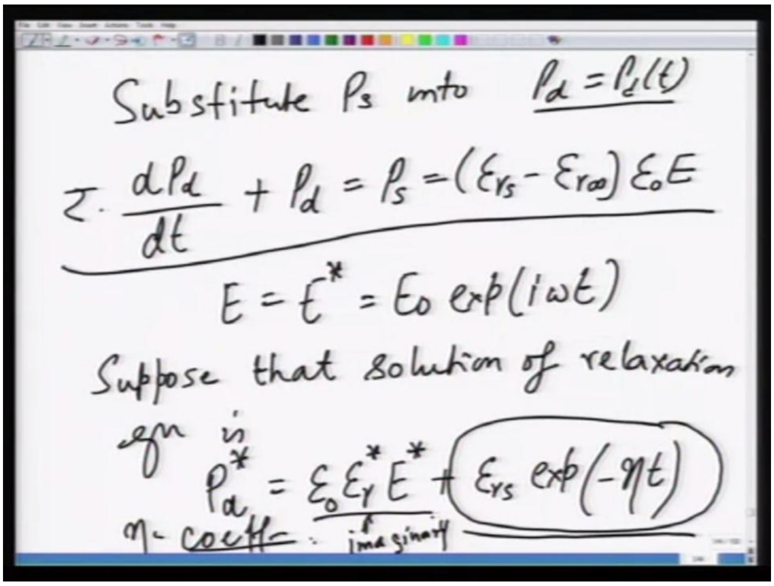

Substitute P_s into $P_d = P_d(t)$

$$\tau \cdot \frac{dP_d}{dt} + P_d = P_s = (\varepsilon_{rs} - \varepsilon_{r\infty})\,\varepsilon_0 E$$

$$E = E^* = E_0 \exp(i\omega t)$$

Suppose that solution of relaxation eqn is

$$P_d^* = \underbrace{\varepsilon_0 \varepsilon_r^* E^*}_{\text{imaginary}} + \boxed{\varepsilon_{rs} \exp(-\eta t)}$$

η - coeff. :

So, when you make the substitution of p s into the expression of in the relaxation equation. What you get is tau d p d by d t plus p d is equal to p s as we know is equal to epsilon r s minus epsilon r infinity epsilon naught E. Now, what we do is that we make a substitution for electric field that is E is equal to E star E naught exponential i omega t. And since, field is a complex quantity; you would notice that many of these things have got these stars as a superscript. And this is star is nothing but a symbol of complex nature of the quantity.

So, when you make this substitution then let us assume now, solution of this equation is pretty complex. So, we assume that the solution of this equation was p d star was equal to epsilon naught epsilon r star E star plus epsilon r s exponential of minus eta t. So, there is another term this that comes into picture. And then we took what is d p d star by d t, to now when the solution, if the solution is like this. Then what we want to do is that if you want to calculate d p d by d t and we put the p d. And then we want to relate then we want to substitute back and calculate some quantities.

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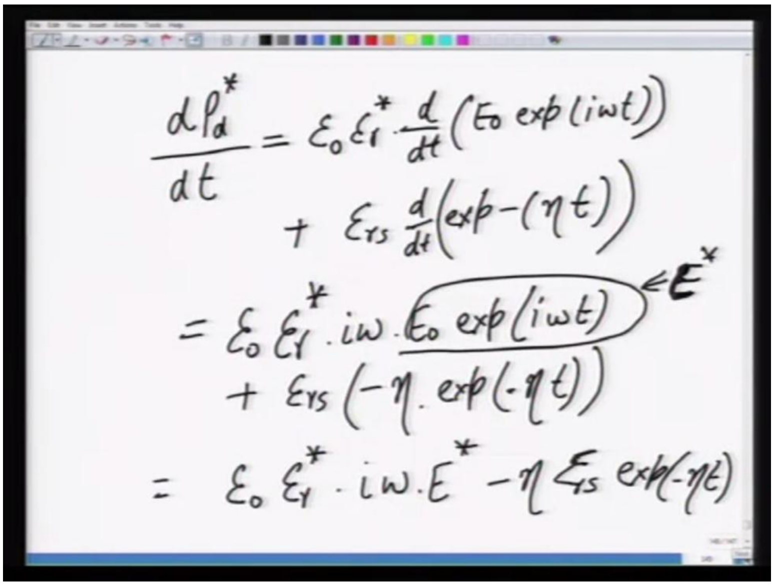

$$\frac{dP_d^*}{dt} = \varepsilon_0\varepsilon_r^* \frac{d}{dt}\left(E_0 \exp(i\omega t)\right)$$

$$+ \varepsilon_{rs}\frac{d}{dt}\left(\exp-(\eta t)\right)$$

$$= \varepsilon_0\varepsilon_r^* . i\omega . \boxed{E_0 \exp(i\omega t)} \leftarrow E^*$$

$$+ \varepsilon_{rs}\left(-\eta . \exp(-\eta t)\right)$$

$$= \varepsilon_0\varepsilon_r^* . i\omega . E^* - \eta\, \varepsilon_{rs} \exp(-\eta t)$$

So, d p d star by d t will become epsilon naught epsilon i star d by d t of exponential E naught exponential i omega t plus epsilon r s d by d t of exponential minus eta t. And then you do this further what do you come, what you come after, what you come about is epsilon naught epsilon r is star i omega E star minus eta epsilon r s exponential of minus eta t.

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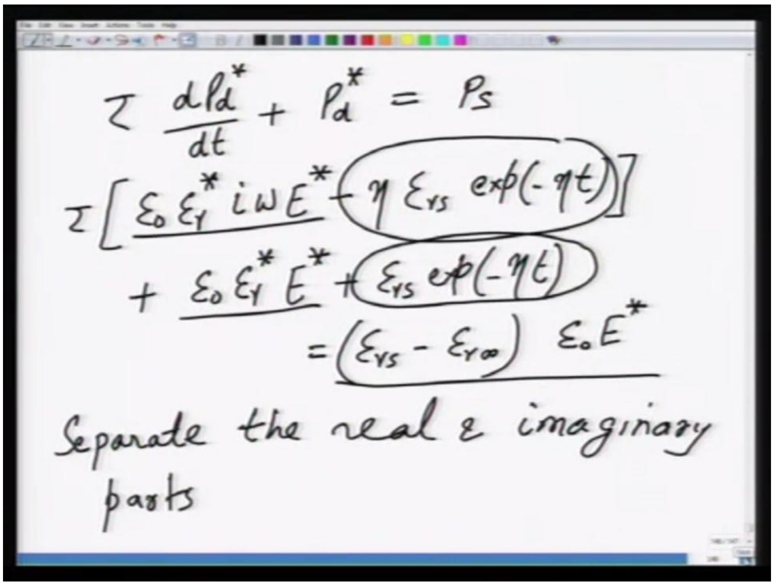

$$\tau \frac{dP_d^*}{dt} + P_d^* = P_s$$

$$\tau\left[\underline{\varepsilon_0\varepsilon_r^*\, i\omega E^*} \boxed{-\eta\, \varepsilon_{rs} \exp(-\eta t)}\right]$$

$$+ \underline{\varepsilon_0\varepsilon_r^* E^*} \boxed{+\varepsilon_{rs} \exp(-\eta t)}$$

$$= \underline{\left(\varepsilon_{rs} - \varepsilon_{r\infty}\right) \varepsilon_0 E^*}$$

Separate the real & imaginary parts

And so finally, when you when you make the substitutions, what you come about is tau multiplied by epsilon naught epsilon r star i omega E star minus eta epsilon r s exponential of minus eta t.

This is one term multiplied by tau, and this another term epsilon naught epsilon r star E star plus epsilon r exponential of minus eta t is equal to epsilon r s minus epsilon r infinity into epsilon naught E star. So, you got some real terms, some imaginary terms, and what now, we will do is that we will now differentiate this equation. So, what we will do is that we will just go back here; you must be wondering what eta was. So, here eta is some coefficient which is to be determined by the solution.

So, we do not worry about it at the moment. Now, this is where we ended. Now, we have to what we have to do is that we have to separate the real and imaginary parts from this equation. So, you can see what the real and imaginary part in this equation is; this is the real part; this is the real part, and this is imaginary; this is imaginary and this is imaginary. So, whatever is underlined is imaginary, whatever is rounded is real.

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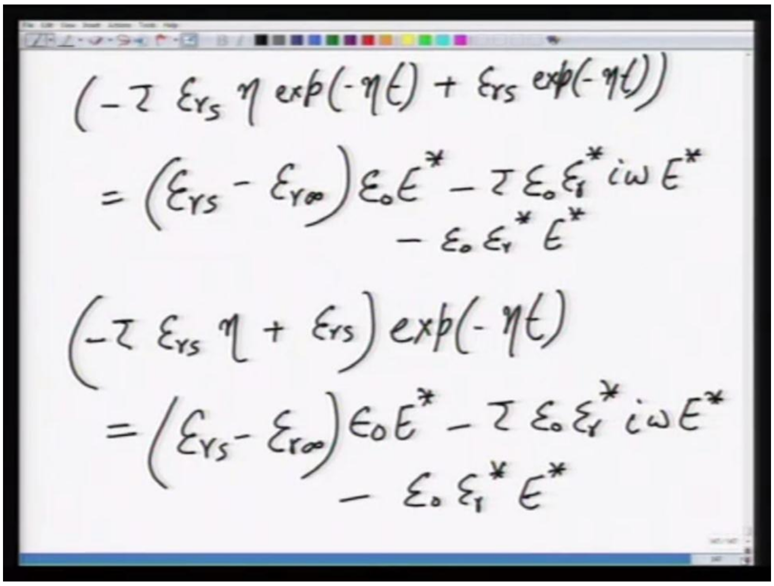

$$\left(-\tau \varepsilon_{rs} \eta \exp(-\eta t) + \varepsilon_{rs} \exp(-\eta t)\right) = \left(\varepsilon_{rs} - \varepsilon_{r\infty}\right)\varepsilon_0 E^* - \tau \varepsilon_0 \varepsilon_r^* i\omega E^* - \varepsilon_0 \varepsilon_r^* E^*$$

$$\left(-\tau \varepsilon_{rs} \eta + \varepsilon_{rs}\right) \exp(-\eta t) = \left(\varepsilon_{rs} - \varepsilon_{r\infty}\right)\epsilon_0 E^* - \tau \varepsilon_0 \varepsilon_r^* i\omega E^* - \varepsilon_0 \varepsilon_r^* E^*$$

So, when we separate these parts. So, what you get is minus of tau epsilon r s eta exponential minus eta t plus epsilon r s exponential minus eta t is equal to epsilon r s minus epsilon r infinity multiplied by epsilon naught E star minus of tau epsilon naught epsilon r star i omega E star minus of epsilon naught epsilon r star E star. And then again make the substitute then again make some of the jumble up this equation again. So, what you get is minus of tau epsilon r s eta plus epsilon r s exponential of minus eta t is equal to epsilon r s minus epsilon i infinity multiplied by epsilon naught E star, and minus of tau epsilon naught epsilon r star i omega E star minus again of epsilon naught epsilon r star E star. So, this is now we need to make now, we need to now, we are coming close to the solution so which means, if you want to solve this then the both sides.

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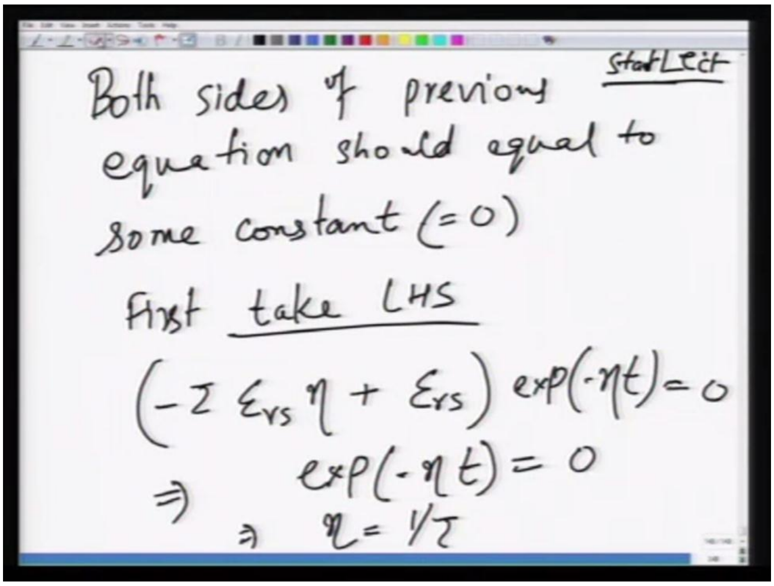

Start Lect

Both sides of previous equation should equal to some constant (= 0)

First take LHS

$$\left(-\tau \varepsilon_{rs}\eta + \varepsilon_{rs}\right)\exp(-\eta t) = 0$$

$$\Rightarrow \quad \exp(-\eta t) = 0$$

$$\Rightarrow \quad \eta = 1/\tau$$

So, this is new lecture. So, we are starting today. So, both sides of previous equation should be should equal to some constant. And what would that constant be looking at the equation. If you look at these equations the only constant, if they are equal to a constant, the only constant can be equal to 0 and that is nothing but 0. So, if we do that now. So, first take left hand side of the equation, if you go back to left hand side of the equation which is minus tau epsilon r eta plus epsilon r s minus tau of epsilon r s eta plus epsilon r s multiplied by exponential of minus eta t is equal to 0.

Now, this naturally gives you, and from this you can work out what is eta is equal to 1 over tau or rather.

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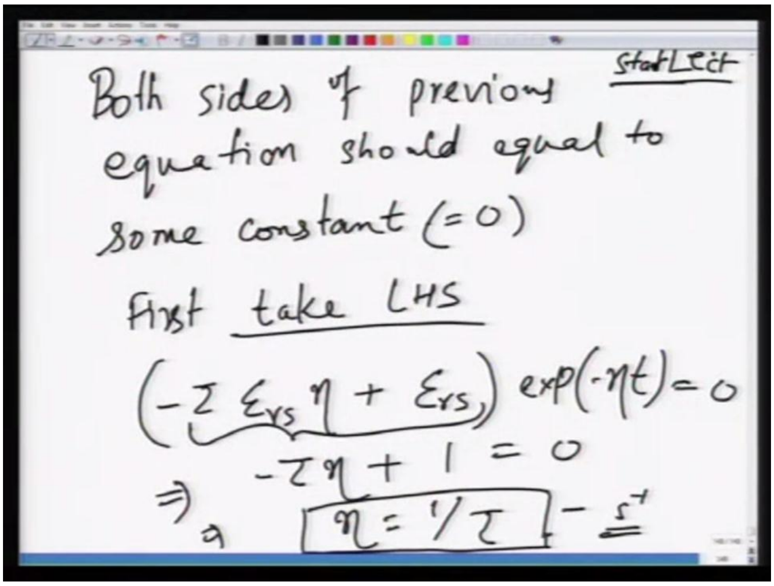

StatLect

Both sides of previous equation should equal to some constant (= 0)

First take LHS

$$\left(\underbrace{-\tau\,\varepsilon_{rs}\,\eta + \varepsilon_{rs}}\right)\exp(-\eta t) = 0$$

$$-\tau\eta + 1 = 0$$

$$\Rightarrow \boxed{\eta = 1/\tau} - s^{-1}$$

Just to make it simpler, you can write it as, so this epsilon r of course, so minus of tau eta plus 1 is equal to 0. So, this side and eta becomes equal to 1 over tau. So, this is the first. So, you get an. Now, you get a hang of eta, what eta is? Eta is nothing but it corresponds to 1 over relaxation time. So, the units of eta would be in second minus 1, it is something like frequency or characteristic frequency.

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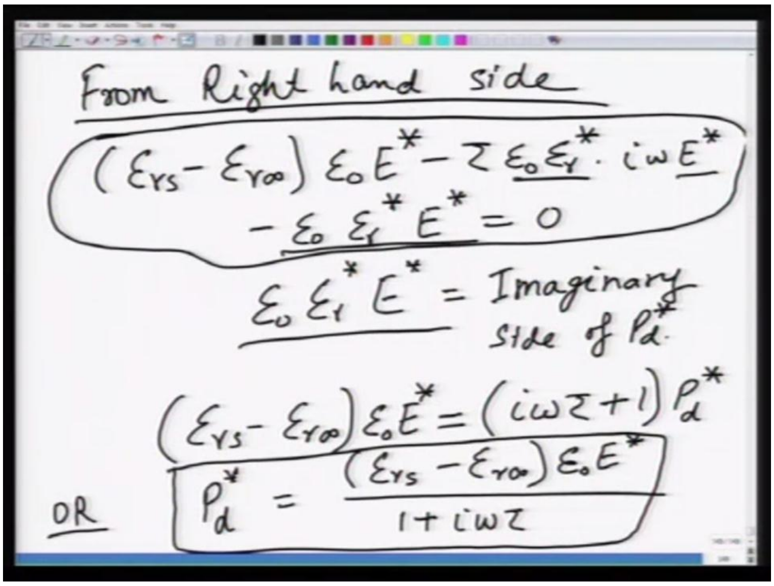

From Right hand side

$$(\varepsilon_{rs} - \varepsilon_{r\infty})\,\varepsilon_0 E^* - \tau\,\underline{\varepsilon_0 \varepsilon_r^*}\cdot i\omega\,\underline{E^*} - \underline{\varepsilon_0\,\varepsilon_r^*\,E^*} = 0$$

$\underline{\varepsilon_0\,\varepsilon_r^*\,E^*}$ = Imaginary side of P_d^*.

$$(\varepsilon_{rs} - \varepsilon_{r\infty})\,\varepsilon_0 E^* = (i\omega\tau + 1)\,P_d^*$$

OR $$\boxed{P_d^* = \frac{(\varepsilon_{rs} - \varepsilon_{r\infty})\,\varepsilon_0 E^*}{1 + i\omega\tau}}$$

And from now, use right hand side. So, using right hand side, if you look at the right hand side of the equation, which was epsilon r s is minus epsilon r infinity multiplied by epsilon naught E star minus of this and minus of that.

So, when we take this now. So, this becomes equal to epsilon r s minus into epsilon naught E star minus of tau epsilon naught epsilon r star into i omega E star. And then minus of epsilon naught r star. This is equal to again 0. Now, what is p d star? Now, what is epsilon naught epsilon r? This is nothing but imaginary side of p d star. So, this is so, we can use the imaginary side, in the imaginary side, because this is imaginary. So, we can make this as epsilon r s minus epsilon r infinity multiplied by epsilon naught E star to be equal to i omega tau. So, this and this and this will make p star plus 1 multiplied by p d star or you can change it as or p d star. In that case will become epsilon r s minus epsilon r infinity multiplied by epsilon naught E star divided by 1 plus i tau. So, you must be wondering how, why did we write this? This we are able to write, because this side that we have taken is the imaginary side? And if you only take the imaginary side then the imaginary component of p d star is epsilon naught epsilon r star E star.

So, we can write this p d star as epsilon naught epsilon r star E star so perfectly logical in that sense.

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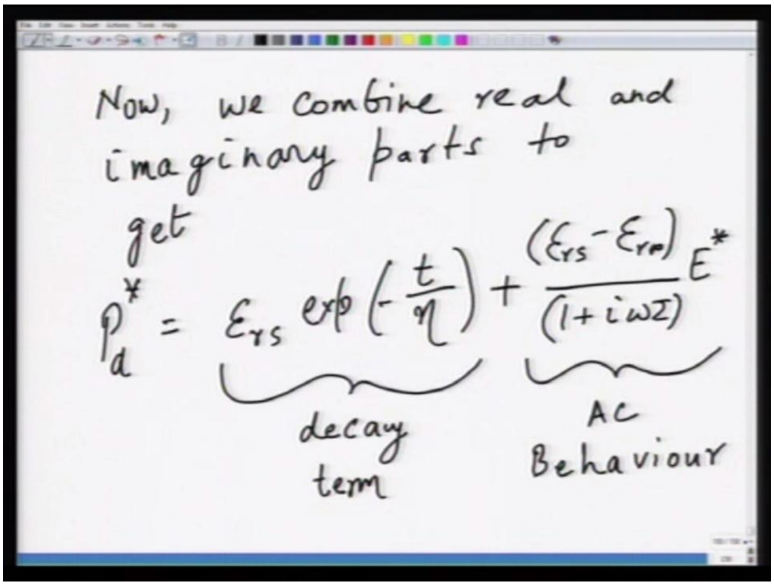

Now, we combine real and imaginary parts to get

$$p_d^* = \underbrace{\varepsilon_{rs} \exp\left(-\frac{t}{\eta}\right)}_{\text{decay term}} + \underbrace{\frac{(\varepsilon_{rs}-\varepsilon_{r\infty})}{(1+i\omega\tau)} E^*}_{\text{AC Behaviour}}$$

Now, we combine real and imaginary parts. And what we get is get an overall expression of P d star. And this overall expression of P d star now becomes epsilon r s. So, if you go to previous equation. So, the expression was P d star was epsilon naught epsilon r star E star. So, this is the imaginary part; this is the real part epsilon r s exponential of minus eta t. So, now, we are making substitutions for each of these things, and writing in overall equations. So this is the imaginary. So, we have worked out what imaginary component would be, and what the real component would be.

So, we put them together what we get is epsilon r s into exponential of minus of t by tau. This term is time dependant decay term, plus another term which is the A C which represents the A C behaviour which is epsilon r s minus epsilon r infinity multiplied by 1 plus i omega tau into E star. And this represents your A C behaviour. So, you got these two terms, just to take it further. So, now what we can do is that, we can we can also separate the time dependant. And time independent part from the dipolar polarization that is the electronic and ionic polarization parts. So, how we do that?

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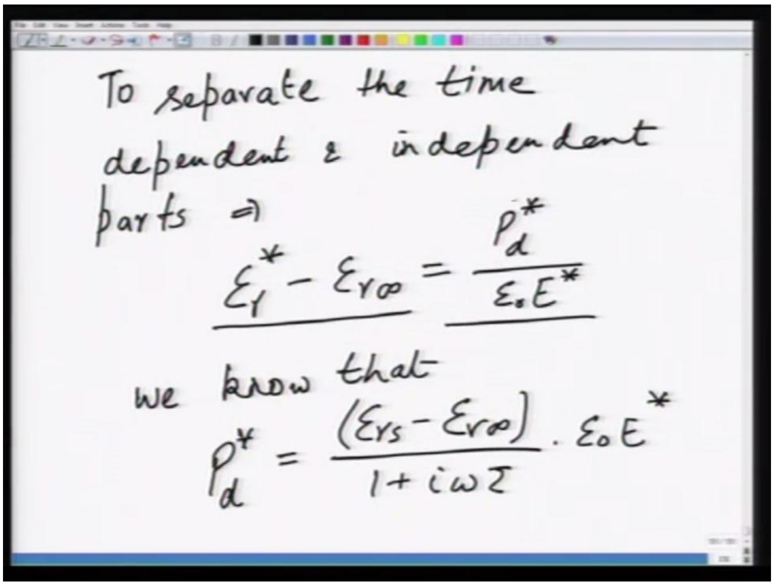

To separate the time dependent & independent parts ⇒

$$\underline{\varepsilon_r^* - \varepsilon_{r\infty}} = \underline{\frac{P_d^*}{\varepsilon_0 E^*}}$$

we know that

$$P_d^* = \frac{(\varepsilon_{rs} - \varepsilon_{r\infty})}{1 + i\omega\tau} \cdot \varepsilon_0 E^*$$

So, this epsilon r star minus epsilon r infinity will be equal to P d star minus of epsilon naught E star. Now since we know, that, so this is the difference, just to separate out the time dependent independent part. So, this is the total dielectric constant complex minus the static dielectric constant at high frequencies that will be sum of your dipolar contribution complex divided by E epsilon naught epsilon naught E star and p d star. We know is this the complex part of p d star. So, what we can write is.

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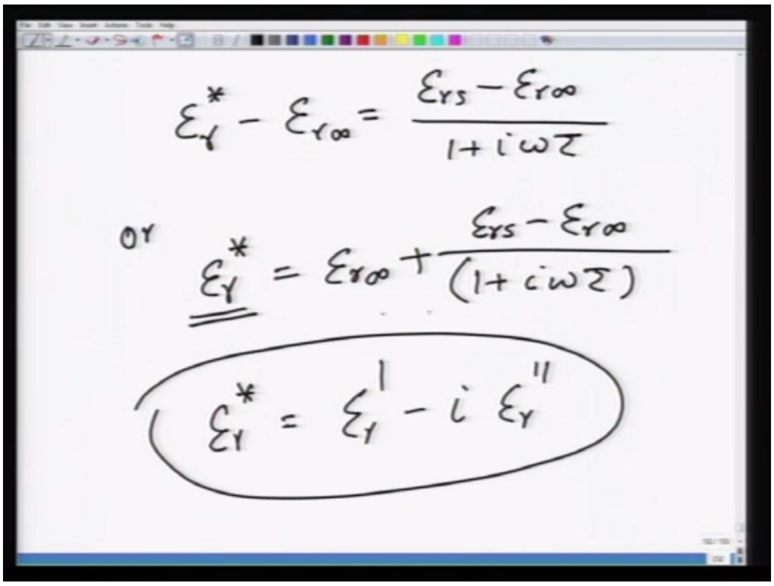

$$\varepsilon_r^* - \varepsilon_{r\infty} = \frac{\varepsilon_{rs} - \varepsilon_{r\infty}}{1 + i\omega\tau}$$

or

$$\underline{\underline{\varepsilon_r^*}} = \varepsilon_{r\infty} + \frac{\varepsilon_{rs} - \varepsilon_{r\infty}}{(1 + i\omega\tau)}$$

$$\boxed{\varepsilon_r^* = \varepsilon_r' - i\,\varepsilon_r''}$$

So, then now we can write a epsilon r star minus of epsilon r infinity to be equal to epsilon r s minus of epsilon r infinity divided by 1 plus i omega tau or omega tau. Now, we this is the complex from of dielectric constant. So, we know that epsilon r star is can be written as epsilon r prime minus of i epsilon r double prime, if that is true then since this can be written like that. So, we can break this up. So, if this can be written in this form then we can break this epsilon r star into epsilon r prime epsilon r double prime.

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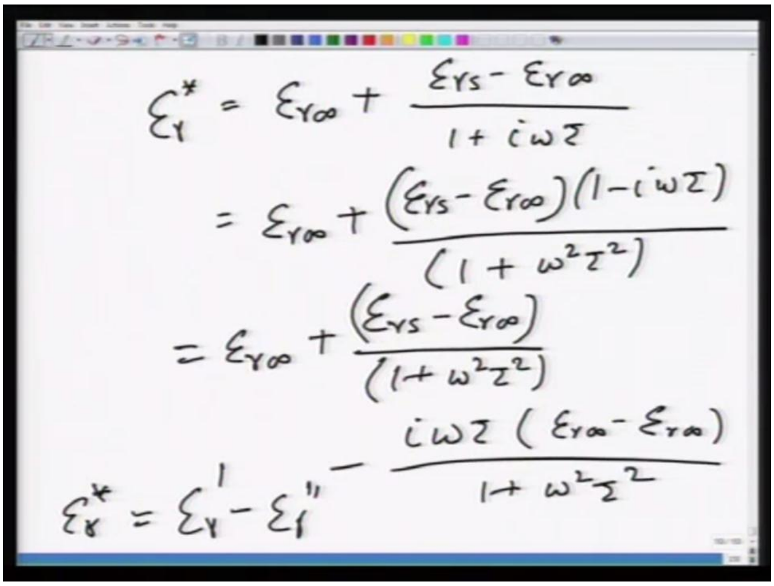

$$\varepsilon_r^* = \varepsilon_{r\infty} + \frac{\varepsilon_{rs}-\varepsilon_{r\infty}}{1+i\omega\tau}$$

$$= \varepsilon_{r\infty} + \frac{(\varepsilon_{rs}-\varepsilon_{r\infty})(1-i\omega\tau)}{(1+\omega^2\tau^2)}$$

$$= \varepsilon_{r\infty} + \frac{(\varepsilon_{rs}-\varepsilon_{r\infty})}{(1+\omega^2\tau^2)} - \frac{i\omega\tau\,(\varepsilon_{r\infty}-\varepsilon_{r\infty})}{1+\omega^2\tau^2}$$

$$\varepsilon_r^* = \varepsilon_r' - \varepsilon_r''$$

So, epsilon r star is equal to as we know is epsilon r infinity plus 1 plus i omega tau i multiply, and divided by 1 minus i omega tau. And this becomes 1 plus omega square tau square as i square is equal to minus 1. So, I take the real and imaginary parts out divided by 1 plus omega square tau square, and then minus of i omega tau into epsilon r s minus epsilon r divided by 1 plus omega square tau square.

So, now it is easy to separate out, we know that epsilon r star is equal to epsilon r prime minus epsilon r double prime. So, we can write this as.

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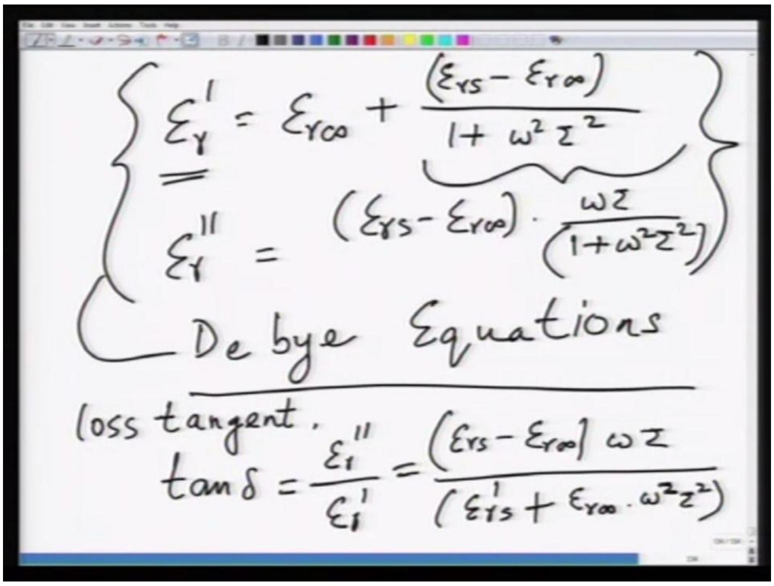

$$\underline{\underline{\varepsilon_r'}} = \varepsilon_{r\infty} + \frac{(\varepsilon_{rs}-\varepsilon_{r\infty})}{\underbrace{1+\omega^2\tau^2}}$$

$$\varepsilon_r'' = (\varepsilon_{rs}-\varepsilon_{r\infty}) \cdot \frac{\omega\tau}{(1+\omega^2\tau^2)}$$

Debye Equations

loss tangent.

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'} = \frac{(\varepsilon_{rs}-\varepsilon_{r\infty})\,\omega\tau}{(\varepsilon_{rs}' + \varepsilon_{r\infty}\cdot\omega^2\tau^2)}$$

So, epsilon r prime would be equal to epsilon r infinity. So, this is the static high frequency part plus epsilon r s minus epsilon r infinity divided by 1 over omega square tau square. And this is your contribution which is coming from the dipolar part. And this is the net dielectric constant, and epsilon r double prime would be epsilon r s minus epsilon r infinity multiplied by omega tau divided by 1 over omega tau square omega square tau square.

So this, these are very fundamental equations which show dipolar contribution very explicitly, and these are called as your Debye equations. These are fundamental equations and for dielectric materials showing dipolar behaviour. And from this you can also determine, what is loss tangent? So, loss tangent can be written as tan delta, and what is that as you know that is nothing but epsilon r double prime divided by epsilon r prime. And this will be, you just have to make a division here. And this would be equal to epsilon r s minus epsilon r infinity multiplied into omega tau divided by epsilon r s prime minus of epsilon r infinity plus of epsilon r infinity into omega square tau square. So, you just have to work this way out. This is your loss tangent. Now, if you want to plot these this is the.

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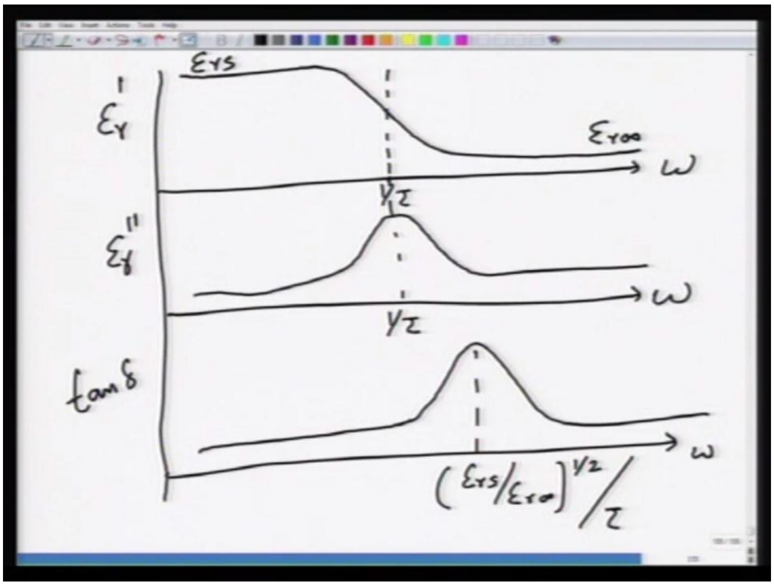

It is interesting to plot these equations. So, let us say, so if you plot just the dielectric constant first versus frequency. So, this goes as something like this behaviour, and this frequency corresponds to 1 over tau. And so, naturally this part would be epsilon r s, and this part would be epsilon r infinity. If you plot the correspondingly epsilon r double prime plot; this plot goes as with the maxima at a frequency 1 over tau. And if you plot tan delta correspondingly, its peak is a little bit shifted towards higher frequencies. And this is your, and this corresponds to epsilon r s divided by epsilon r infinity to the power half divided by tau.

So, this is these are the; so, you can see that, this curve looks very different from the curve that we looked for ionic and electronic polarizations. You do not have a resonance term rather you have a relaxation phenomena here. The dielectric constant falling from a static value r s slowly around a value slowly to epsilon r infinity and these phenomena happens at low frequencies correspondingly, at a frequency 1 over tau, you have a maximum in epsilon r double prime. And correspondingly you can also have a maximum in tan delta which is slightly shifted to higher frequencies as a result of the expression that you get.

Now, these equations are extremely important equations from the point of view of fundamental understanding or dipolar polarization or polarization mechanisms. And you should go through this analysis a little bit more carefully to understand what is happening. So, as you can see. So, if you now draw the so, you have 3 pictures; you have dielectric; you have polarization which is composed of electronic component of polarization, ionic component of polarization, and dipolar component of polarization. And all these three have different behaviour on one hand where P i, and P E are extremely fast make almost instantaneous polarization. And they happen at very high frequencies. So, as a result they become almost instantaneous in contrast p d is at time dependant phenomena which happen slowly. So, development of saturation polarization is relatively slow as compared to these two. So, as a result the, you have this behaviour.

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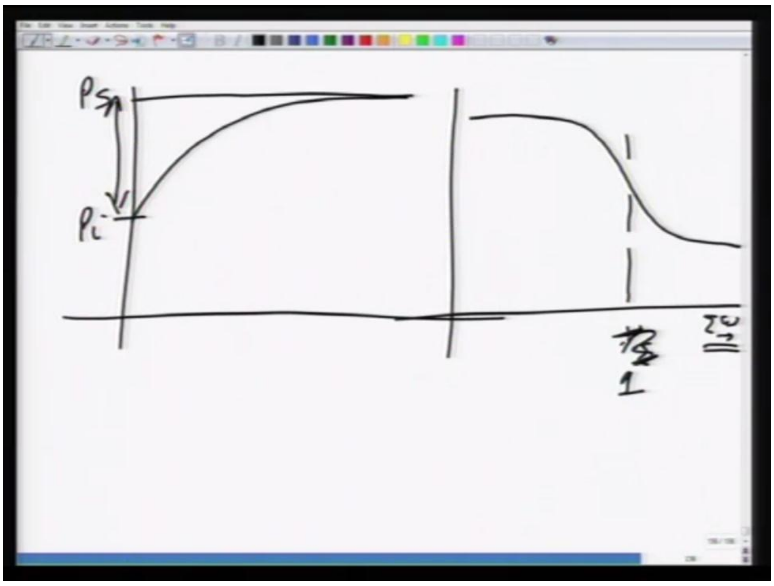

Which goes as, so it develops into P s a little bit slowly from P i to P s, it takes bit of time. And as a result of this, you have when you plot dielectric constant. It goes like this through a frequency 1 over tau, and or if you plot omega versus tau, if you plot frequency is 1 over tau, but if you plot omega tau then it becomes equal to 1.

So, depending upon which book do you follow it depend in some books, they use on x axis omega tau. And in some books they use omega. So, when you have omega then this inflection point is at 1 over tau, when you use omega tau the inflection point would be at 1. So, this time dependent behaviour is a characteristic of dipolar materials which have which again happens, because of i mean ability of dipole to move instantaneously with the applied field.

Since the mass of the entities involved here is much higher, so, and what it means is that at omega is equal to tau the charges are coupled together with the applied field. And absorb maximum energy. So, as a result the losses are maximum at tau is equal to 1 over tau omega is equal to 1 over tau. And this phenomenon, since it is a time dependent phenomena, but it is also a temperature dependent phenomena.

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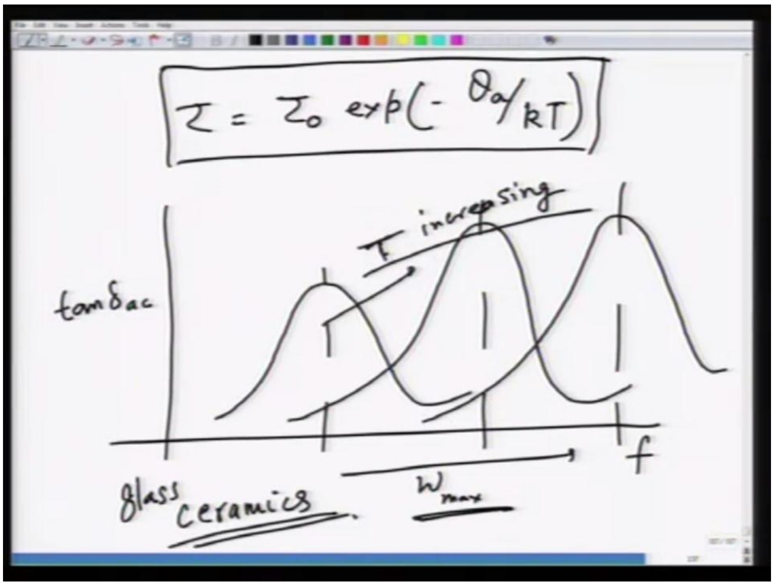

This tau has a relation tau naught exponential minus Q a by k T. And this is kind of Arrhenius relationship which means this the there will be a shift in tau as you change the temperature. And this has a dependence on the activation energy.

So, when you plot for instance the tan delta versus tan delta versus frequency then the shift the maxima that you observe, this maxima shifts to higher frequencies as you increase the temperature. So, this is increasing the temperature, and this you would expect, because as the temperature increases you provide thermal energy to the system. And when you provide thermal energy to the system, the relaxation time should go down which means this maxima, if tau goes down this frequency omega. Let us say omega max will shift towards the right which means relaxation will occur much faster, and as a result this peak in tan delta will or loss will shift towards higher frequencies.

So, this explains the Arrhenius or diffusion behaviour of the relaxation time in dipolar materials. So, this is typical behaviour for example, for glass ceramics, if you want to go through some more plots, I would suggest for few plots on.

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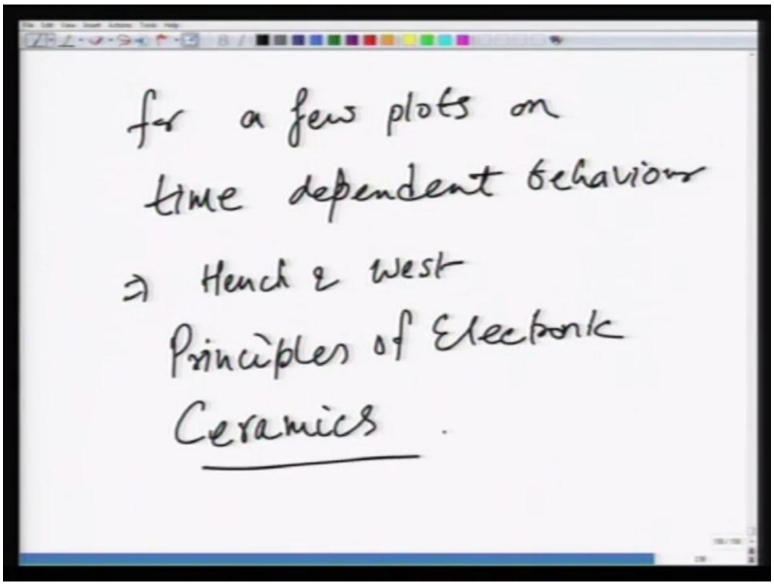

for a few plots on
time dependent behavior
⇒ Hench & West
Principles of Electronic
Ceramics.

I would suggest you go to the book of Hench and west principles of electronic ceramics. And this gives a very nice explanation of the temperature dependence of relaxation time. So, now as you have seen that the peak in the maxima shifts towards right on the frequency plot as the temperature is increased.

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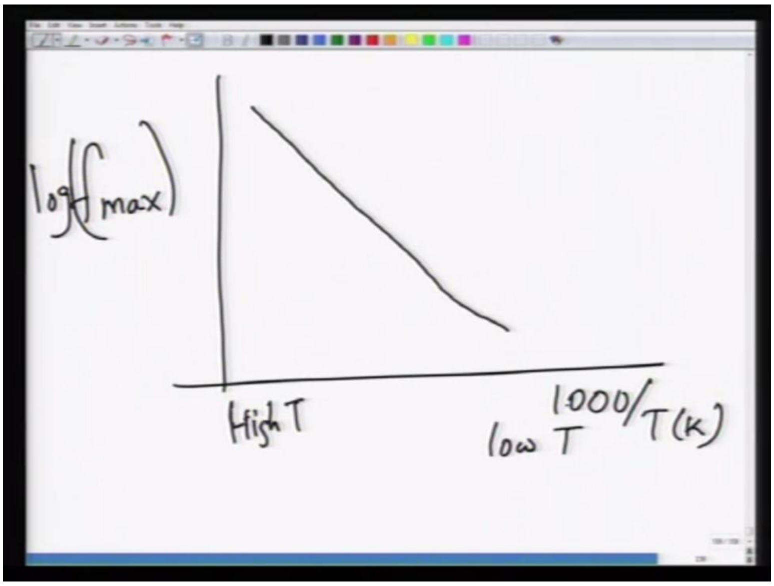

You can also plot this f max log of f max versus, and as you can see that this would be as the temperature is as the temperature is. So, this is your low temperature, high temperature side, this is your low temperature side. So, at higher at higher temperature you will have f max occurring at higher frequencies whereas, at lower temperature Maxwell occur at a lower frequencies, because the relaxation time will be larger at these frequencies. So, the complete picture is now rather different.

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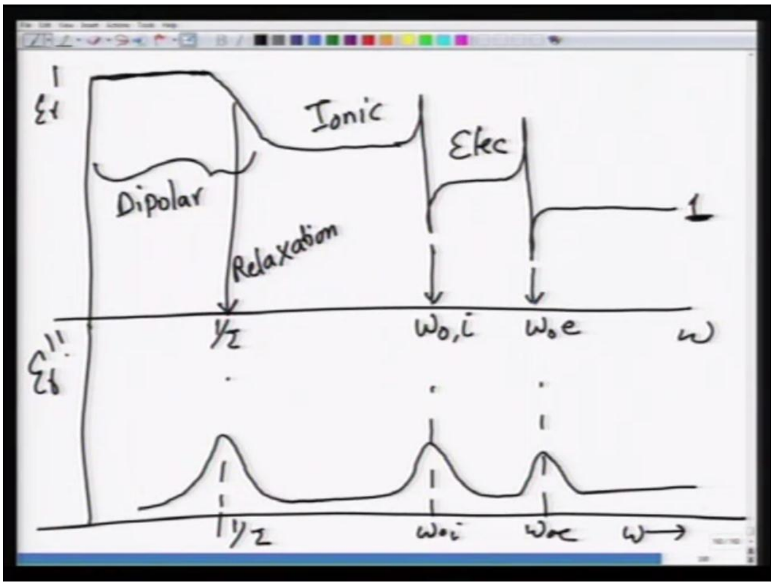

It looks; it looks pretty much complete now. So, the overall behaviour now is you have, if you now go from. So, this is omega; this is epsilon r prime. So, you have epsilon r coming from epsilon r s going through relaxation. Then it goes to resonance another resonance, so, this will be your dipolar component. This would be 1 over tau, and relaxation frequency. This is your omega naught i; this is your omega naught e. So, here you have, in this you have ionic polarization here, you have electronic, and this is your 1 or n square; 1 just 1 as if no dielectric was present, and correspondingly if you plot the loss all of these go through a maxima. So, this is 1 over tau omega naught i omega naught e omega, and this is epsilon r double prime.

So, you can see now the presence of various features in this dielectric constant versus frequency plot, and the loss versus or the imaginary part of dielectric constant, which represents the loss versus frequency. The thing to remember is in for an ionic, and electronic mechanism the. These are resonance dependent mechanisms as a result the resonant frequency is on higher sides greater than 10 to the power 11 Hertz in most cases. And for ionic polarization since, it is a for a dipolar polarization since, it is a time dependent phenomena the, it is a relaxation kind of phenomena. And the relaxation frequency happens at much more smaller frequency, because it takes bit of time to for dipoles to relax, because they are heavier than just 2 ions and or an atom.

So, this sort of completes very important part of this module, where we took, where we looked into the frequency dependence on the dielectric constant considering ionic electronic and dipolar mechanisms. If you want to read about interface mechanisms, it is a little bit more complex in beyond the scope of this course. But if you want to read, you can again follow the book of Hench and west and where.

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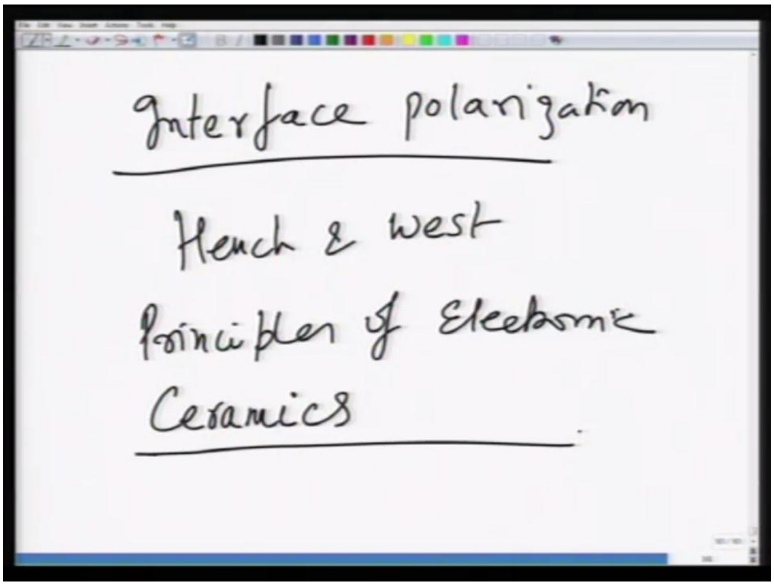

Interface polarization

Hench & west

Principles of Electronic Ceramics

So, which is given in some detail? So, otherwise we have completed the important segments of this phenomenon. In the next class, what we will do is that we will look at some of some other methods which are used to characterize the dielectric materials by using the same dielectric data. And can be used much more effectively to represent the behaviour of dielectrics much more easily. And finally, we will look at some other breakdown mechanisms or dielectric materials in the same module.

Thank you.