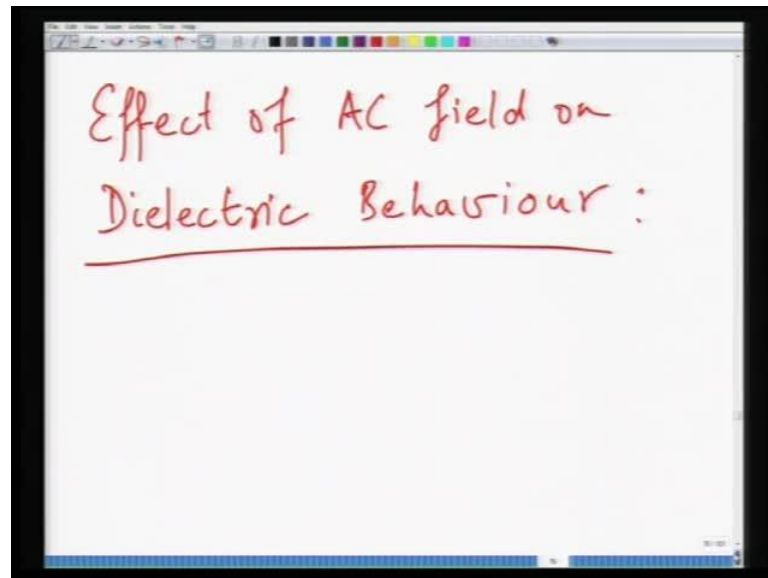


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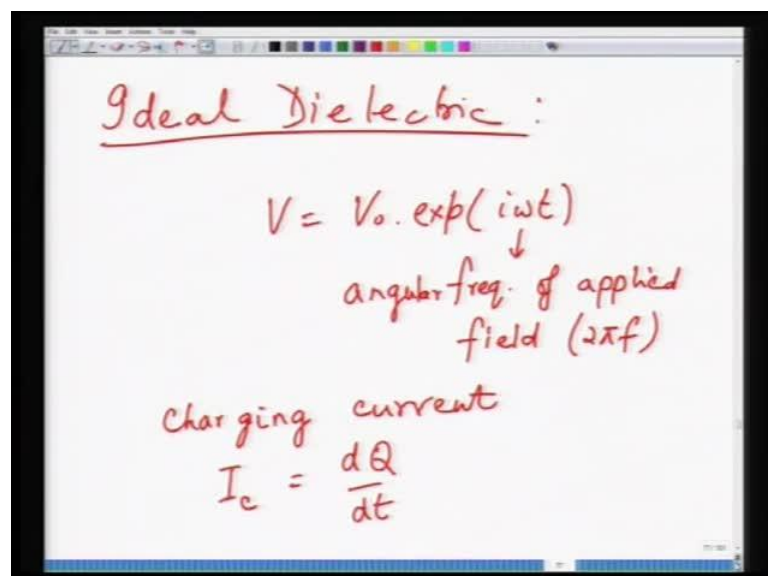
**Lecture - 21**

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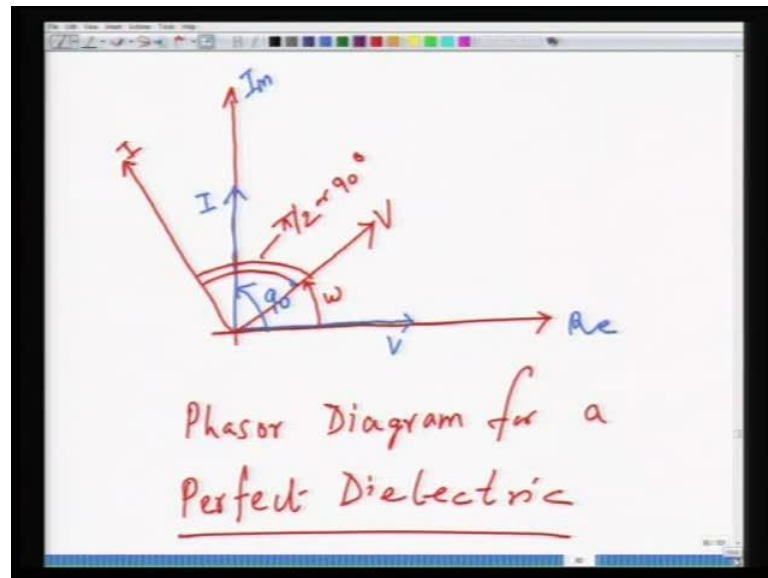
So, this is the again start of a new lecture. So, what we will first do is that just go through the last lecture. So, in the last lecture we essentially looked at what is the response of dielectric materials in alternating electric field.

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And we looked at the case of ideal dielectric, where power dissipation is 0 and current leads the voltage by 90 degrees.

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So, this is the phasor diagram for an ideal dielectric where current leads a voltage by 90 degrees and this results in 0 power loss.

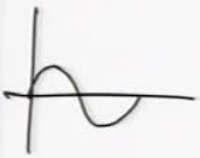
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Power Dissipation

$$P_{av} = \int_0^T \frac{I_c V}{T} dt \quad T = \text{time period} = \frac{2\pi}{\omega}$$

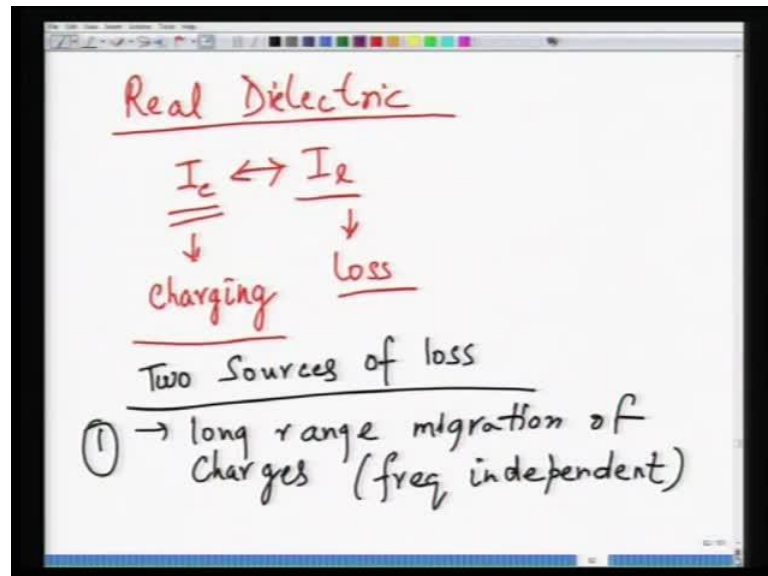
$$= \frac{1}{T} \int_0^T I_c V dt$$

$$= \frac{1}{T} \int_0^T \frac{I_c}{i\omega C} V_0 \exp(i\omega t) dt$$

$$= 0$$


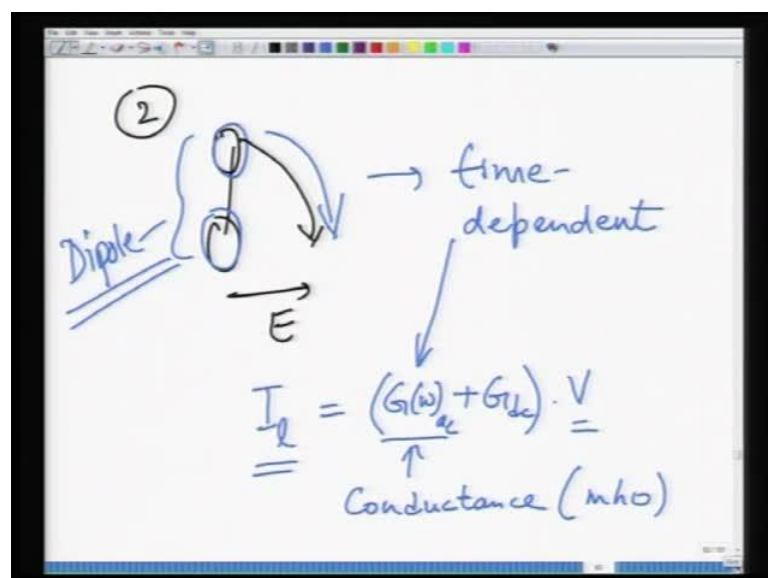
So, and then we, but in reality most of the materials are not ideal, they have some sort of long range migration of charges and which results in a power dissipation.

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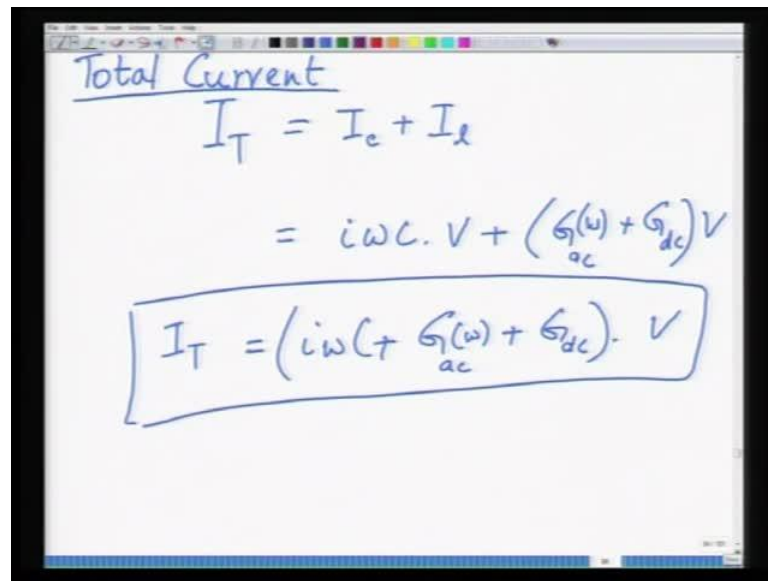
So, as a result they have certain loss in terms of charges storage. So, what basically means is that some sort of the charge leaks out of dielectric material and which is represented by a quantity called as dissipation factor or loss factor.

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So, here in real dielectrics you have, in addition to charging current we have loss current and this loss current can be because of either long range migration of charges or ... Or it could be a time dependent process such as in case of dipole, dipolar relaxation and this so this loss current consists of two terms. One is the AC term which is the frequency dependent term, and second is the DC term and we represent this in terms of quantity called as conductance which is nothing but inverse of resistance.

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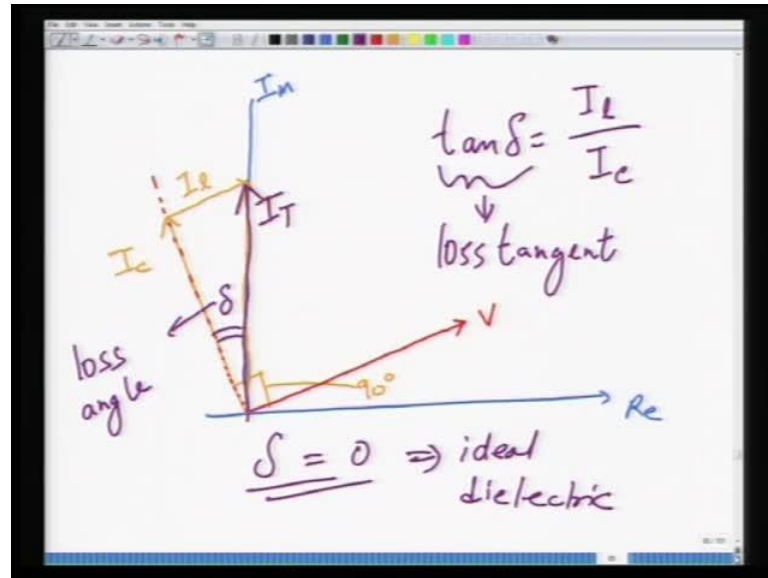


The image shows a digital whiteboard with handwritten equations. At the top, 'Total Current' is underlined. Below it, the equation  $I_T = I_c + I_l$  is written. This is followed by  $= i\omega C \cdot V + (G_{ac}(\omega) + G_{dc})V$ . The final equation,  $I_T = (i\omega C + G_{ac}(\omega) + G_{dc}) \cdot V$ , is enclosed in a hand-drawn rectangular box.

$$\text{Total Current}$$
$$I_T = I_c + I_l$$
$$= i\omega C \cdot V + (G_{ac}(\omega) + G_{dc})V$$
$$I_T = (i\omega C + G_{ac}(\omega) + G_{dc}) \cdot V$$

So, the total current in for a real dielectric would be charging current plus loss current and as a result we get an expression for this.

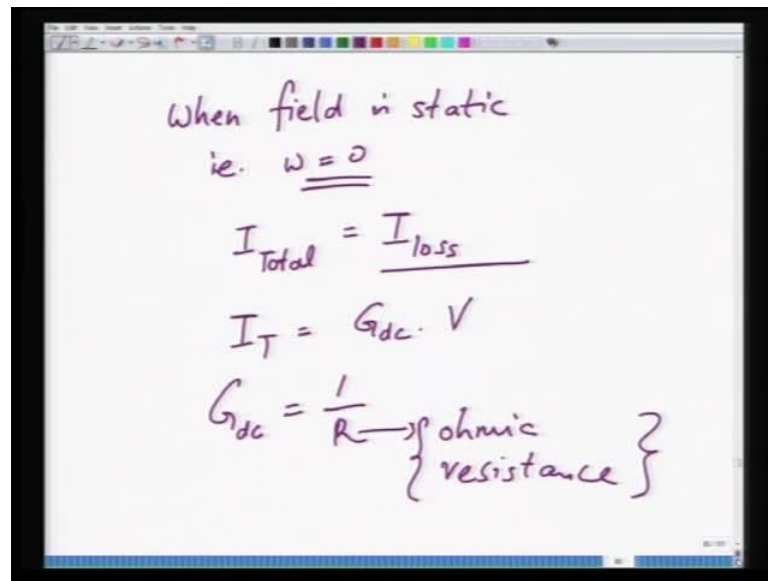
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And what we do the next is and now basically this presence of this loss current how how can it be schematically shown? So, if you have so this if this is the voltage in the charging current, which is 100 percent current the ideal ideal dielectric leads the voltage by 90 degrees. In addition you have a loss current which is in phase with the applied field and which is depicted by this vector. Sum of these two vectorial sum of these two  $I_c$  plus  $I_l$  gives rise to  $I_T$ .

Now, you see that this  $I_T$  makes an angle  $\delta$  with  $I_c$  and this  $\delta$  is called as loss angle. So, naturally as you can see larger  $\delta$  is which means larger  $I_l$  would be and hence larger the loss would be. So, this  $\tan \delta$  is nothing but ratio of loss current to charging current and this is what we have done in the next few slides.

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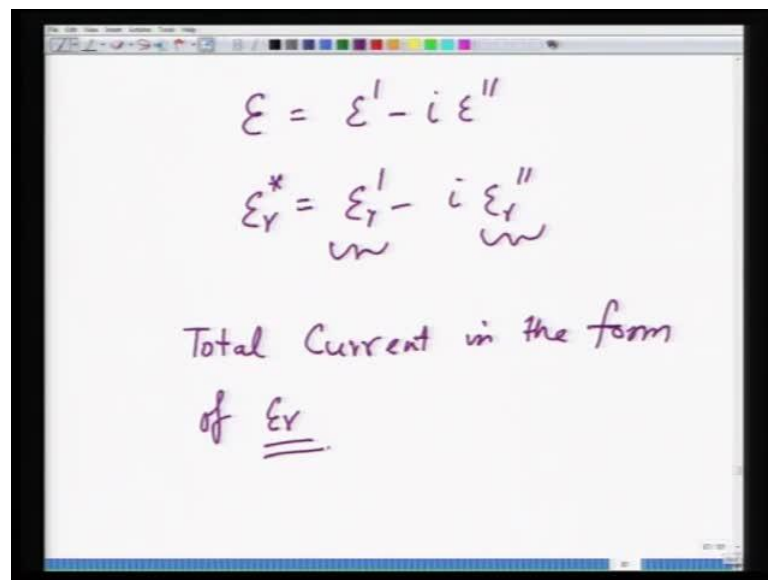


When field is static  
ie.  $\omega = 0$

$$I_{\text{Total}} = I_{\text{loss}}$$
$$I_T = G_{dc} \cdot V$$
$$G_{dc} = \frac{1}{R} \rightarrow \left. \begin{array}{l} \text{ohmic} \\ \text{resistance} \end{array} \right\}$$

So, then we looked at variety of cases. I will not go through all of them, but some of them.

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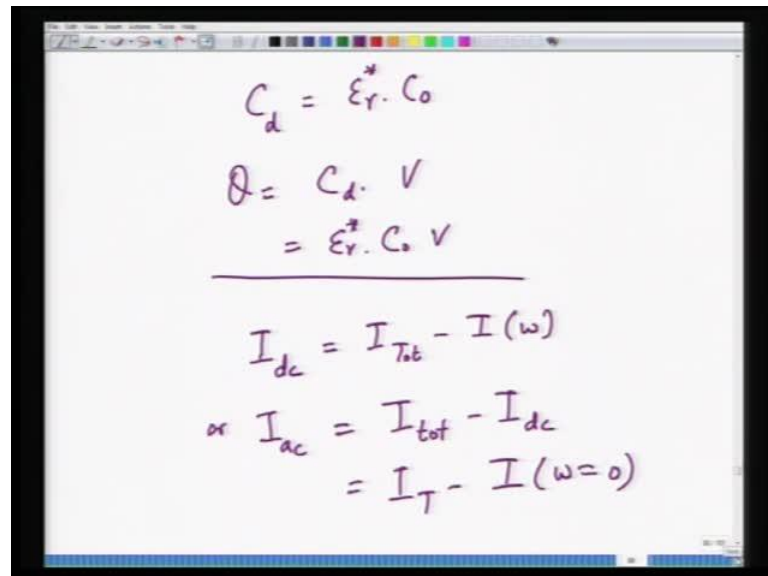

$$\epsilon = \epsilon' - i\epsilon''$$
$$\epsilon_r^* = \epsilon_r' - i\epsilon_r''$$

Total Current in the form  
of  $\epsilon_r$ .

Another thing that is done in in case of real dielectrics is now that we know that there is a charging current, there is a loss current and since we know that current is a complex quantity here the dielectric constant is also a complex quantity and it can be it can be represented in the complex form as epsilon is equal to epsilon prime minus i epsilon double prime or epsilon r star which is the complex dielectric constant is nothing but has

contribution from real dielectric constant, which is  $\epsilon_r'$  minus  $i$  into  $\epsilon_r''$  which is the imaginary part of dielectric constant.

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$$C_d = \epsilon_r^* \cdot C_0$$

$$Q = C_d \cdot V$$

$$= \epsilon_r^* \cdot C_0 \cdot V$$


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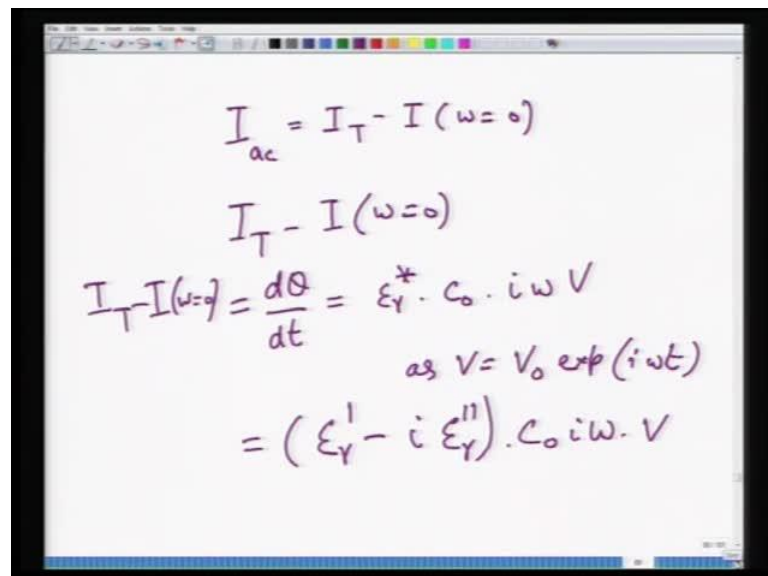

$$I_{dc} = I_{tot} - I(\omega)$$

$$\text{or } I_{ac} = I_{tot} - I_{dc}$$

$$= I_T - I(\omega=0)$$

And then we looked at the current expressions.

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$$I_{ac} = I_T - I(\omega=0)$$

$$I_T - I(\omega=0)$$

$$I_T - I(\omega=0) = \frac{dQ}{dt} = \epsilon_r^* \cdot C_0 \cdot i\omega V$$

as  $V = V_0 \exp(i\omega t)$

$$= (\epsilon_r' - i\epsilon_r'') \cdot C_0 i\omega \cdot V$$

And what we worked out in the then was if we equate this current expression to dielectric constant by inserting dielectric constant expression then we get a term for

current which is equivalent to  $\epsilon_r' - \epsilon_r''$  into  $C$  naught  $i \omega V$ .

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Handwritten notes on a whiteboard showing the derivation of total current  $I_T$  as a function of the complex permittivity  $\epsilon_r^*$ .

$$I_T = \underbrace{i\omega \cdot C_0 \cdot \epsilon_r' V}_{\text{Out of phase } I_c} + \underbrace{\omega \epsilon_r'' \cdot C_0 \cdot V}_{\text{in phase AC } (I_d)} + \underbrace{G_{dc} \cdot V}_{\text{dc current } (I_{dc})}$$

Below the equation, it is noted:  $I(\omega=0) \rightarrow \text{dc current}$

The final result is summarized as:

$$I_T = f(\epsilon_r^*)$$

And here we can see from this expression that out of phase term is  $i \omega$  term and the  $\omega$  term is the, so this is a charging current  $i \omega C$  naught  $\epsilon_r' V$ , this is the out of phase charging current term and the in phase current terms are  $\omega \epsilon_r'' C$  naught  $v$  and this first term is the since it is frequency dependent this is the AC term and then you have loss current which is a DC term  $F d c$  into  $V$ . So, total current  $I_T$  can be represented as function of this  $\epsilon_r'$ .



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Handwritten derivation on a whiteboard:

$$I_T = (i\omega C_0 + G_{ac}(\omega) + G_{dc}) V$$

$$I_T = \underbrace{(i\omega C_0 \epsilon_r')}_{I_{ch}} + \underbrace{(\omega \epsilon_r'' C_0 + G_{dc})}_{I_{loss}} V$$

$$\Rightarrow G_{ac}(\omega) = \omega \epsilon_r'' C_0$$

$$\tan \delta = \frac{I_{loss}}{I_{charging}} = \frac{I_L}{I_C}$$

Now, you can from this analysis further if you delve you find out an expression for tan delta which is nothing but i loss divided by i charging.

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Handwritten derivation on a whiteboard:

$$\tan \delta = \left( \frac{G_{dc} + \omega \epsilon_r'' C_0}{\omega \epsilon_r' C_0} \right)$$

if  $G_{dc} \ll \omega \epsilon_r'' C_0$

$$\tan \delta = \frac{\cancel{\omega \epsilon_r'' C_0}}{\cancel{\omega \epsilon_r' C_0}} = \frac{\epsilon_r''}{\epsilon_r'}$$

$$\epsilon_r'' \rightarrow \epsilon_r' \approx \epsilon_r'' \approx \epsilon_r' \cdot \tan \delta$$

And this tan delta can be expressed in terms of  $G_{dc}$  plus  $\omega \epsilon_r'' C_0$  divided by  $\omega \epsilon_r' C_0$ . So, here let us say in to to make our life little easier if we make  $G_{dc}$  much smaller than the  $\omega \epsilon_r'' C_0$  which is the DC contribution of loss current is much smaller than the AC contribution of loss current then this tan delta becomes equal to  $\epsilon_r''$

divided by epsilon r prime which is nothing but equivalent to I l divided by I c. So, what basically it means is that that epsilon r double prime is equivalent to loss current and epsilon r prime is equivalent to charging current and epsilon r double prime is or you can say epsilon r double prime is equal to epsilon r prime into tan delta.

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Handwritten notes on a whiteboard:

- $\tan \delta \rightarrow$  loss tangent
- $\rightarrow$  Dissipation factor.
- $\epsilon'' = \epsilon' \tan \delta$
- $\uparrow$  Dielectric loss factor

So, tan delta is often called as loss tangent or dissipation factor or epsilon r double prime is called as dielectric loss factor which is which is product of epsilon r prime and tan delta. So, naturally higher tan delta is higher epsilon r double prime would be.

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Material	D.C. ( $\epsilon'$ )	$\tan \delta (\times 10^{-4})$
Alumina	$\sim 10$	5-20
$\text{SiO}_2$	3-8	2
$\text{BaTiO}_3$	500	150
PVC	3	160

Then we looked at some material. So, these are the dielectric constant losses of few materials.

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Power Dissipation: Real Dielectric

AC Conductivity + DC Conductivity

$$G_{tot} = G_{dc} + \omega \cdot \epsilon_r'' C_0$$

$G_{dc} \approx 0$  as  $G_{dc}$  is small

$$G_{tot} = G_{ac} = \omega \cdot \epsilon_r'' C_0$$

$$G_{tot} = \omega \cdot \epsilon_r'' \cdot \tan \delta C_0$$

And then we looked at the power dissipation.

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$$P_{avg} = \frac{1}{T} \int_0^T \overbrace{I_{loss}}^{I_T = I_C + I_L} V dt$$

$$= \frac{1}{T} \int_0^T (\omega \epsilon_r'' C_0 + G_{dc}) V_0 \exp(i\omega t) \times V_0 \exp(-i\omega t) \cdot dt$$

$$G_{dc} \ll \omega \epsilon_r'' C_0$$

And what we found here was by using the same approach as we took in case of ideal material. So, we just integrated the current for the time period so this is the average power loss.

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The image shows a handwritten derivation of average power loss. The first equation is:

$$P_{avg} = \frac{1}{T} \int_0^T (\omega \epsilon_r'' C_0) V_0^2 \exp(2i\omega t) dt$$

The second equation shows the simplification:

$$= \frac{1}{2} (\omega \epsilon_r'' C_0) V_0^2$$

The term  $(\omega \epsilon_r'' C_0)$  is bracketed and labeled  $G_{ac}$ . The final result is boxed:

$$P_{avg} = \frac{1}{2} G_{ac} \cdot V_0^2$$

So, average power loss becomes half of  $G_{ac}$  into  $V_0$  square. So, this  $G_{ac}$  is the conductance related to AC component of loss current.

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The image shows a handwritten derivation of average power loss. The first equation is:

$$P_{avg} = \frac{1}{2} (\omega \epsilon_r' \tan \delta C_0) V_0^2$$

The second equation is boxed:

$$\underline{P_{avg} = \frac{1}{2} V_0^2 \omega C_d \tan \delta}$$

The third equation shows the relationship between capacitance and area:

$$C_0 = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = \frac{V_0}{d}$$

The final equation shows the power density:

$$\text{Power Density} = \frac{P_{avg}}{V} = \frac{1}{2} \omega \epsilon_0 \epsilon_r' \tan \delta \epsilon_0^2$$

Higher it is higher would be the the average dissipation of, average power dissipated and you can see that this is equivalent to writing half  $V_0$  square omega  $C_d$ .  $C_d$  is a capacitor of a dielectric multiplied by tan delta. So, higher omega is higher  $P_{avg}$  and higher tan delta is higher the losses. So, basically it scales with tan delta so higher the loss factor is higher the power dissipation would be.

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if  $\omega = 0$

$$G_{\text{total}} = G_{\text{dc}} = \frac{1}{R}$$
$$P_{\text{avg}} = I^2 R$$

$\tan \delta = \frac{I_C}{I_R} = \frac{\epsilon''}{\epsilon'}$

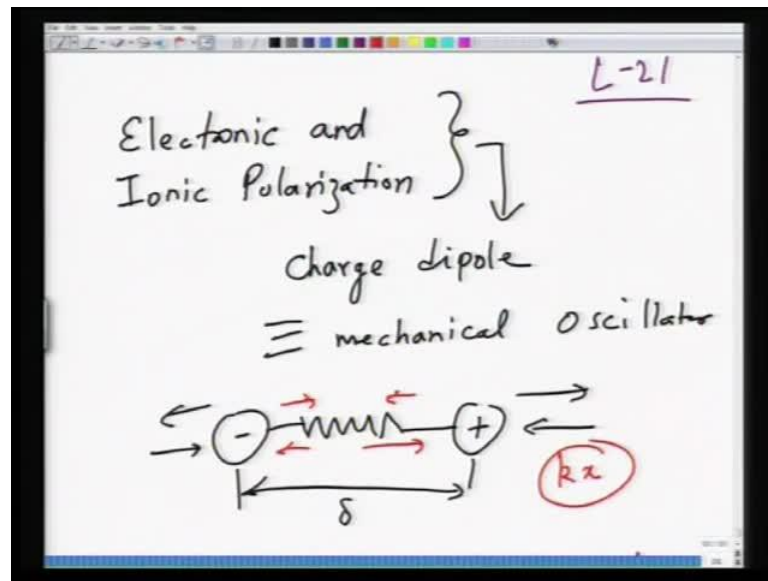
$I_T = I_C + I_R$

Diagram showing a vector  $I_T$  at an angle  $\delta$  to the horizontal axis  $I_R$ , with a voltage vector  $V$  also shown.

So, now what we will do is that we will, this is, so, this is a new lecture. So, what we will do is that we will start this is lecture 21. So, we will start with something else now. So, we have looked that variety of different things about dielectric, we have looked that how they work in DC fields, what different polarization mechanisms are, what is the concept to local field and then what is how can polarization be explained analytically? So, we got expressions for variety of polarizabilities and then we moved on into the AC field dependence.

Now, this frequency dependence of dielectric properties I can, before that taken in, so that we can, can be taken further so that we can get an expression for now polarizability in terms of frequency because, in the previous part we looked at polarizability that was in the DC form. How will polarizability expression looks, how will this expression look like when there is a frequency dependence? So, now there are, as we know there are two variety of polarization mechanisms.

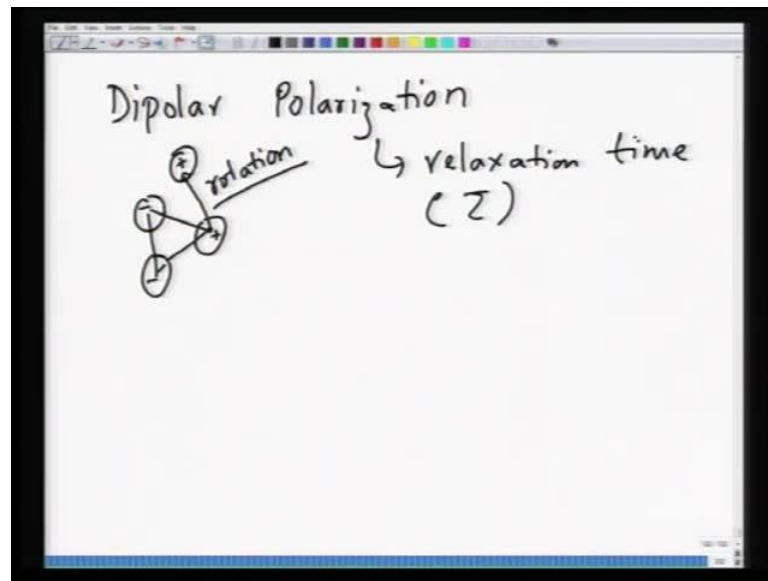
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We know that you have electronic and ionic polarization mechanisms. Now, in these in these two polarization mechanisms the charged dipoles which means a positive charge and the negative charge, this charged dipole can be considered as is if it is a mechanical oscillator. So, so which means you have this negative charge, you have this positive charge and we know that this is the distance delta between them. So, what it means is that basically center of a negative charge and center of positive charge are displaced by distance delta giving rise to creation of a dipole moment and this charge dipole can be considered as if it is a mechanical oscillator where these two charges are attached to each other by a linear spring.

So, here this restoring force of the spring. So, when he pull it apart or when you pull it in depending upon the direction of whether you whether you extend or whether you compress there is a restoring force. So, for the top part you will have restoring force like this and for this you will have a restoring force like this which is which is nothing but your spring restoring force and this is nothing but your  $kx$ , scales as  $kx$  and where  $k$  is the spring constant. And the characteristic of such an oscillating system, such a mechanical oscillator is the, is is that they exhibit the resonance at certain frequency. So, these two mechanisms are characterized by something called as a resonant frequency. So, the so this is the characteristic of this kind of polarization.

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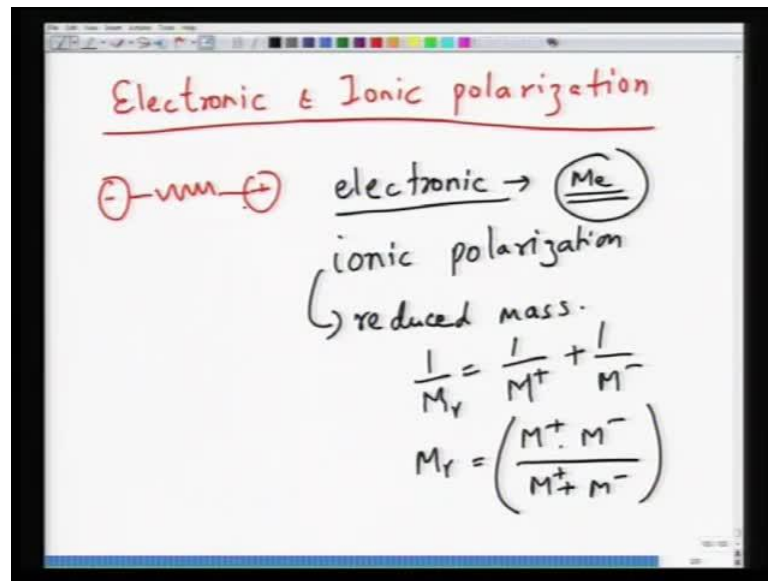


Then third polarization mechanism that we looked at was dipolar polarization. Now, in case of dipolar polarization you have a molecule like this. So, let us say this is minus, this is plus and this goes from one state to another state where this rotates, let us say.

So, this rotation here is, so, when you apply electric field the the molecule turns into a direction of applied field giving rise to a net dipole moment in the direction of applied field, when you take it back then the molecule relaxes itself back to either original position or to some other statistically equivalent position in order to, is such that that it then then then that the net dipole moment becomes equal to either 0 or something else. It is different in case of ferroelectrics, but typically it would become 0, if it is if it is a normal dielectric material. So, as a result what happens is that these dielectric materials are characterized by something called as relaxation time which is often called as tau. Basically, the time which is taken by this dipole or the ensemble of atoms to relax back to their either the original position or a new position which is a new equilibrium position.

So, these processes, these, this phenomena does not have any any resonance phenomena rather it is characterized by what is called as relaxation. So, you understand the term relaxation, they relax to a either to original position or to a new position which is equivalent or which is again a equilibrium position. So, what we will do is that first we will describe. How can we, how can we take up this in a solution for electronic and ionic polarizations.

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The treatment for both of these cases is fairly similar that is why I am treating them in a same manner. The only difference is when you, when you consider a charged dipole like this minus plus and consider it to be connected by a spring, then in case of electronic polarization you take basically mass related to mass of electron. So, in this case basically the mass of whatever this atom would be and in case of ionic polarization so this is at the electronic level, so, masses are much more smaller, ionic electron, ionic polarization happens at the level of lattice or in in case of ions so here we take what is called as reduced mass and this reduced mass is nothing but basically  $\frac{1}{M_r}$  is equal to  $\frac{1}{M^+} + \frac{1}{M^-}$ . So,  $M_r$  will be equal to  $M^+ \cdot M^-$  divided by  $M^+ + M^-$ . So, where  $M^+$  is the mass of cations,  $M^-$  is the mass of anions and this is the effective mass and here we are taking in terms of mass of electron.

So, the the the the methodology is similar except that the masses change. So, we will we will treat both of them in a similar fashion. So, consider this is the linear harmonic oscillator where two charges or center of two charges negative and positive are connected by a linear spring. So, if they if they follow like a linear harmonic oscillator they follow what is called as equation of motion.



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The image shows a whiteboard with the following handwritten content:

Equation of motion friction term

$$q_i E = M_i^* \frac{d^2 x}{dt^2} + M_i^* \gamma_i \frac{dx}{dt} + M_i^* \omega_{0,i}^2 x$$

$\gamma_i$  term  $\rightarrow$  acceleration term  
 $\gamma_i \rightarrow$  friction coefficient  
 $\omega_{0,i} \rightarrow$  natural frequency of dipole

Spring restoring force

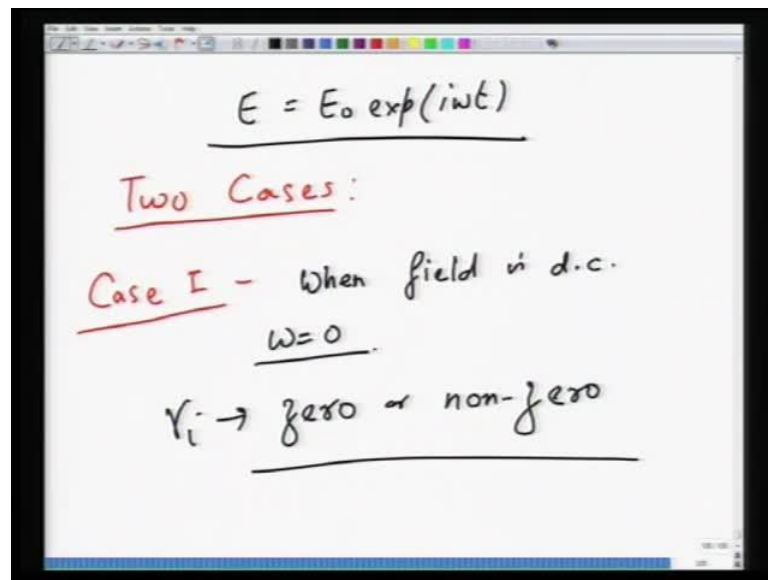
So, this equation of motion is basically it describes the oscillating system by a, via sinusoidal force and this can be written as when you apply a sinusoidal field to it this can be written as so this is the applied field  $q_i E$  let us say. This balanced by what is called as  $M_i$  multiplied by this is the mass multiplied by  $d^2 x$  by  $dt^2$  plus second term which is  $M_i$  into  $\gamma_i$  into  $dx$  by  $dt$  plus third term  $M_i$ , you can write  $M_i$  star actually just to differentiate, this is the, from, just from the mass of a species, it does not really matter, it is a just a terminology. So,  $M_i$  multiplied by  $\omega_{0,i}^2$ . So,  $i$  is the  $i$ th entity let us say square into  $x$ .

Now, what these three terms are you can see that the first term is, first term as you can see is the term which is so which is due to acceleration acceleration of particles of mass  $M_i$  star and and  $x$  as you can see when you apply a electric field which is sinusoidally varying  $x$  can be displacement from the equilibrium. So,  $x$  is the displacement and since field is sinusoidally varying with time or as a frequency then there is a time term as well.

So, the the first term represents the acceleration of this particle of mass  $i$  star and then second term includes this  $\gamma$  term.  $\gamma$  is nothing but your friction factor and this represents this represents what is called as the damping term because the the oscillations do not finish off suddenly, they they damp slowly and this is because of friction in the in the lattice or in the media. So, this is  $M_i$  star  $\gamma_i$   $dx$  by  $dt$  is the friction term.

And the third term is  $M \ddot{x}$  and this is so this term is the friction term and this third term is your due to spring restoring force and where  $\omega_0$  is the natural frequency of this dipole, so, the frequency at which this dipole or this linear harmonic oscillator undergoes resonance.

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Handwritten notes on a whiteboard:

$$E = E_0 \exp(i\omega t)$$

Two Cases:

Case I - When field is d.c.  
 $\omega = 0$   
 $\gamma \rightarrow \text{zero or non-zero}$

Now, what we will do is that we will we will insert this let us say the field is field applied is  $E$  is equal to  $E_0 \exp(i\omega t)$  and we will treat this system like a gas. So, it contains like  $N$  molecule and these all these molecules are non interacting molecules. So, there is there is no interaction force of these molecules on each other.

So, basically we are treating each of them as an individual. So, consider two cases. So, we consider here two cases. Let us say case 1. Case 1 is when field is DC or a static field what happens in such a situation?  $\omega$  is equal to 0. If  $\omega$  is equal to 0 then you when you apply field and when you remove the field, then the the spring because of a spring storing force it comes back and the and the oscillations slowly die off, if the friction is not 0, if the friction friction is 0 there will be no damping of oscillation. So, depending upon the  $\gamma$ 's magnitude whether it is 0 or non 0 you will have oscillations which are damped or not damped.

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Case II - when  $\omega \neq 0$

$$E = E_0 \exp(i\omega t)$$
$$x = x_0 \exp(i\omega t)$$
$$q_i E = m_i^* \frac{d^2 x}{dt^2} + m_i^* \gamma_i \frac{dx}{dt} + m_i^* \omega_{0,i}^2 x$$

The third second case is second case is when omega is not equal to 0 which means the field is sinusoidally varying field and this as I said is E is equal to E naught exponential i omega t. So, what happens when you apply this field? So, when you apply this field there is going to be some displacement and this corresponding displacement let us say is x is equal to x naught exponential i omega t. So, this field which is sinusoidally varying gives rise to a displacement which is also of the same nature.

Now, if you insert this. So, what you will have basically if you look at the expression you, we had q i multiplied by E into M i star into del 2 x by del x square, del del 2 x by del t square sorry plus M i star gamma i into d x by d t plus M i star into omega naught i square into x. So, what you basically you need to do is that you need to now put the value of E here which is E naught exponential i omega t. You put the value of x which is x so you you have to differentiate this twice with respect to time.

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$$\begin{aligned}\frac{dx}{dt} &= \frac{d}{dt} (x_0 \exp(i\omega t)) \\ &= i\omega \cdot x_0 \exp(i\omega t) \\ &= i\omega \cdot x \\ \frac{d^2x}{dt^2} &= (i\omega)^2 \cdot x = -\omega^2 x \\ \text{Ignore the transient terms} \\ x(t) &= \frac{q_i E_0 \exp(i\omega t)}{M_0 [( \omega_{0i}^2 - \omega^2 ) + i \gamma_i \omega]}\end{aligned}$$

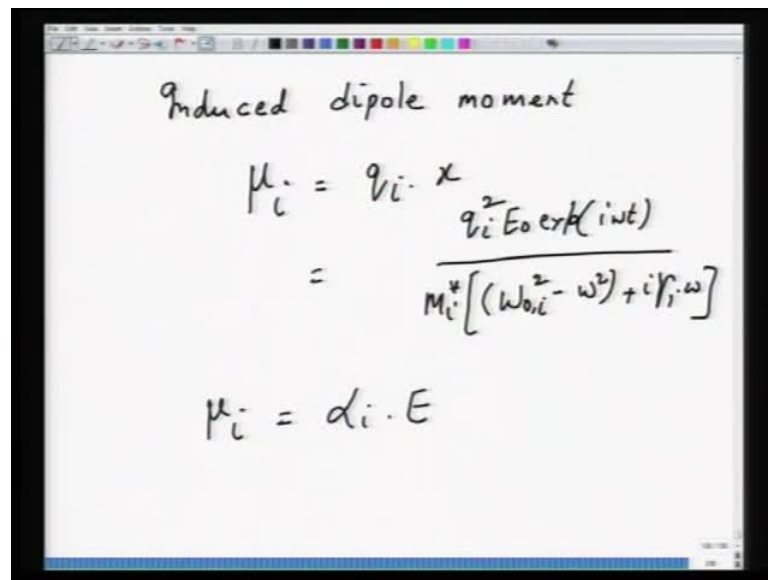
So, of course this will become, so, this term will become so  $d^2x$  by  $dt^2$  will become, so, let us go one by one. So, so let us first go for  $dx$  by  $dt$  and this would be  $d$  over  $dt$  exponential  $x$  naught exponential  $i\omega t$  and this will be  $i\omega$  into  $x$  naught exponential  $i\omega t$ . So, this would be  $i\omega$  into  $x$  again and then  $d^2x$  by  $dt^2$  this would be  $i\omega$  square into  $x$ . So, this is the first term. So, all you need to do is that you need to put in the values of  $x$  and basically for the restoring force term  $x$  for the damping force term  $i\omega x$  and for the acceleration term  $i\omega$  square into  $x$  which basically becomes minus  $\omega$  square  $x$ .

So, you just put in these terms and and you put in the term for  $E$  and if you ignore the transient term the solution of this equation is basically in terms of  $x$ . So,  $x(t)$  what you get is  $q_i$  into  $E$  naught exponential of  $i\omega t$  divided by  $M$  into  $\omega$  naught  $i$  square minus  $\omega$  square plus  $i$  into  $\gamma_i \omega$ . So, this is the expression. So, here what we done is you just ignore the... So, all you need to do is do is that you just need to put in the values of, you need to put in the value of  $dx$  by  $dt$  and  $d^2x$  by  $dt^2$  and get an expression in terms of  $x$  in terms of  $E$ , this is  $E q_i$  divided by what you will get is so you can see here the first term is  $i\omega$  square which is minus  $\omega$  square.

So, this is this is minus  $\omega$  square, the the third term would have become  $M$  naught  $\omega$  naught square multiplied by  $x$ . So, this  $x$  goes there and the fourth and the and the

first term was and the second term was  $i\omega$  into  $\gamma$  and this  $x$  comes out. So, if you, so, this is the solution for this expression which we looked earlier which is very simple just by replacing these values of  $d \times b$  by  $d \times t$  and  $d^2 \times b$  by  $d^2 \times t$  you can receive this.

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Induced dipole moment

$$\mu_i = q_i \cdot x$$

$$= \frac{q_i^2 E_0 e^{i\omega t}}{M_i^* [(\omega_{0,i}^2 - \omega^2) + i\gamma_i \omega]}$$

$$\mu_i = \alpha_i \cdot E$$

So, what is the induced dipole moment? Induced dipole moment per particle or per oscillator which is equal to  $\mu_i$  is equal to  $q_i$  into  $x$ . So, this is equal to  $q_i$  multiplied by  $q_i$  into  $E$  naught exponential  $i\omega t$  divided by  $M_i$  star into  $\omega$  naught  $i$  square minus  $\omega$  square plus  $i\gamma_i \omega$ . So, this is the expression for  $\mu_i$ . So, this naturally makes this go as  $\mu_i$  square. If I just take this off, so, this will become  $q_i^2$  and this and what is  $\mu$  in terms of polarizability it goes as  $\alpha_i$  into  $E$ .

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Polarizability

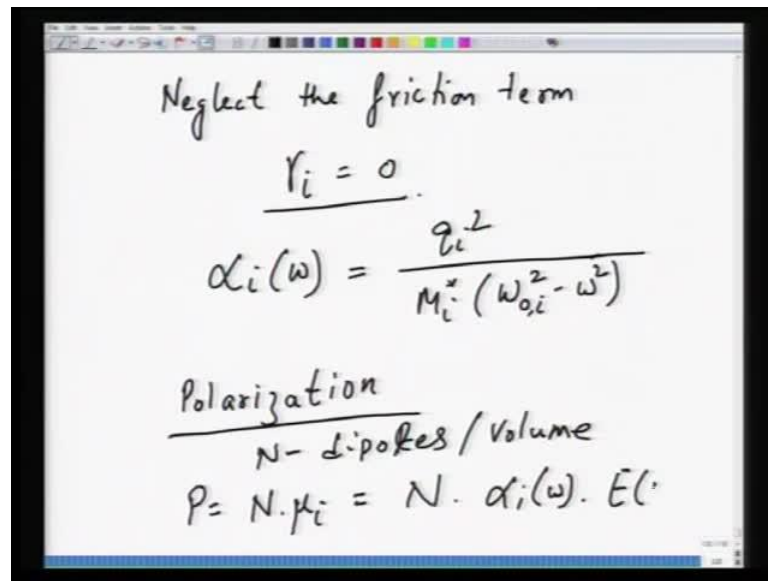
$$\alpha_i(\omega) = \frac{q_i^2}{m_i \left[ (\omega_{oi}^2 - \omega^2) + i\gamma_i\omega \right]}$$

↑  
(natural freq.  
of the particular  
dipole)

So, from this expression you can get the expression for alpha i which is the polarizability. So, alpha i is given as q i square divided by M i star into omega naught i square minus omega square plus omega i gamma i omega. So, this is the expression for alpha i where this omega naught i is the natural frequency of the particular dipole.

So, this is the frequency at which you will have resonance and gamma i is the friction factor. So, what you get here is you get an expression for alpha i which is nothing but dependent upon the frequencies so you can write it as alpha i omega which is equal to q i square divided by M i star into omega naught i square minus omega square plus i gamma i omega and if... So, this is the, so, gives you, this expression gives you dependence of polarizability of of this particular system of of of a particular system whether it is electronic system, whether it is ionic system in terms of frequency.

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Neglect the friction term

$$\gamma_i = 0$$

$$\alpha_i(\omega) = \frac{q_i^2}{M_i (\omega_{0i}^2 - \omega^2)}$$

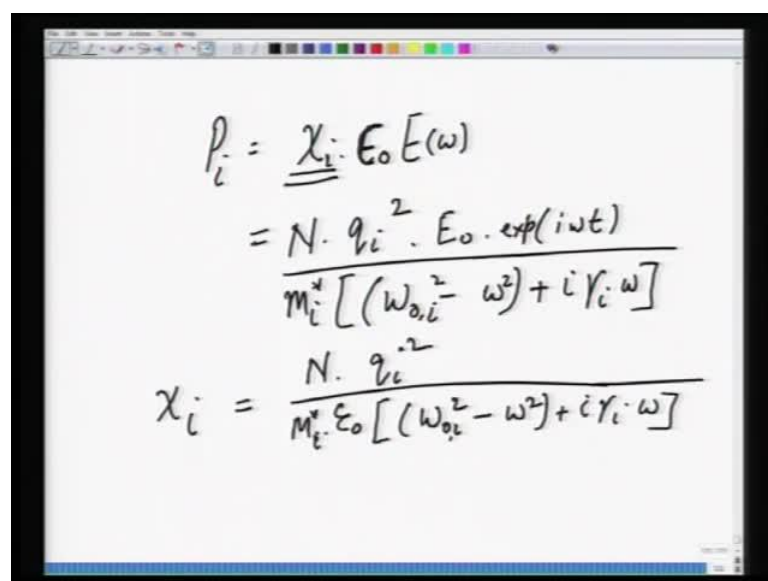
Polarization

N - dipoles / volume

$$P = N \cdot \mu_i = N \cdot \alpha_i(\omega) \cdot E(\omega)$$

Now, if you neglect friction term what it would mean is that gamma i will be equal to 0. If gamma i is equal to 0 then alpha i omega would become equal to q i square divided by M i star into omega i square minus omega square and how can you write now polarization? So, polarization assume that there are N N dipoles. So, as a result polarization will be equal to N into mu i per unit volume force and this will be equal to N into alpha i omega into E omega.

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$$P_i = \chi_i \cdot \epsilon_0 E(\omega)$$

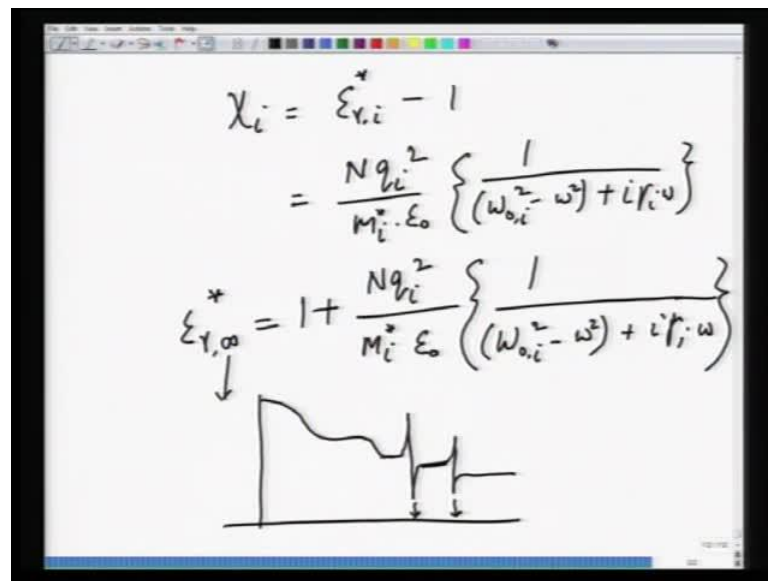
$$= \frac{N \cdot q_i^2 \cdot E_0 \cdot \exp(i\omega t)}{M_i [(\omega_{0i}^2 - \omega^2) + i\gamma_i \omega]}$$

$$\chi_i = \frac{N \cdot q_i^2}{M_i \cdot \epsilon_0 [(\omega_{0i}^2 - \omega^2) + i\gamma_i \omega]}$$

So, polarization will be equal to or let us say  $P_i$  will be equal to and this is also equal to as we know is  $\epsilon_0 \chi_i$  into  $E$ . Now, if you do that so polarization was equal to  $N q_i^2$  divided by  $M_i$  star into  $\omega_i^2 - \omega^2 + i \gamma_i \omega$  into  $E$  which is  $E e^{i \omega t}$ . Now, what you can do is that since you know that, so, this is your  $\chi_i$ .

So,  $\chi_i$  becomes equal to from this expression,  $\chi_i$  will become equal to  $N q_i^2$ . So, you just have to make this equivalent. So, this  $E$  is nothing but  $E e^{i \omega t}$ . So, these two will cancel each other so what you will have is  $N q_i^2$  divided by  $M_i$  star into  $\omega_i^2 - \omega^2 + i \gamma_i \omega$  and you, now, you can guess where we are heading to, this is essentially to work out the real and imaginary parts of dielectric constant.

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$$\chi_i = \epsilon_{r,i}^* - 1$$

$$= \frac{N q_i^2}{M_i^* \epsilon_0} \left\{ \frac{1}{(\omega_{0,i}^2 - \omega^2) + i \gamma_i \omega} \right\}$$

$$\epsilon_{r,i}^* = 1 + \frac{N q_i^2}{M_i^* \epsilon_0} \left\{ \frac{1}{(\omega_{0,i}^2 - \omega^2) + i \gamma_i \omega} \right\}$$

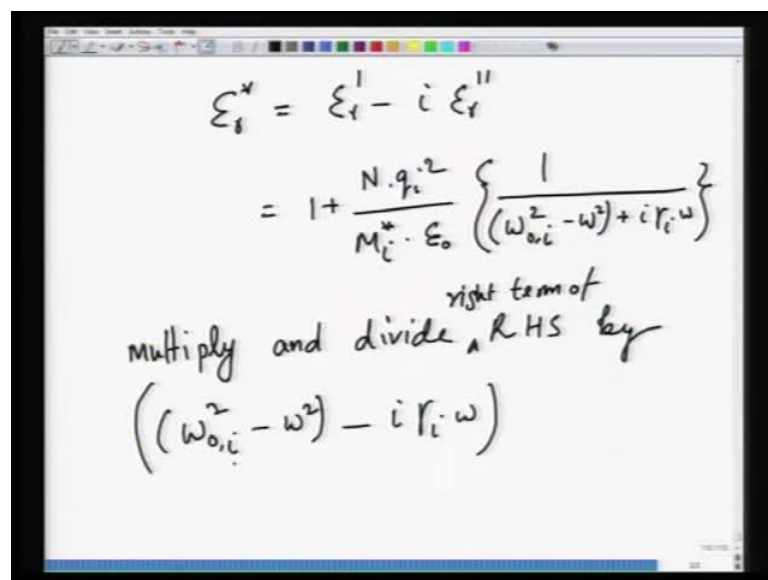
So, we know that  $\chi_i$  is equal to  $\epsilon_{r,i} - 1$  or in case of complex we write it as  $\epsilon_{r,i}^*$ . So, even in here we write this as star because this is a complex quantity. So, what you do is that now so this is equal to, this was equal to  $N q_i^2$  divided by  $M_i$  star into  $\epsilon_0$  multiplied by 1 divided by  $\omega_i^2 - \omega^2 + i \gamma_i \omega$  and if you just make this equivalent then what you are getting is  $\epsilon_{r,i} - 1$  infinity. Infinity terms basically means the static dielectric constant and this is equal to 1 plus  $N q_i^2$  divided by  $M_i$  star



epsilon naught into 1 divided by omega naught i square minus omega square plus i. So, this is the expression for your dielectric constant.

Now, so this is, this this infinity term basically implies that we are taking susceptibility as well as dielectric constant at frequencies below the resonance frequency which means they are static. So, as we looked earlier the the dielectric constant curve looks like something like this, like that. So, these are the resonance frequencies and so we are taking this range in which the dielectric dielectric constant is flattish or static. Now, from this you can say that ,you can see that epsilon r has a dependence upon the frequency. Now, the question is how can we separate the real and imaginary parts of dielectric constant?

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$$\epsilon^* = \epsilon' - i \epsilon''$$

$$= 1 + \frac{N \cdot q_i^2}{M_i^* \cdot \epsilon_0} \left\{ \frac{1}{(\omega_{0,i}^2 - \omega^2) + i \gamma_i \omega} \right\}$$

multiply and divide <sup>right term of</sup> R.H.S by

$$((\omega_{0,i}^2 - \omega^2) - i \gamma_i \omega)$$

So, we know that epsilon r star is equal to epsilon r prime minus epsilon r double prime and this is equal to 1 plus N q i square divided by M i star into epsilon naught into 1 divided by omega naught i square minus omega square plus i gamma i omega. So, what you do is that you multiply this equation. So, you multiply this right hand side by... So, multiply the right term of right hand side, so, right term of right hand side by omega naught i square minus omega square minus of i gamma i omega. So, you can see why we are doing it? We are doing it so that we are able separate the terms.

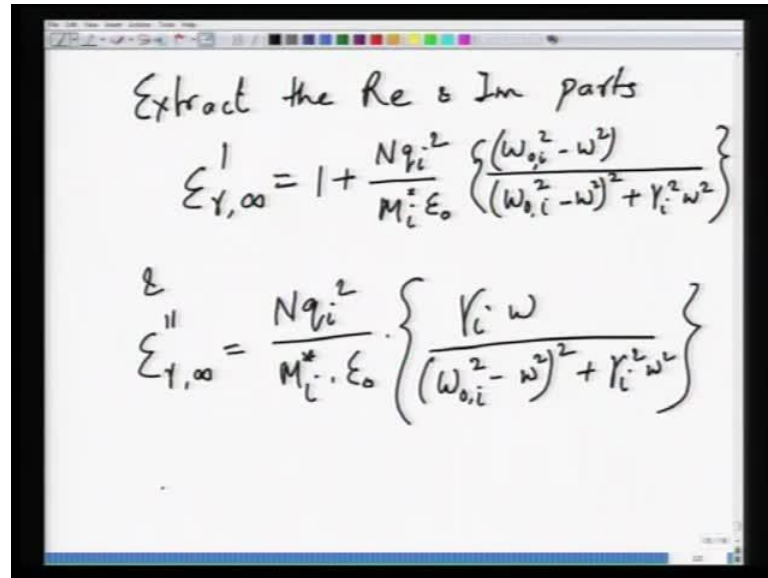
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$$\begin{aligned}\epsilon_r^* &= \epsilon_r' - i\epsilon_r'' \\ &= 1 + \frac{Nq_i^2}{M_i^* \epsilon_0} \left[ \frac{(\omega_{0,i}^2 - \omega^2) - i\gamma_i \omega}{(\omega_{0,i}^2 - \omega^2)^2 - (\gamma_i \omega)^2} \right] \\ &= 1 + \frac{Nq_i^2}{M_i^* \epsilon_0} \left[ \frac{(\omega_{0,i}^2 - \omega^2) - i\gamma_i \omega}{(\omega_{0,i}^2 - \omega^2)^2 + \gamma_i^2 \omega^2} \right]\end{aligned}$$

So, when you do that then you get epsilon r star is equal to epsilon r prime minus epsilon r double prime and this is equal to 1 plus q I, N q i square divided by M let us say M i for that the term star and now this becomes omega o i square minus omega square minus of i gamma i omega divided by what is the bottom term become? The bottom term becomes omega naught i square, minus of omega square. So, basically it is like a minus b plus a minus, a minus b multiplied by a plus b and that becomes a square minus b square. So, this is a. So, this becomes square minus of i gamma i omega square. So, this since we know that this i is equal to, so, this becomes 1 plus N q i square divided by M i star epsilon naught into omega naught i square minus omega square minus of i gamma i omega divided by omega naught i square minus omega square to the power 2 plus gamma i square omega square.

So, the life is simpler now. Now, you can see here what we have been able to do is that we have removed the i term from the bottom or the denominator and we have taken into the numerator. Now, it is easy to separate the real part and imaginary part. So, as you can see here if you compare we will go to the next page now.

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Extract the Re & Im parts

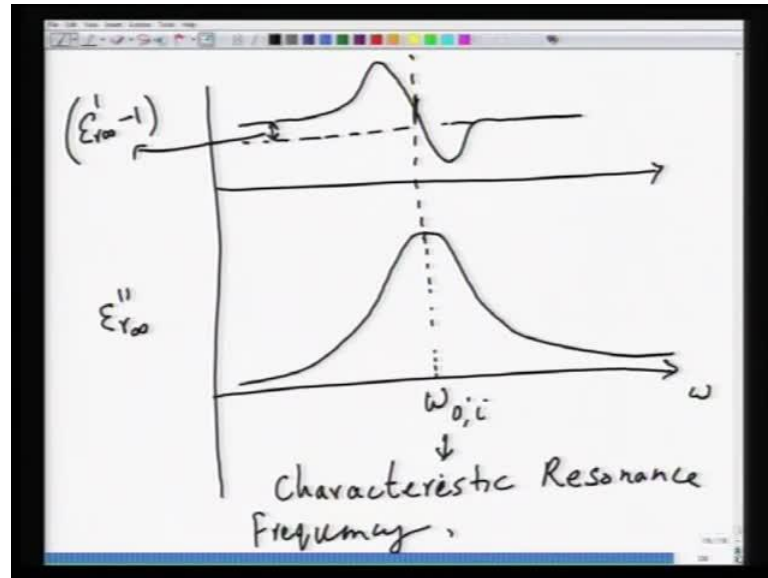
$$\epsilon'_{r,\infty} = 1 + \frac{Nq_i^2}{M_i^* \epsilon_0} \left\{ \frac{(\omega_{0,i}^2 - \omega^2)}{(\omega_{0,i}^2 - \omega^2)^2 + \gamma_i^2 \omega^2} \right\}$$

$$\epsilon''_{r,\infty} = \frac{Nq_i^2}{M_i^* \epsilon_0} \left\{ \frac{\gamma_i \omega}{(\omega_{0,i}^2 - \omega^2)^2 + \gamma_i^2 \omega^2} \right\}$$

So, if you compare extract the real and imaginary parts. So, this gives rise to epsilon r infinity which is the static, real part of a static dielectric constant as 1 plus N q i square divided by M i epsilon naught into omega naught i square minus omega square divided by omega naught i, square minus omega square to the power 2 plus gamma i square omega square. So, this is the real part of dielectric constant and you get imaginary part which is equal to N q i square divided by M i star into epsilon naught into omega naught i square sorry that should come in real part, this would be gamma i omega divided by the same thing omega naught i square minus omega square square plus (( )).

So, here we are able to separate the real and imaginary parts of the dielectric constant. Now, what we will do is that after having received these two terms for epsilon r prime and epsilon r double prime in the static region of dielectric constant that is below the resonant frequency we will plot them. So, when you plot them they look something like this.

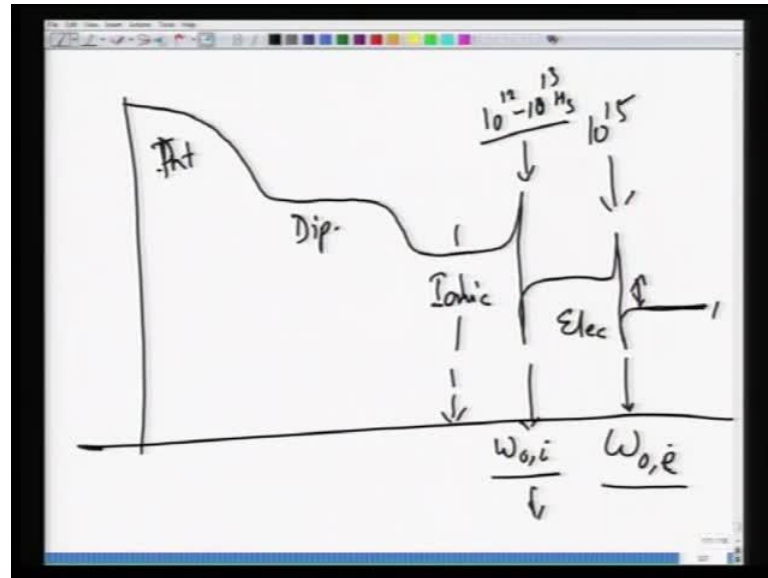
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So, we will have two lines. So, this is the frequency axis  $\omega$ . So, in the first part, so, first we will for the sake of simplicity we will first plot the  $\epsilon''$ . So,  $\epsilon''$  goes like this. The peak happening at, what is the frequency? This would be a characteristic frequency of resonance and this would be  $\omega_0$ . So,  $\omega_0$  is the characteristic resonance frequency. So, whether it is an electronic or ionic it will, the magnitude will change and corresponding to this you will have a plot for  $\epsilon'$ .

So, let us say in this free plot  $\epsilon' - 1$ . So, we take 1 on the other side. So, what happens here is so this is the difference between the dielectric constant. So, this difference would be that and this middle point where the slope changes, this would correspond to  $\omega_0$  and  $\omega_0$  is the characteristic resonance frequency. So, this is how it is going to look like.

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Now, you can very well conceive that when we plotted this diagram in the previous slides so we showed this was interface, this was dipolar and then we said this was so dipolar sorry interface, dipolar, this is ionic and this is electronic and this was 1. So, this is where the resonance occurs. So, this is omega omega naught electronic and this would be omega naught ionic. So, as you can see that omega naught ionic is smaller than omega naught electronic and that you can understand because the masses have become higher.

So, once the masses have become higher in the first case you are only treating electrons with respect to nuclei and here you, in the second case you are treating cation with respect to anion. So, as a result of this increase in mass since system has become heavier it takes, it requires more time to move and resonate and as a result the omega naught i shifts to the lower sides. So, this omega naught e typically, this is of the order of 10 to the power of 15 hertz and this is roughly 10 to the power 12 to 13 hertz typically, these are typical values they are not absolute values, but these are typical values. It will dependent upon system to system. So, what happens here is above 10 to the power 15 hertz since the frequency is are too high no charges will respond. So, as a result dielectric constant will remain 1 as if there was no dielectric present.

The moment the frequency goes below 10 to the power of 5, 15 hertz the the electron, the electronic system, the the charges at the electronic level starts to, start to respond as a result you have some contribution to the dielectric constant and the moment you reach

omega naught i then ions also start responding because the frequency has become smaller. So, as a result your ionic polarization mechanism also starts contributing. So, here when you are operating at some frequency like this both ionic and electronic mechanisms operate and this 10 to power 13 hertz as you can see what is 10 to power 13 hertz. 10 to power 13 hertz corresponds to typically the natural frequency of lattice vibration which is around 10 to the power 13 hertz.

So, so this can be related to your natural frequency vibration itself. So, and you can also see from the previous plot that when frequency becomes very high then you are in the regime of getting dielectric constant which is equal to 1. So, now this relation we have described it only for system, assuming that the local field was equal to the applied field.

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if we treat  $E_{loc}$  diff from  $E_{app}$ .

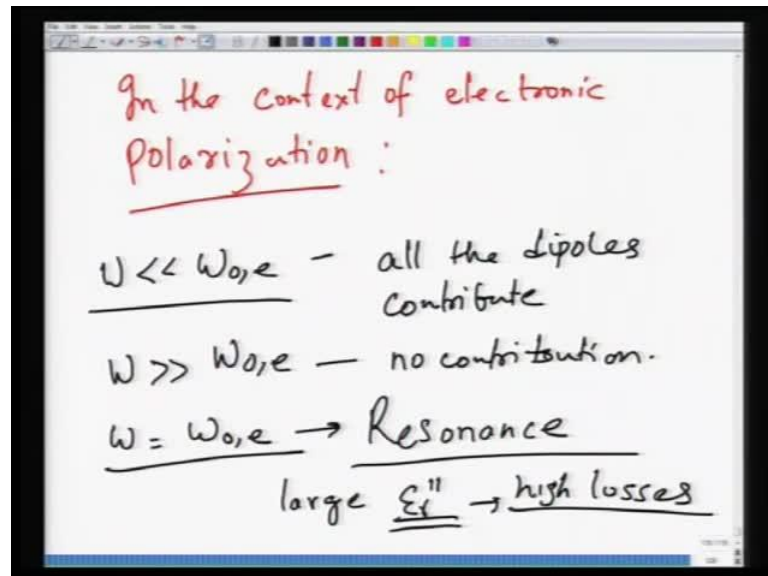
$$E_{loc} = \frac{E}{3} (\epsilon_r + 2)$$

$$\omega_{0,e}' = \sqrt{\omega_{0,e}^2 - \frac{Ne^2}{3M\epsilon_0}}$$

Now, we knew if if we treat E local different from E applied, in that case we need to take E local as equal to E by 3 epsilon r epsilon r plus 2 and if we do that then there will be a shift in the corresponding frequency. The the good thing is that you do not have you do not have you do not have to change the form of expression. All it does is, it only changes the omega naught so this shift, so, omega naught prime let us say the changed frequency because of this, because of this E local this is now omega naught i square minus N e square divided by 3 M.

So, this the dielectronic level of course, so, for the instance for the electronic level let us say  $\omega_e$ . It becomes  $\omega_e^2 - N_e^2$  by divided by  $3 M_e \epsilon_0$ . So, all you have some change in the shift in the frequency rather than any massive change. So, now you can, typically it goes to the lower side, the frequency because the field has become higher.

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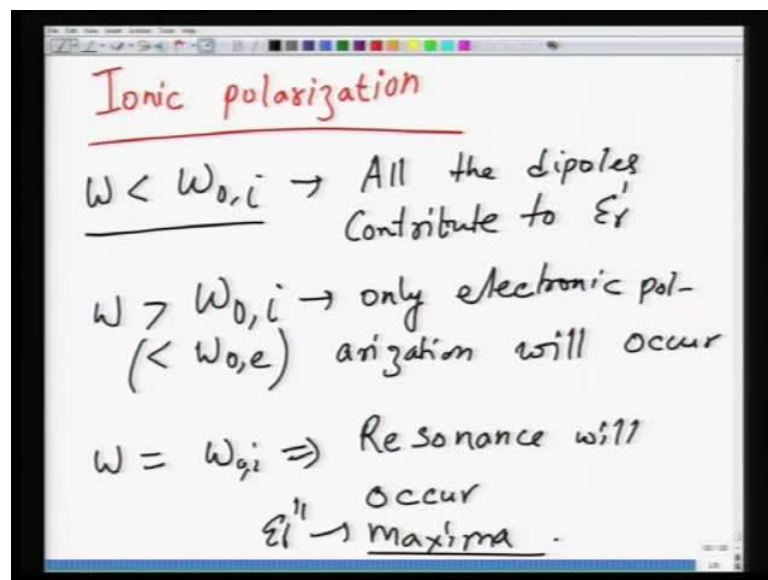


So, now what we will do is we will do is that we will we will take, we will just discuss in this context of electronic and ionic polarization mechanisms. So, first we take the, this equation in the context of electronic polarization. So, in the context of electronic polarization the first thing is when  $\omega$ ,  $\omega$  is much smaller than  $\omega_e$ . Now, if this is the scenario then applied frequency is smaller than natural frequency of vibration of this dipole, frequency of resonance let us say, resonant frequency.

So, if this is the case then the field is being applied slower than the resonant frequency as a result all the dipoles contribute and they are in parallel with the, they they basically oscillate in phase with the applied field and contribute complete dielectric constant and if you have  $\omega$  much greater than  $\omega_e$  then field is too fast, fields is, field is switching too fast than the rate at which charges can switch back and forth. As a result no contribution and when  $\omega$  is equal to  $\omega_e$  then this is equal to natural frequency of vibration of this particular system and when this happen the resonance occurs.

So, this resonance is like you can understand in terms of you know the the swing. So, when you when you throw a swing, if you whether you throw it too fast or too slow you have to always apply more force, but when the frequency matches when the resonance occurs then this swing moves effortlessly as if you you have to apply very little force. So, here what happens is that in such a context, when this happens. when the resonance happens then charges remain 90 degree out of phase with the applied field and do not contribute to any dielectric constant and we witnessed very high loss as well. So, this is characterized by large  $\epsilon''$  because of high losses due to resonance because they are resonating with the applied field.

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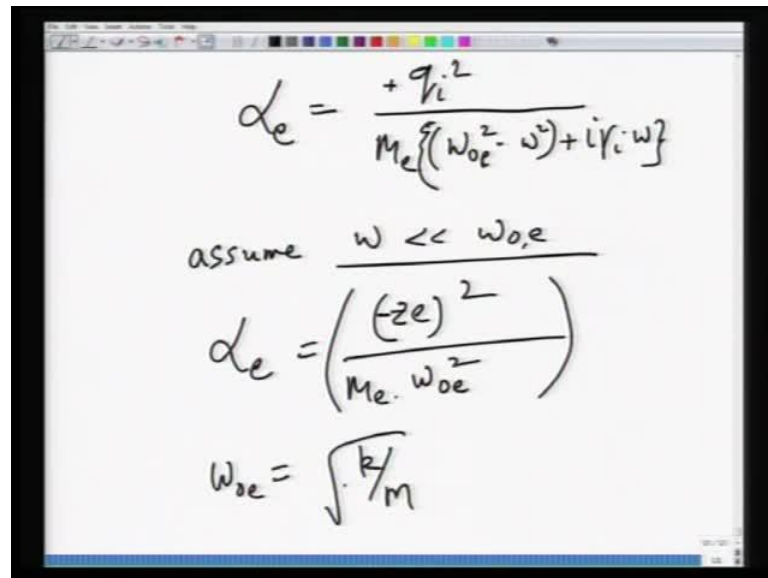
And then in the context of ionic polarization, similar things happens basically. So, when  $\omega$  is less than  $\omega_{0,i}$  then we are operating at a frequency which is smaller than the resonant frequency which means all the dipoles contribute to  $\epsilon'$  and  $\omega$  greater than  $\omega_{0,i}$ , what will happen in such a case? In such a case only electronic polarization will operate because the frequencies, so, we can say  $\omega_{0,i}$  is larger than  $\omega_{0,e}$ , but smaller than  $\omega_{0,e}$ .

So, which means only electronic polarization will occur and when again  $\omega$  is equal to  $\omega_{0,i}$  then again resonance will occur and again for the same reason charges



will be 90 degree out of phase with the applied field and this will show a maxima in  $\epsilon''$ . So, this is how it will behave.

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$$\alpha_e = \frac{+q_i^2}{m_e \{(\omega_{oe}^2 - \omega^2) + i\gamma_i \omega\}}$$

assume  $\omega \ll \omega_{oe}$

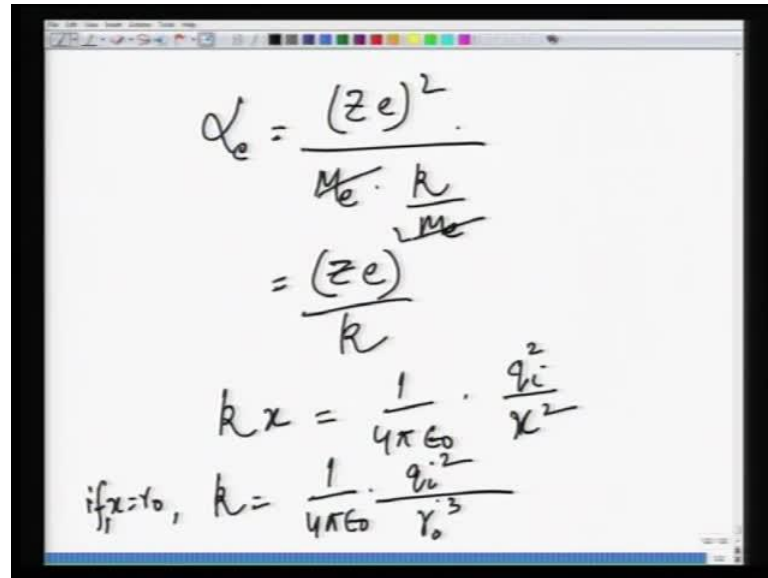
$$\alpha_e = \left( \frac{(-ze)^2}{m_e \cdot \omega_{oe}^2} \right)$$

$$\omega_{oe} = \sqrt{k/m}$$

And then finally, what you can do is that if you can convert for instance if we take the expression for electronic polarizability  $\alpha_e$  let us say, then this is equal to minus of, this is equal to  $q^2$  divided by  $M_e$  into  $\omega_{oe}^2$  minus  $\omega^2$  plus  $i\gamma\omega$ . Now, let us say, assume  $\omega$  is smaller than significantly smaller than  $\omega_{oe}$ . So, as the result  $\alpha_e$  will become and  $q$  will be equal to  $ze$ . So, minus of  $ze^2$  divided by  $M_e$  into  $\omega_{oe}^2$ .

So, now if this is the case then you can convert this expression into the expression that we got for using the DC field. Now, let us say  $\omega_{oe}$ , how can this be represented by for a spring? This is simply equal to  $\sqrt{k/M}$  where  $k$  is the spring constant and  $M$  is the mass and what you do is that you use this expression.

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$$\alpha_e = \frac{(ze)^2}{M_e \cdot \frac{k}{M_e}}$$

$$= \frac{(ze)^2}{k}$$

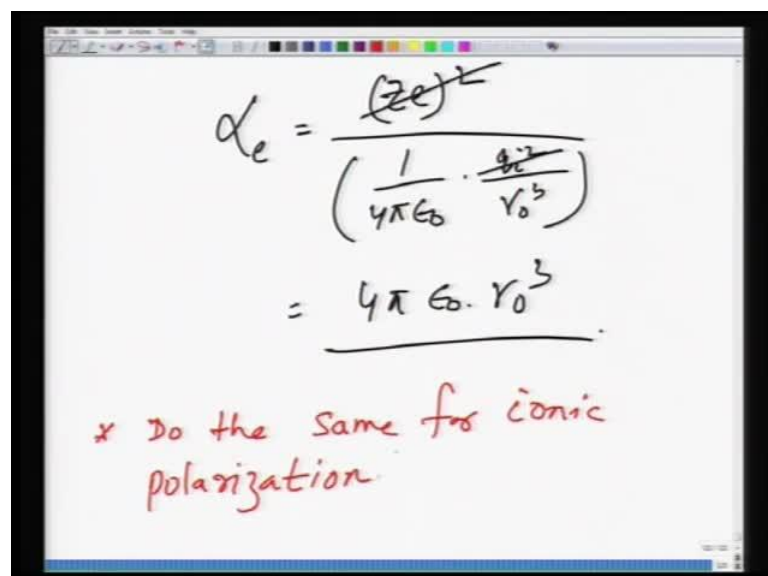
$$kx = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_i^2}{x^2}$$

$$\text{if } x = r_0, k = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_i^2}{r_0^3}$$

So, what you get is alpha e is equal to z e square divided by M e into k. So, root k by M so M e. So, this is z e square divided by k and now, what you do is that you just equate the spring force which is the k x to the Coulombic force which is 1 4 pi epsilon naught into 1 divided by q i square divided by i square or x square.

So, where x can be taken as r in this case r naught. So, basically k will become equal to, if x was r naught then k becomes equal to 1 over 4 pi epsilon naught q i square which is z e square divided by r naught cube.

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$$\alpha_e = \frac{(ze)^2}{\left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q_i^2}{r_0^3}\right)}$$

$$= 4\pi\epsilon_0 \cdot r_0^3$$

x Do the same for ionic polarization

And if you do that, if you substitute in the previous expression and  $\alpha_e$  was equal to, if you go to previous expression  $\frac{ze^2}{k}$  and if i replace the value of k which is  $\frac{1}{4\pi\epsilon_0}$  into  $\frac{q_i^2}{r^3}$  and these two cancel each other and this becomes equal to  $4\pi\epsilon_0 r^3$  and this is the same expression that we got earlier.

So, you can convert this expression which we obtained using the linear harmonic oscillator model into the same model that we operated, that we used, using the simple analysis of just a spring. So, you know this shows that both the models that we have taken into consideration, they are not in, they are they are in reasonable synchronization with each other making certain assumptions we can reach to the same conclusion from both of them.

So, so whether you get the polarizability from the force balance or by writing a equation of motion in the in the end eventually you are operating in the same fashion. The only difference is that when you do the equation of motion you are able to get a magnitude for  $\omega_i$  which is important.

So, same analysis can be done for ionic polarization. So, I will just leave it as a as a home work. So, do the same for ionic polarization and satisfy yourself whether you get the similar expression for, similar looking expression for the  $\alpha_i$  as well. So, we will we will stop here. In the next class we will take the discussion on dipolar materials or dipolar relaxation and we will we will we will look at the importance of that mechanism in the context of AC field and time.

Thank you.