

Electroceramics
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Lecture - 20

Today, we start a new lecture. Before we start the contents of this lecture, we will just review the previous lectures. So, this module is basically about dielectric materials in which we introduced essential concepts of dielectric materials like what is polarization? What is dipole moment? What is susceptibility? What is a relative dielectric constant? And then we looked into the origin of polarization. And we looked at various kinds of polarization mechanisms. And there are many essentially 4 different kinds of polarization mechanisms. You start with the most, the fastest one which happens at the smallest scale, tiniest scale at the atomic scale which is called as electronic and ionic polarization, electronic polarization or atomic polarization. And then we come to ionic polarization which happens in the solids which have cations and anions put together. And what how the polarization happens is basically when you apply electric field, these the centres of negative and positive charges displace with respect to each other.

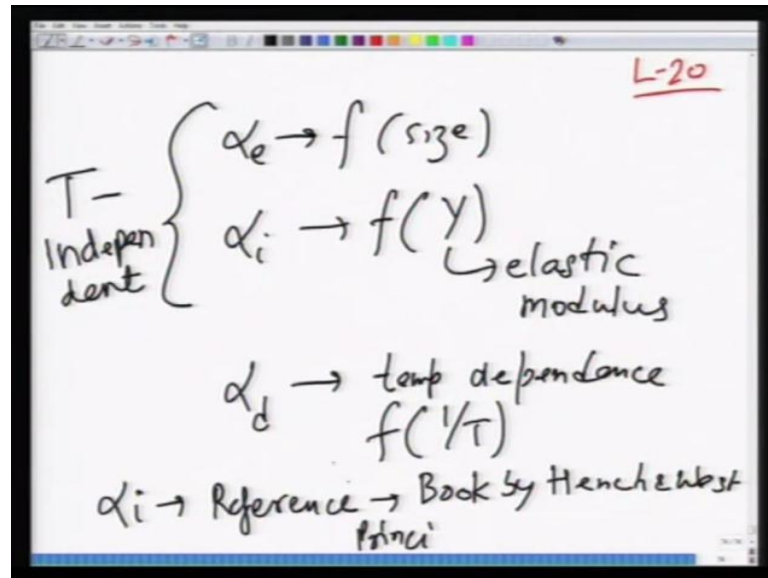
So, in a naturally in an ionic solid, you will have electronic component as well because all the solids contain atoms. So, atomic or electronic polarization is fundamental to all of them, and then ionic solid on top of that that you will get ionic polarization contribution which is due to displacement of ions with respect to each other. And as a result you have net displacement in one direction as compared to another, so as a result you have dipole moment. The third mechanism is a dipolar or polarization, dipolar or orientational polarization which is because of rotation of dipoles typically in polar solids.

So, substances like water or barium titanate or any other substances which has permanent dipolar moment gives rise to dipolar polarization. So, molecules tend to rotate along the direction of applied field to align the dipole moment in the direction of applied field. And all these, and fourth mechanism was interface polarization which could be because of presence of various defects and electrodes.

Now, we took them sort of analytical treatment of electronic, ionic, and dipolar polarization. And there we calculated the polarizability of all these 3 mechanisms. And we find that the polarizability of electronic and ionic polarization mechanism is

dependent upon parameters like, so electronic for instance electronic polarizability, α_e is function of size. So, and in the other case α_i was function of γ which is the elastic modulus.

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So, for instance α_e which is the electronic polarizability is the function of size as it went as there was a dependence on the radius of the atom. And typically larger atoms would give rise to larger polarization. So, as a result anions typically have higher polarizability as compared to cations and we looked at certain examples.

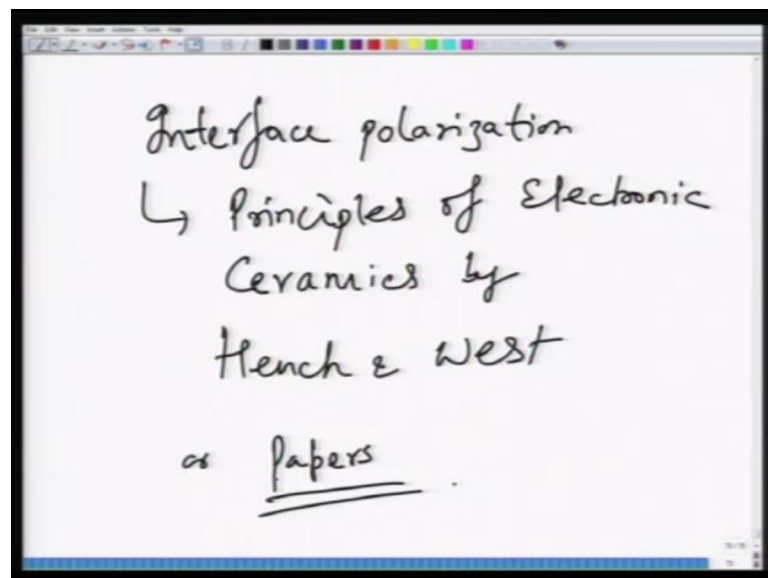
Then we looked at α_i which was ionic interface, ionic polarization which dependent upon parameters like γ which is the elastic modulus. And what it suggested was higher the elastic modulus, smaller the ionic polarization is that makes sense, because higher the elastic modulus which is because of higher bond strength. And so typically solids with higher modulus which is because of higher body strength will tend to have a smaller ionic polarizability, and that makes sense. And then finally, we looked at α_d which is the dipolar and that was that had temperature dependence.

These two were temperature independent, and this was inversely proportional to temperature. And what they suggested was as the temperature increases the polarizability decreases or and vice versa. So, this also makes sense because as a temperature increases the tendency of dipoles to align along the applied field will be reduced because

of higher thermal forces. And so either you increase the either increase the magnitude of field or you decrease the temperature in order to increase the tendency of alignment.

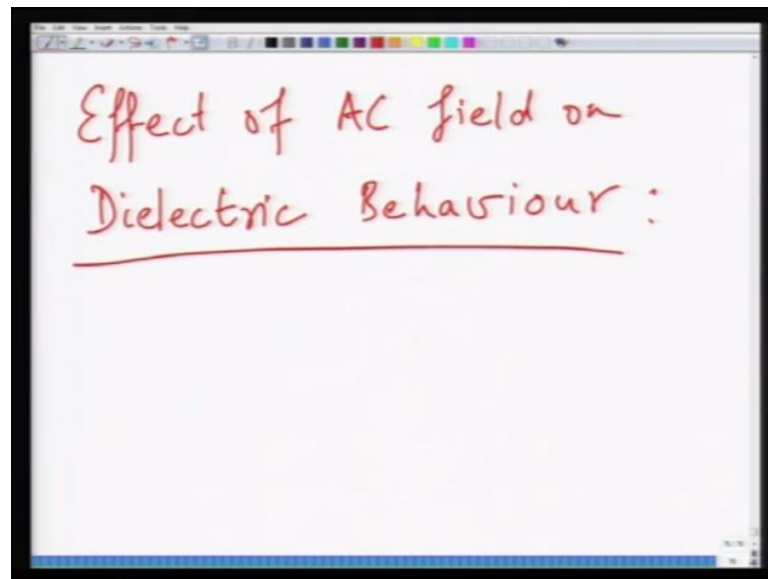
So, these were the 3 polarizabilities that we looked upon in terms of analytical treatment. We did not go into details of interface polarization that is the analysis of that is slightly out of scope of this course. However, if there, if you are interested in going into treatment of this I can suggest some references, you can look at. So, for α_i , you can a reference would be the book by Hench and west which is principles of or i will just write in another slide.

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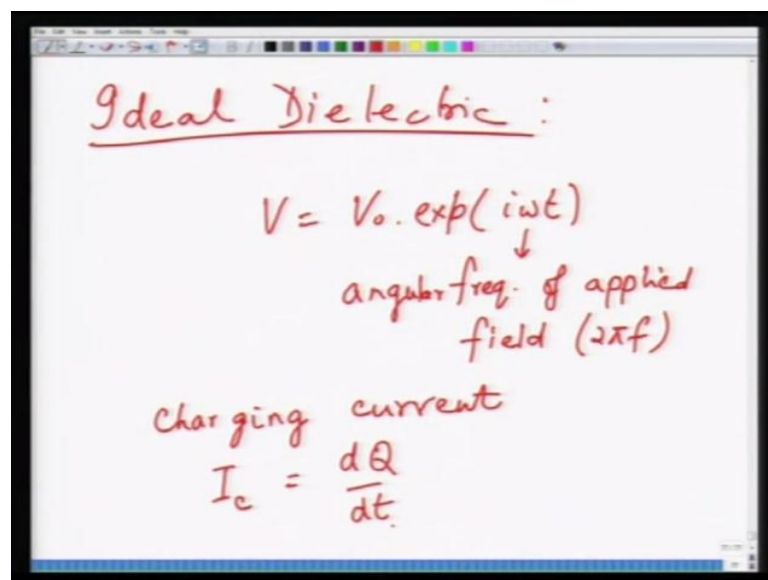
So, in this book you can find a reasonably analytical treatment or you can look at papers in various journals. So now, this establishes. So now, you understand the importance of polarizability, and how it is dependent upon physical parameters as well as temperature. Now, so far we have not considered any analytical treatment on the effect of frequency on the dielectric. So, what we will do now is, we will look at the behaviour of dielectric materials under the influence of alternating field that is the AC field.

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So, what we will do now is we look at effect of. This will be the theme of next this lecture as well as may be part of next lecture. So, in this context we will first look at that behaviour of ideal dielectric.

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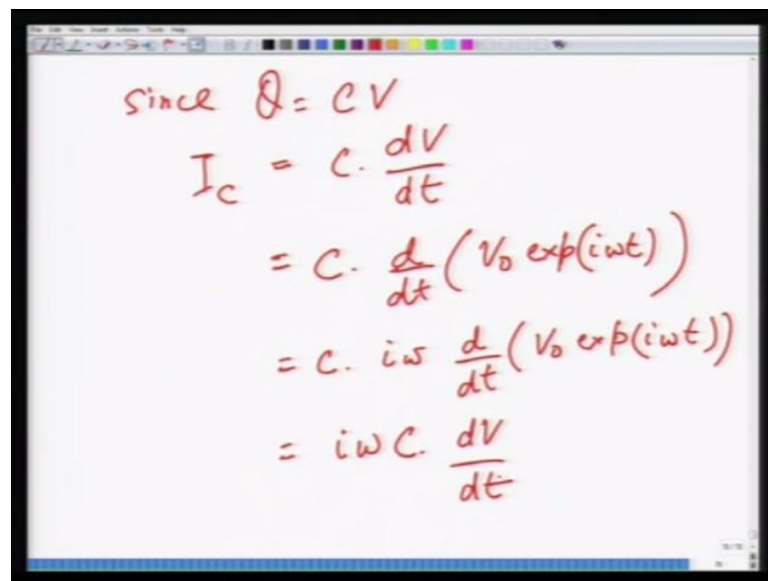


This is important because in most of the applications dielectric materials are used under the influence of AC field. As a result we need to understand how they behave what are various parameters which are influenced by application of AC field that is for instance AC frequency. And at what frequency we should be able to operate the material etcetera.

So, while most of the discussion that we did in past few lectures was basically on a DC or a static field. Here we will take up AC field effect. So, let us say we apply a sinusoidal field. And this field is given as a voltage is given as $V = V_0 \exp(i\omega t)$, where ω is the frequency of angular frequency of applied field basically $2\pi f$, you can write angular frequency.

Now, the moment you apply this sinusoidally varying applied electric field that this V is equal to $V_0 \exp(i\omega t)$. This gives rise to development of charging current. So this charging current I_c . Now, why does this develop, this develops because as you apply a voltage to a dielectric there is a change in charge as a function of time. And this change in charge as a function of time gives rise to what is called a charging current and this I_c is given as dQ/dt . So rate of change of charge per unit with respect to time gives you charging current. Now, what is Q ?

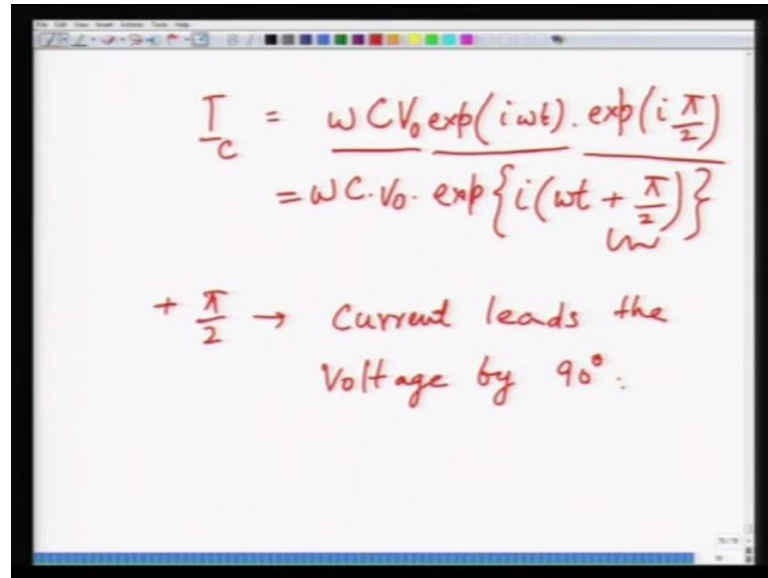
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$$\begin{aligned}
 \text{since } Q &= CV \\
 I_c &= C \cdot \frac{dV}{dt} \\
 &= C \cdot \frac{d}{dt} (V_0 \exp(i\omega t)) \\
 &= C \cdot i\omega \frac{d}{dt} (V_0 \exp(i\omega t)) \\
 &= i\omega C \cdot \frac{dV}{dt}
 \end{aligned}$$

We know that q is equal to $c v$. So as since q is equal to $c v$ I_c can be written as c into $d v$ by $d t$ and that makes sense, because c is a constant which is a capacitance of the material. And so this $d v$ by $d t$ $V_0 \exp(i\omega t)$, and if you differentiate this will become c into $i\omega$ $d v$ by $d t$ $V_0 \exp(i\omega t)$. So, this will become $i\omega c$ into $d v$ by $d t$ all right so because $V_0 \exp(i\omega t)$ will make v .

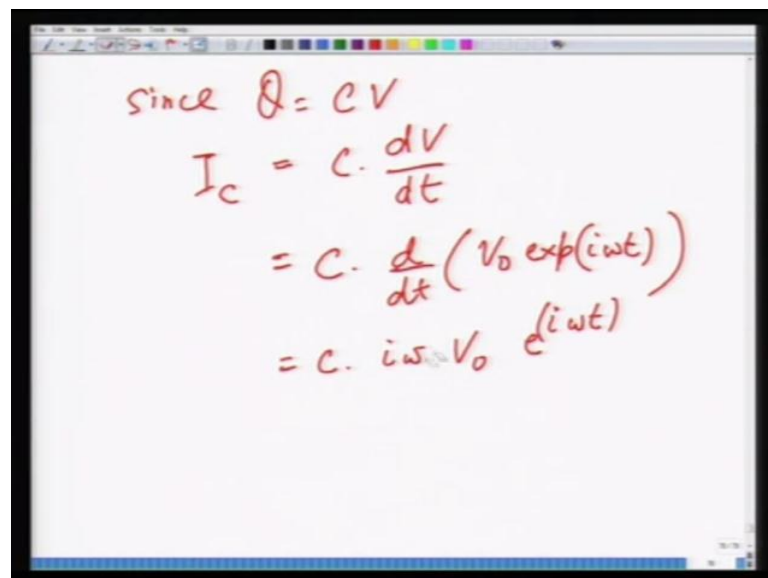
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The image shows a handwritten derivation on a whiteboard. The first line is
$$\frac{I_c}{C} = \frac{\omega C V_0 \exp(i\omega t) \cdot \exp(i\frac{\pi}{2})}{\omega}$$
. The second line is
$$= \omega C \cdot V_0 \cdot \exp\left\{i\left(\omega t + \frac{\pi}{2}\right)\right\}$$
. Below this, it says $+ \frac{\pi}{2} \rightarrow$ Current leads the Voltage by 90° .

So, I_c can further be written as. So I_c will be equal to ωC into exponential of $i\omega t$ into exponential of $i\pi/2$. And this you can see from the previous expression ωC into V_0 hang on there was some.

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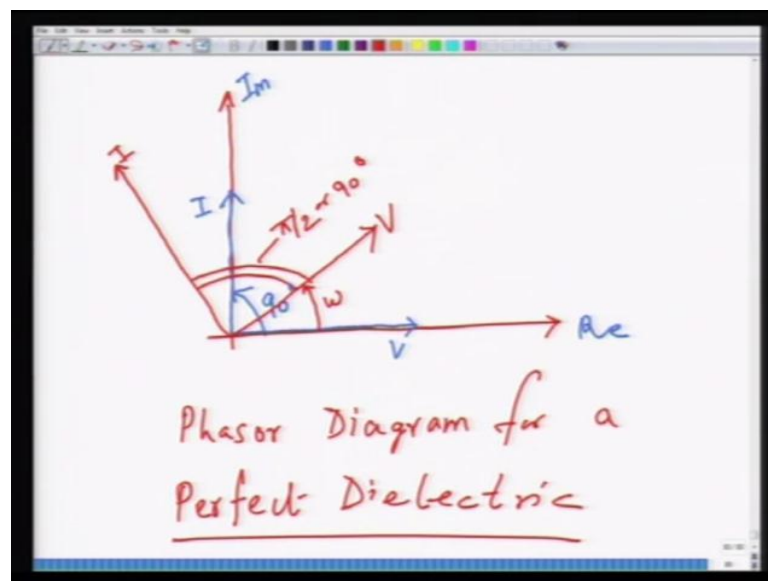
The image shows a handwritten derivation on a whiteboard. The first line is
$$\text{since } Q = CV$$
. The second line is
$$I_c = C \cdot \frac{dV}{dt}$$
. The third line is
$$= C \cdot \frac{d}{dt} (V_0 \exp(i\omega t))$$
. The fourth line is
$$= C \cdot i\omega V_0 e^{i\omega t}$$
.

There was some mistake here. So, $i\omega$ comes out sorry this last line is. So, this $i\omega$ comes out and V_0 comes out. So, what you have here is exponential $i\omega t$, e to the power $i\omega t$, V_0 e to the power $i\omega t$, we will just leave it as it is. So, $i\omega C$ into V_0 exponential $i\omega t$. So, I_c can write this as that

particular equation as ωc , $\omega c v_0 \exp(i\omega t)$, and I can multiply by another factor which is $\exp(i\pi/2)$ which is nothing but $\cos \pi/2 + i \sin \pi/2$, and that is nothing but one. And this will be written as, and this can be further written as $\omega c v_0 \exp(i\omega t + i\pi/2)$.

So if I just rub this out into $\exp(i\omega t + i\pi/2)$. What this means? This means that the charging current which is developed due to rate of change of charge versus time, gives rise to a current that leads the voltage by $\pi/2$. So, the charging current so this $\pi/2$ term means that current leads the voltage by 90 degrees. So, how do you draw, and this is this we are considering in a perfect dielectric which means there are no losses in the dielectric. So, this you can draw this you can express in terms of phasor diagram.

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So, if you have, let us say these are. So, if this is your v then I will be led by 90 degrees in this direction. So, this will be I , and this will be v . So, on a real and imaginary plot if you draw I versus v that is how it would look like. So, this would be for instance R , R is real axis, this would be imaginary axis. So, no matter how you take it v will, the, I will always. So, if you take for instance if I change the angle of v , if v was like this and I would always be this would be. So, this is some angle ωt , and this would be $\pi/2$ or 90 degrees. So, on a real and imaginary plot v and i relationship would appear something like that. So, this is the phasor diagram for a perfect dielectric.

Now, what we will do is that we look at the behaviour of real dielectrics. Now, real dielectrics hardly follow this 90 degree relationship what so. But before we do that we also would like to look at what is the power which is dissipated in a ideal dielectric. Now, what would be the power dissipated?

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The image shows a handwritten derivation on a whiteboard titled "Power Dissipation". The derivation calculates the average power P_{av} over one time period T . It starts with the formula $P_{av} = \frac{1}{T} \int_0^T I_c V dt$. The current I_c is given as $i\omega C V_0 \exp(i\omega t)$ and the voltage V is given as $V_0 \exp(i\omega t)$. The integral of the product of these two expressions over one period $T = \frac{2\pi}{\omega}$ is shown to be zero. A small sketch of a sine wave is also present on the left side of the derivation.

$$\begin{aligned}
 P_{av} &= \frac{1}{T} \int_0^T I_c V dt \quad T = \text{time period} = \frac{2\pi}{\omega} \\
 &= \frac{1}{T} \int_0^T \underbrace{I_c}_{i\omega C V_0 \exp(i\omega t)} \underbrace{V}_{V_0 \exp(i\omega t)} dt \\
 &= 0
 \end{aligned}$$

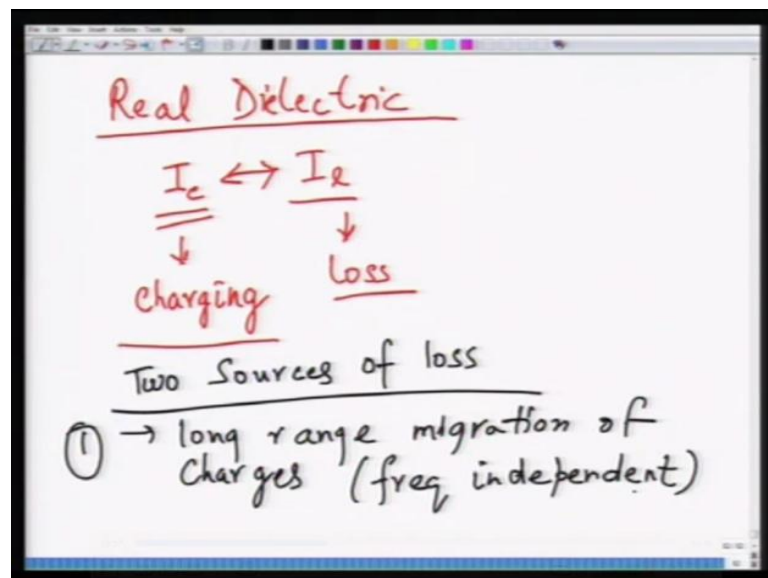
So, just now, looking at power dissipation just a small exercise before, we move to real dielectric. So, power dissipated, let us say the average power is p average. This p average can be written as over the time period τ . So, τ is equal to time period, and this can be written as 2π over ω . So, if we average the power that is dissipated over this time period, this can be written as 0 to τ , I_c into V divided by τ dt . And so this can be written as 1 over τ since, τ is a constant 0 over τ $I_c V dt$.

Now, since the voltage, since the current leads the voltage by 90 degrees in a ideal dielectric p average has to be equal to 0. So, you can put the expression for I_c which is I_c is equal to $i\omega C$ into $V_0 \exp(i\omega t)$. And v will be $v_0 \exp(i\omega t)$. And if you do the multiplication, and do the integral, you will find out that the net average the average power which is dissipated in an ideal dielectric is equal to 0. So, what it means is that, that when you apply during the cycle, let us say so during this cycle. So, A C field is applied like this. So, during this cycle the capacitor charges completely, and discharges completely without any dissipation of without any loss of charge. So, this is like a perfectly oscillating, perfectly oscillating oscillator under

the for example, on a perfect spring under gravity. So, without so you have a spring, you attach a mass to it, and it perfectly oscillates without any dumping or losses.

So, in this case what the analogy is that when you apply this voltage v , oscillating field v is equal to $v \sin \omega t$ to the dielectric in the first cycle. Let us say charges completely in the next cycle, it discharges completely, in the next half cycle it discharges completely. So, charging and discharging of this net charging, and discharging of dielectric does not lead to any dissipation of charge. As a result the net power dissipation is equal to 0. So, this is what, this is about perfect dielectric.

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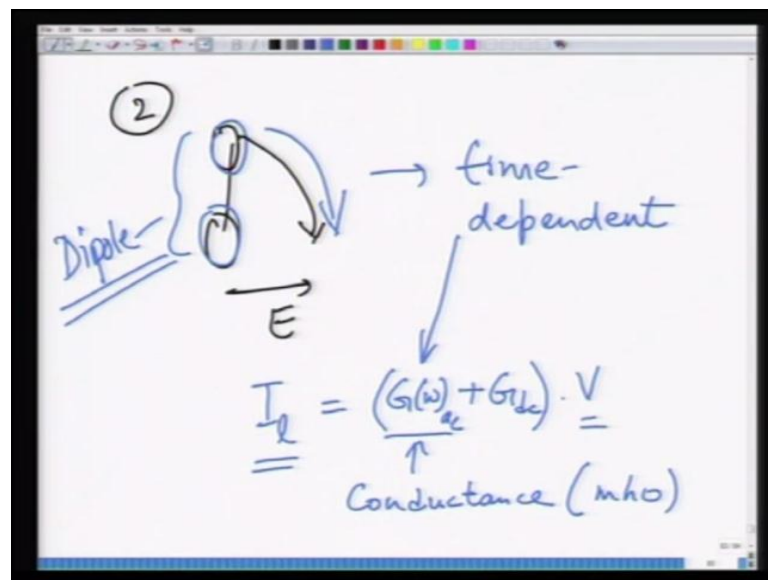


Now, the case is slightly different in case of real dielectrics. Or rather quite different, in case of real dielectrics, what happens is that this charging current which is I_c is also the, so for this charging current I_c there is an accompanying current which is called as I_l . So, this is your charging current, and then you have accompanying it is called as loss current. So, which means in real dielectric there are some finite losses of charge in the system, and which would happen because of variety of reasons. And whatever be the reasons are the current associated with them is called as loss current. And this is and there are typically two sources.

So, two sources of loss and this loss first it could be long range movement of migration of charges. So, this is number 1 which means just like a normal material when you apply

voltage there is something like ohmic conduction that is taking place which means a long charges are able to move. So, in for a long distance, unlike in polarization where I told you that in normal dielectric charges are supposed to move only a tiny distance. So, that to so that they give rise to what is called as polarization. But when you have a real dielectric not only you have that polarization taking place, but also you have what is called as long range migration of charges or ohmic condition which gives rise to which gives rise to loss, and this is typically frequency independent. So, like a D C loss. So, this is typically frequency independent.

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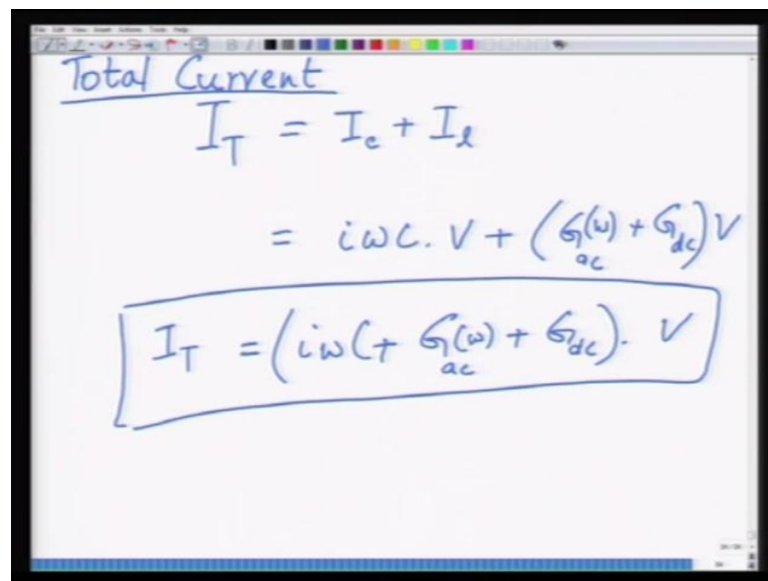
In the second term is now, when you have these dipoles which they have which have to rotate in the direction of light field. So, you have the dipoles like this, and if they have to rotate in the direction of applied field, let us say this is e so this is I will use different colours. So, this is the dipole, and when you applied field E , it has to rotate in this direction, and this rotation is resisted by what is called as inertia. And this inertia has to be there in a system if the entities involve some sort of mass. And the moment mass comes into picture you have a inertia, and this inertia opposes the rotation of these dipoles in the direction of applied field, and this is typically time dependent loss.

So, some sort of loss of energy which happens as a result of resistance to the movement of this dipole in the direction of applied field. And so there are two components one is the d c component which is at frequency independent you have another component

which is frequency dependent or time dependent, and this is a c component. So, this i can be expressed as so something multiplied by V . And this i when you so how can you we can we can calculate i something multiplied by V , and that something is called as G_{ω} a c which is the, a c part, the time dependent part. And then you have G_{dc} , and what is G ? G is the inverse of resistance or conductance in the simplest terms, because we know from ohm's law v is equal to $i r$.

So, if you so if so i would be equal to 1 over r into v , and sought of one over r is called as conductance whose units are in mho. So, you have these two components of conductance one is the, a c component of conductance which is the time dependant part which is because of inertial a resistance to the movement of dipoles to the direction of applied field. And second is the d c component which is because of long range movement of charges.

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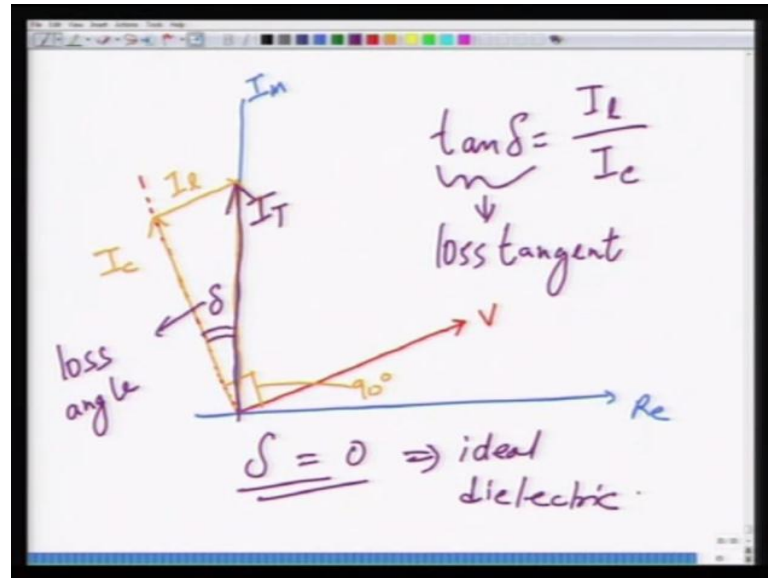


The image shows a whiteboard with handwritten equations for Total Current. The title "Total Current" is underlined. Below it, the equation $I_T = I_c + I_l$ is written. This is followed by $= i\omega C \cdot V + (G_{\omega} + G_{dc})V$. The final equation, $I_T = (i\omega C + G_{\omega} + G_{dc}) \cdot V$, is enclosed in a hand-drawn rectangular box.

$$\begin{aligned} \text{Total Current} \\ I_T &= I_c + I_l \\ &= i\omega C \cdot V + (G_{\omega} + G_{dc})V \\ \boxed{I_T &= (i\omega C + G_{\omega} + G_{dc}) \cdot V} \end{aligned}$$

So, the total current I_T total current, I_T can be written as I_c plus I_l . I know I_c is $i\omega C$ into V plus I know this is G , G_{ω} a c plus G_{dc} into v . So, this is $i\omega C$ plus G_{ω} a c plus G_{dc} into V . So, this is the expression for total current for a real dielectric. And what it means in terms of phasor diagram is.

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So, in it was a phasor diagram. So, if this was let us say V , for a perfect dielectric I would expect this I to be developed. So, perfect dielectric will have I . Now, since we are saying that we do not have a perfect dielectric, what is this component? This component will be I_c which is a charging current. And you have so in the perfect dielectric this would be 90° degrees. But my overall current is not made up of I_c it is smaller than I_c . So, actually the charging current is not equal to I_c in a real dielectric it is smaller, total current is made up of I_c and another component which is called as I_l . So, this I_l is in phase with the applied field. So, this is your, I_l and this would be your, I_t .

So, this is your, I_t for a real dielectric. And now, you can see you have the charging current which is at 90° degree to the applied voltage. And then you have I_l which is in loss current which is in phase with the applied field or applied voltage. And this angle made between I_t and I_c is called as δ and this is called as loss angle. So, you can see that δ . So, you can represent δ as $\tan \delta$ which is called as loss tangent, this is equal to I_l divided by I_c . So, what it shows basically is larger the loss current is larger your $\tan \delta$ would be which means, total current would move to closer and closer to the voltage. So, this is the deviation which happens in a real dielectric as compared to the ideal dielectric.

The total current in the real dielectric is I_c only so as a result the angle is 90° degrees whereas, in total current in a ideal dielectric, I am total current in the ideal dielectric is I

c which is which leads the voltage by 90 degrees whereas, total current in the real dielectric is I_t which is made up of I_c which anyway leads to the voltage by 90 degrees. But also I_l which is parallel to v , and as a result this gives rise to δ , and this is defined by a quantity which is called as delta or loss angle.

So, the total current makes an angle δ to the, I_c in the direction of ω . And in the direction of V , sorry voltage and this δ is called as loss angle. And \tan of this δ is nothing but I_l by I_c as you can see from this. So, higher the, I_l is higher $\tan \delta$ would be which means worse which means more the losses would be in a dielectric. So, higher $\tan \delta$ means more loss in the dielectric is. So, basically δ is equal to 0 would mean an ideal dielectric, and higher the δ is more loss in the dielectric is. So, when field is a static.

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When field is static
 ie. $\omega = 0$

$$I_{\text{Total}} = I_{\text{loss}}$$

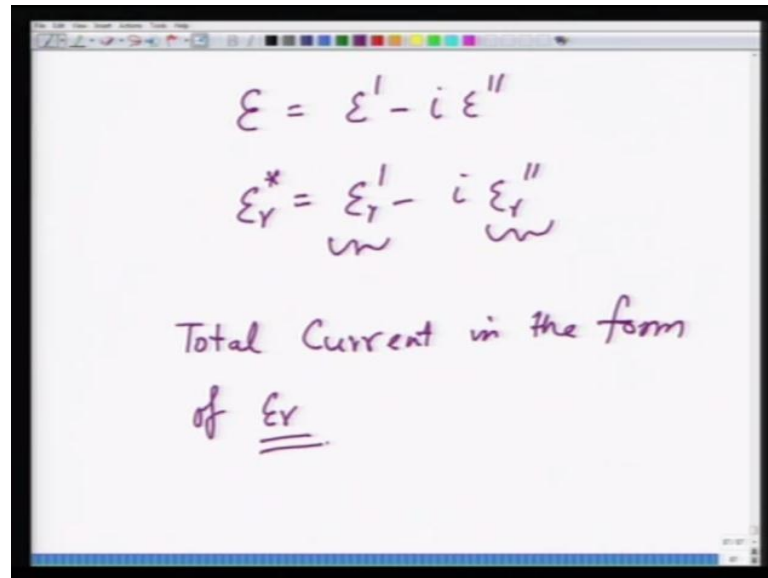
$$I_T = G_{dc} \cdot V$$

$$G_{dc} = \frac{1}{R} \rightarrow \left. \begin{array}{l} \text{ohmic} \\ \text{resistance} \end{array} \right\}$$

Let us say static field would mean that ω is equal to 0. And when ω is equal to 0, I_{total} will be equal to I_{loss} multiplied by V . And this you can see because if you go to previous expression I_t was equal to $I_{\omega c}$, a c component & $d c$ component into V so a c and $d c$ components of conductances. So, if you make ω is equal to 0 both of these ωc , a c components disappear as a result I becomes equal to I_{total} becomes equal to I_{loss} multiplied by V . And this so I_t is equal to G_{dc} into, sorry what did I. So, as a result I_{total} will be equal to I_{loss} only, and I which means I_t will be equal to G_{dc} multiplied by V .

So, G & C will be equal to nothing but $1/R$ and this is your ohmic resistance. So, another way to express these real dielectrics is to use to depict this charging in loss current is by using permittivity as a complex quantity. So, we can also express.

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$$\epsilon = \epsilon' - i\epsilon''$$

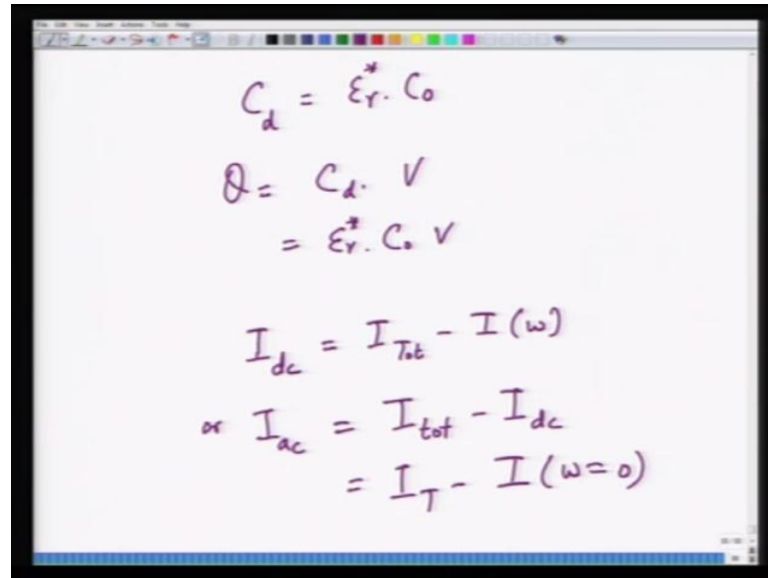
$$\epsilon^* = \epsilon' + i\epsilon''$$

Total Current in the form of ϵ_r

So, this epsilon which is a permittivity that can be expressed as epsilon prime minus of i into epsilon double prime. So, what it means is that basically given the fact that you have now two components of current, one is the charging current; one is the loss current or discharging current as a result. So, basically the current has both d.c. part frequency dependent part as well as frequency independent part, given the fact that that happens the permittivity of the material has to be, has to have a real part as well as the imaginary part. So, this can be written as ϵ_r^* is equal to ϵ_r' minus i epsilon ϵ_r'' .

So, this will be called as your real part of dielectric constant, and this will be called as imaginary part of dielectric constant. And this represents the charging as well as loss currents accurately. So, idea behind doing this is so that we can express the total current in a dielectric in terms of a single parameters, and that is in the form of epsilon ϵ_r . So, one single term takes care of the overall current in the dielectric material. So, we can write now.

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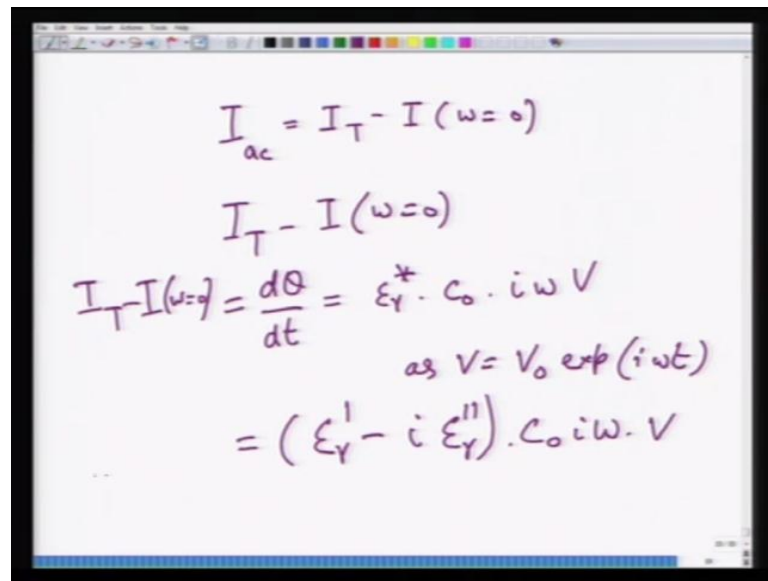
The image shows a whiteboard with handwritten equations in purple ink. The equations are:

$$C_d = \epsilon_r^* \cdot C_0$$
$$Q = C_d \cdot V$$
$$= \epsilon_r^* \cdot C_0 \cdot V$$
$$I_{dc} = I_{tot} - I(\omega)$$
$$\text{or } I_{ac} = I_{tot} - I_{dc}$$
$$= I_T - I(\omega=0)$$

The capacitance C dielectric would be equal to epsilon r into C_0 which is the capacitance of the vacuum capacitance. And we can write Q as we know is equal to C_d multiplied by V . So, this will be epsilon r . So, this epsilon r will be epsilon r star since, we have charging in loss current. And assuming that we are taking that dielectric constant is a complex quantity having a real and imaginary part. So, we can write Q in this manner which will be equal to epsilon r C_0 into V .

Now, we can express now, we again get back to the current equation. And total current we can write as or I let say the $d c$ current, I_{dc} will be equal to I_{total} minus of I of frequency dependent part. So, what about the frequency dependent part of current is since, we have so. What we have done is instead of taking charging and discharging current separately, we have taken $d c$ and $a c$ component. And this will be equal to or alternatively, you can write I_{ac} is equal to I_{total} minus I_{dc} . And what would I_{dc} mean so I_t minus I_{ω} is equal to 0, this would be the current.

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The image shows a whiteboard with handwritten mathematical equations. The equations are as follows:

$$I_{ac} = I_T - I(\omega=0)$$

$$I_T - I(\omega=0)$$

$$I_T - I(\omega) = \frac{dQ}{dt} = \epsilon_r^* \cdot c_0 \cdot i\omega V$$

as $V = V_0 \exp(i\omega t)$

$$= (\epsilon_r' - i\epsilon_r'') \cdot c_0 i\omega \cdot V$$

So, I_{ac} is I_T minus I of ω is equal to 0, and this is nothing but your. So, this, we take this equation as it is at the moment. And we write this I_T minus I of ω is equal to 0 as what can this be? This could be written as dQ by dt . And this is so from the previous expression, we know that Q is equal to $\epsilon_r^* \cdot c_0 \cdot V$. So, if we plug this definition of Q in this equation. So, this I_T minus I of ω is equal to 0 is dQ by dt , and this becomes $\epsilon_r^* \cdot c_0 \cdot i\omega V$ because V , as V is equal to $V_0 \exp(i\omega t)$. So, that is stays the same as we took earlier. Now, what you can do is that you can write this ϵ_r^* as $\epsilon_r' - i\epsilon_r''$ multiplied by $c_0 i\omega V$. So I_{ac} total will become.

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$$\begin{aligned}
 \underline{I_T} &= \underbrace{i\omega \cdot \epsilon_0 \epsilon_r' V}_{\text{Out of phase } I_c} \\
 &+ \underbrace{\omega \epsilon_r'' \cdot \epsilon_0 \cdot V}_{\text{in phase } AC(I_l)} \\
 &+ \underbrace{G_{dc} \cdot V}_{\text{dc current } I(\omega=0)} \\
 \underline{I_T} &= \underline{f(\epsilon_r^*)}
 \end{aligned}$$

So, I total will become now, $i\omega \epsilon_0 \epsilon_r' V$ plus $\omega \epsilon_0 \epsilon_r'' V$ plus I have taken the d c component on right side that will be $G_{dc} V$. So, this is your, I ω is equal to 0 basically the d c current. Now, what we have done is we have represented this total current as a function of ϵ_r^* . So, a single quantity gives rise to the total expression for current. And this single quantity can also be measured experimentally. So, the first term so which is the in phase current, and which is the outer phase current. The first term is out of phase charging current term. So, this is out of phase charging current, because this is you know imaginary part is there $i\omega$ is there. And then this would be the in phase a c loss current I_l , and this would be in phase I_{dc} . So, these are different components of current that you can write in terms of dielectric constant.

So, by invoking this relation, that the dielectric constant of a real dielectric material also has a real and imaginary part. And if you so this helps you to write the total current expression in terms of that quantity. So, if you compare some of these equations earlier specifically the total current expression. If you compare the total current expression so we have two expressions.

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$$I_T = (i\omega C_0 + G_{ac}(\omega) + G_{dc})V$$

$$I_T = \underbrace{(i\omega C_0 \epsilon_r' + \omega \epsilon_r'' C_0)}_{I_{ch}} + \underbrace{G_{dc}}_{I_{loss}} V$$

$$\Rightarrow G_{ac}(\omega) = \omega \epsilon_r'' C_0$$

$$\tan \delta = \frac{I_{loss}}{I_{charging}} = \frac{I_L}{I_C}$$

The first one is, first one, we developed as I_T was equal to, if you go if you go back earlier, we wrote it as I_T is equal to just a second, $i\omega C_0$ plus G_{ac} plus G_{dc} into V . This is first expression we had. And the second expression now, we have is I_T is equal to $i\omega C_0 \epsilon_r'$ plus $\omega \epsilon_r'' C_0$ plus G_{dc} into V . So, if I just take V out plus $\omega \epsilon_r'' C_0$ plus G_{dc} into V .

So, here I get G_{ac} as equal to $\omega \epsilon_r'' C_0$. And this by definition gives you C_0 is equals to $C_0 \epsilon_r'$ which is true. And G_{dc} is nothing but G_{dc} so. And from this you can also calculate what is $\tan \delta$, $\tan \delta$ as we know is equal to I_{loss} divided by $I_{charging}$ which is I_L divided by I_C . So, what is this written as so $\tan \delta$, you can now mention as so what is I_{loss} from the previous expression? This would be your, I_{loss} , and this would be I_{ch} .

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$$\tan \delta = \frac{G_{dc} + \omega \epsilon_r'' C_0}{\omega \epsilon_r' C_0}$$

if $G_{dc} \ll \omega \epsilon_r'' C_0$

$$\tan \delta = \frac{\cancel{\omega} \epsilon_r'' \cancel{C_0}}{\cancel{\omega} \epsilon_r' \cancel{C_0}} = \frac{\epsilon_r''}{\epsilon_r'}$$

$\epsilon_r'' \rightarrow \epsilon_r' \approx \epsilon_r'' \approx \epsilon_r' \cdot \tan \delta$

So, this would be G_{dc} plus $\omega \epsilon_r'' C_0$, if you go back here $\omega \epsilon_r'' C_0$ divided by $\omega \epsilon_r' C_0$. So, $\tan \delta$ is G_{dc} plus $\omega \epsilon_r'' C_0$ divided by $\omega \epsilon_r' C_0$.

So, now, this looks like a much more complicated expression than what we wrote earlier, simply I_{loss} divided by I_C . However, if we assume that G_{dc} is much more smaller as compared to $\omega \epsilon_r'' C_0$ which is not a unreasonable expression assumption, because G_{dc} for most of the dielectric materials which are reasonably insulating can be very small. So, if that is true. And this is nothing but a conductance. So, they are not really conducting, they are insulating materials. So, if r is reasonably high, that means G_{dc} is reasonably is sufficiently low. And we can assume that G_{dc} is much smaller than $\omega \epsilon_r'' C_0$ then you can ignore G_{dc} in the above expression. So, this $\tan \delta$ is equal to $\omega \epsilon_r'' C_0$ divided by $\omega \epsilon_r' C_0$, sorry here you do not need to keep i , because current term will not include, i represents the loss current.

So, ϵ_r'' . So, we replace $\omega \epsilon_r'' C_0$, we cut $\omega \epsilon_r' C_0$ cancel each other; this will become ϵ_r'' divided by ϵ_r' . So, this is a very nice expression, because earlier we started with definition of $\tan \delta$

as you know loss current divided by charging current. And this now, you can represent by a single quantity which is epsilon r star.

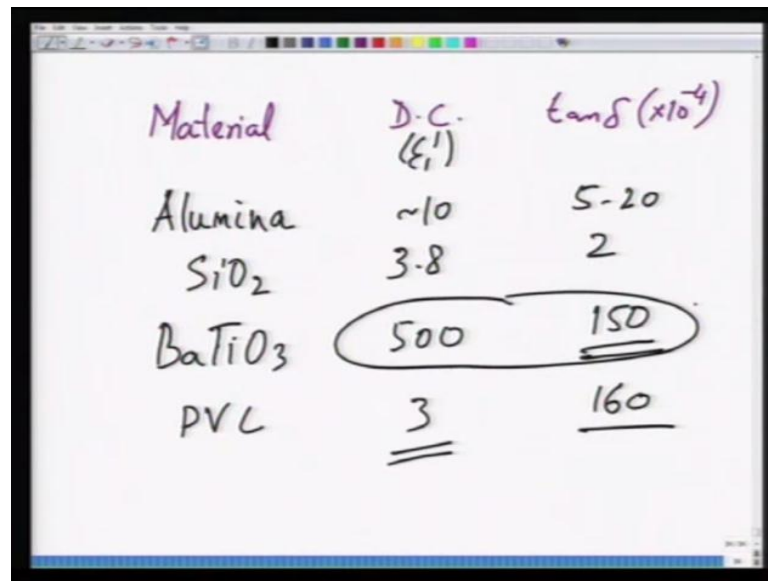
So, if you know epsilon r star, you would know epsilon r prime and epsilon r double prime. And this would give rise to this fundamental quantity which is tan delta. So, higher tan delta would mean higher the imaginary constant, imaginary component of dielectric constant or higher the real part of dielectric constant, smaller tan delta would be. So, alternatively you can write epsilon r double prime to be equal to epsilon r prime into tan delta. So, basically for dielectric materials higher the loss angle is higher epsilon r double prime is, and even higher epsilon r prime gives rise to often higher epsilon r double prime. So, and this epsilon r double prime is called as dielectric loss factor.

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tan δ \rightarrow loss tangent
 \rightarrow Dissipation factor.
 $\epsilon'' = \epsilon' \tan \delta$
 \uparrow
Dielectric loss factor

So, tan delta is called as loss tangent, and epsilon r double prime is equal to epsilon r prime tan delta, and this called as dielectric loss factor. So, tan delta is often called often also called as not only loss tangent, but also dissipation factor. There are various definitions various terms which are used for tan delta. So, I am just giving you some of those terms for these materials. Now, which are used in typically used in material science? So I will give you various some values for dielectric constant and tan delta for some materials. So, for instance;

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Material	D.C. (ϵ_1')	$\tan \delta (\times 10^{-4})$
Alumina	~ 10	5-20
SiO_2	3.8	2
BaTiO_3	500	150
PVC	3	160

So, let us say this is you know material and then dielectric constant and then tan delta into ten to power minus 4. So, for instance alumina, alumina has, and this is the real part epsilon r prime. So, this would be approximately 10, and the dielectric constant, dielectric loss is tan delta is 5 to 20, in to divided, 10 to the power minus 4. A material like silicon oxide, silicon oxide is a very well known dielectric material used in transistor devices, 3.8 is the dielectric constant, and it can have dielectric losses which are pretty low. Similarly, you have barium titanate; this is a very famous material used for variety of ferroelectric and piezoelectric applications, this has high dielectric constant.

So, as you can see it also has a slightly higher tan delta as compared to silicon oxide and alumina. If you look at polymers like P V C, some of the polymers also polymers are insulating materials as a result they are also classified into category of dielectric materials. And so polymers have lower dielectric constant they can have high tan delta. And these two together for instance for barium titrate would result in higher epsilon r double prime as well. So, these are some of those correlations that you need to understand.

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Power Dissipation: Real Dielectric

AC Conductivity + DC Conductivity

$$\sigma_{tot} = \sigma_{dc} + \omega \cdot \epsilon_r'' C_0$$

$\sigma_{dc} \approx 0$ as G_{dc} is small

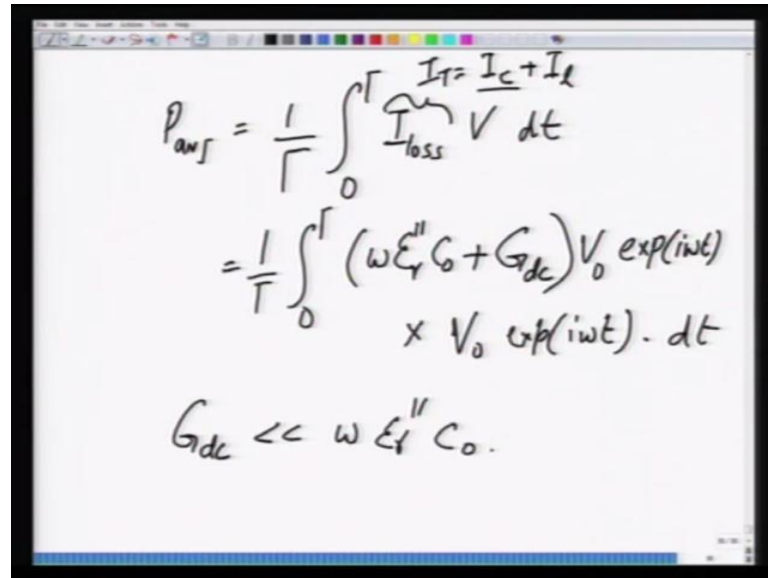
$$\sigma_{tot} = \sigma_{ac} = \omega \cdot \epsilon_r'' C_0$$

$$\sigma_{tot} = \omega \cdot \epsilon_r' \cdot \tan \delta \cdot C_0$$

Now, what would be the power dissipation in, power dissipation in a real dielectric? So, by definition, we had the term p is p average is equal to 1 over τ , and integrated over the whole I c and v as a function of time. Now, first here we need to write what is a c conductivity? So, σ_{ac} which is the c conductivity, σ_{ac} is nothing but sum of σ_{dc} plus. So, σ_{dc} plus ω into, so the conductivity term will include the d c term as well as the loss current which is a c .

So, this is ϵ_r'' into C_0 . So, rather I should say this is σ_{total} a c plus d c conductivity will give rise to this total. So, and as I said, as we said that is if σ_{dc} is very small. This σ_{dc} can be ignored, and σ_{dc} is almost equal to 0 as we are saying that G_{dc} is small. So, if that is true then σ_{total} will come equal to σ_{ac} . And that will come equal to ω into ϵ_r'' into C_0 . And this will come equal to ω into ϵ_r' into $\tan \delta$ into C_0 . So, this is how you can determine the conductivity of a dielectric material?

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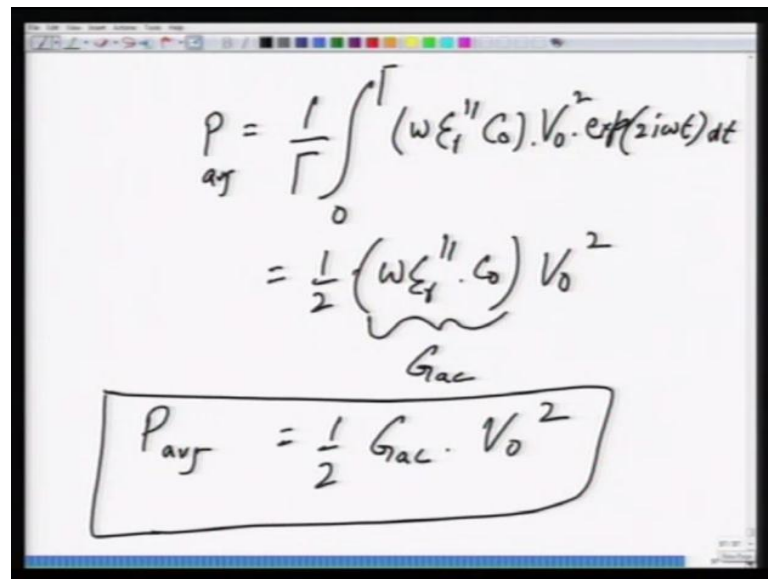


The image shows a handwritten derivation on a whiteboard. At the top, it defines $I_T = I_C + I_L$. The first equation is $P_{avg} = \frac{1}{T} \int_0^T I_{loss} V dt$. The second equation is $= \frac{1}{T} \int_0^T (\omega \epsilon_r'' C_0 + G_{dc}) V_0 \exp(i\omega t) \times V_0 \exp(-i\omega t) dt$. The third equation is $G_{dc} \ll \omega \epsilon_r'' C_0$.

So, the time average power loss P_{avg} will be equal to $\frac{1}{T}$ times the integral from 0 to T of I_{loss} into $V dt$ and because if you take for charging current, it would anyway be equal to 0. So, we only need to consider so basically this would be I_T is equal to I_C plus I_L . So, if you take only for I_C this would be equal to 0. So, we need to consider only I_L in our, in order to work out what is the total power loss.

So, this would be $\frac{1}{T}$ times the integral from 0 to T of $\omega \epsilon_r'' C_0 + G_{dc}$ plus. Now, since, this will have a $V V$ term. So, we can multiply both the V terms. So, this is the conductance multiplied by V . So, or I can just do it later on, exponential of $i\omega t$ multiplied by V_0 naught exponential of $-i\omega t$ into dt . So, and we have already assumed that G_{dc} is equal to $\omega \epsilon_r'' C_0$ naught. So, this takes us to. So, if you assume that.

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The image shows a handwritten derivation on a whiteboard. The first line is the equation for average power P_{avg} as the real part of the complex power integral over one period T :
$$P_{avg} = \frac{1}{T} \int_0^T (\omega \epsilon_r'' C_0) V_0^2 \exp(2i\omega t) dt$$
The second line shows the simplification where the integral of the exponential term over one period is 1, leaving:
$$= \frac{1}{2} (\omega \epsilon_r'' C_0) V_0^2$$
A bracket under the term $(\omega \epsilon_r'' C_0)$ is labeled G_{ac} . The final result is boxed:
$$P_{avg} = \frac{1}{2} G_{ac} \cdot V_0^2$$

Then, this will take us to 1 over p average is equal to 1 over tau. And this would be omega epsilon r double prime c naught into v naught square into exponential of 2 i omega t d t. And if that was the case, if you solve it, what you will get here is half of omega epsilon r double prime into c naught into v naught square. And what is this? This is G a c. So, this is half of G a c into v naught square. This is an important term.

So, the average power which is dissipated in the real dielectric is proportional to the a c conductance, and which is dependent on the epsilon r double prom, double prime as well as the frequency value.

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$$P_{avg} = \frac{1}{2} (\omega \epsilon_r' \tan \delta \epsilon_0) V_0^2$$
$$\boxed{P_{avg} = \frac{1}{2} V_0^2 \omega C_d \tan \delta}$$
$$C_d = \frac{\epsilon_0 A}{d} \quad E_0 = \frac{V_0}{d}$$
$$\text{Power Density} = \frac{P_{avg}}{V} = \frac{1}{2} \omega \epsilon_0 \epsilon_r' \tan \delta E_0^2$$

So, P average now can be written as half of, if you further make substitutions. This would be half of omega epsilon r prime tan delta into c naught into v naught square or half of v naught square omega c tan delta. So, this would be the expression for pure power loss in a real dielectric.

So, naturally if I multiply omega epsilon r prime by c naught, and this will be c, you can say c d, c dielectric. So, naturally higher tan delta is higher your P average would be. Now, if we take c naught is equal to epsilon naught A over d, and E naught is equal to v naught divided by d. Then p average divided by volume will give rise to average power dissipate power density. And this would be half of omega into epsilon naught epsilon naught prime into tan delta into E naught square. So, this is power density.

So, this is quite a nice expression, because average power basically it drives from the same. So, what it tells you is that higher the tan delta value is or higher the angle between the total current, and the charging current more the power loss will be. That make sense, because your I loss component will go bigger and in this expression if you.

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if $\omega = 0$

$$G_{total} = G_{dc} = \frac{1}{R}$$

$$P_{avg} = I^2 R$$

$\tan \delta = \frac{I_l}{I_c} = \frac{\epsilon''}{\epsilon'}$

$I_T = I_c + I_l$

Phasor diagram showing voltage V as the reference vector. The charging current I_c is perpendicular to V . The loss current I_l is in phase with V . The total current I_T is the vector sum of I_c and I_l , making an angle δ with I_c .

Now, just to take it back just to prove that we are right if omega was equal to 0 then if omega was equal to 0. In that case, if you go back to this expression; this a c component will become equal to 0. So, in this case, G total will be equal to G d c that was the case and which is nothing but 1 by r. If this was the case then p average would be equal to nothing but I square R ohmic power loss.

So, we will finish this lecture here, but what we have before we do before we end, what we have done here is we have summarised the behaviour of dielectric materials under a c field. So, for normal for a real dielectric, for an ideal dielectric, for an ideal dielectric the current as we looked leads the voltage by angle 90 degrees. And that is the case for, and this current is called as charging current. And that is the case for most of the, for all the ideal dielectrics which means no loss current. So, perfect phasor diagram where i is perpendicular to v whereas, in the case of real dielectric, this total I is never perpendicular to v. What it does is it makes an angle delta with respect to the charging current which would otherwise be at 90 degrees.

So, this angle between total current, and the charging current delta which is towards the v side. So, I so basically, what you have is, if you have v in that, and this is so this is v; this is I c the, I would be somewhere in this direction. So, this would be delta. So, this total current now will include not only I c, but also a component which is in the direction of v applied field which is called as I l. So, I t will be equal to I c c h plus I l. So, this delta

larger the delta is as you can see from the phasor diagram higher I loss will be. And the higher will be the loss power dissipated in the dielectric, and that we have also verified from various equations. And this equation I_t is equal to I_c plus I_l can be very well represented by taking real and imaginary parts of the dielectric constant, because the given the fact that you have a real part of current and imaginary part of current. You would have real imaginary part of dielectric constant as well. And this total current expression can be expressed very well by real and dielectric parts of dielectric constant.

So, as a result $\tan \delta$ which was equal to I_l divided by I_c becomes ϵ_r'' divided by ϵ_r' . So, basically in the nutshell ϵ_r'' represents I_l , and ϵ_r' represents I_c . So, higher I loss is higher ϵ_r'' will be, because higher $\tan \delta$ would be. So, this is the summary of this lecture which basically shows that dielectric materials in real dielectric materials are lossy. They contain some losses, but those losses can be quantified by various quantities which are measurable. And these ϵ_r' and $\tan \delta$, and ϵ_r'' can be measured by a technique called as by a by an equipment called as impedance analyser or something called as LCR meter, where you measure the capacitance and $\tan \delta$ values. And from that you can determine various real and imaginary parts.

So, these are all. So, $\tan \delta$ as well as the capacitance can be measured, and from that you can determine various properties of the dielectric material. So, we will finish here. In the next class, what we will do is that, we will we will look at some more quantitative analysis of the frequency dependence of dielectric materials.

Thank you.