Introduction to crystal elasticity and crystal plasticity
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Week-03
Lecture-06

Good morning, everybody. Last class we have discussed about the 2 dimensional state of the stress and how we can analyze using the (())(0:34) circle diagram to find out the principle stress, principle stresses and shear stresses as well.

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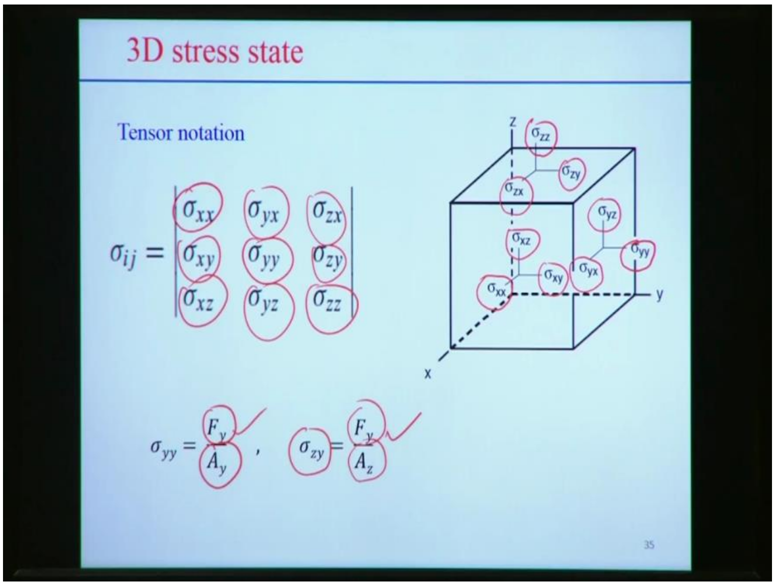

Now this time we try to focus on the 3 dimensional stress state. So far we know that the tensor notation of the stress analysis can be represented like that. Sigma XX, sigma XY sigma ZX, sigma yy and sigma zz. Now this stress notation, this stress tensor notation, we found out that there are 9 components of the stress, in case of 3 dimensional state, and out of that there are 3 normal stresses and 6 of the shear stress components.

Typical, we try to, we represent the state of the stress assuming the element specifically for 3D, some cubic element, and stress components allows X, Y and Z directions. Like that along Y direction sigma YY, that is acting along the direction Y, so it is consider as a normal stress, but on the same plane we define the shear stress, sigma YX and sigma YZ. So here we see sigma XX is also acting which is along the direction X, similarly sigma ZZ also and other components sigma ZX, sigma ZY, sigma XY, sigma XX are the shear stress components.

So basically 0.1 element is subjected to some kind of (())(2:26) arbitrary direction, then we can convert that component of the load one specific direction, maybe in along the direction X along direction Y or along direction Z. So sigma yy in this case is the FY AY, so AY is basically area which is normal to the direction axis Y, similarly (())(2:56) sigma zy this is basically shear stress, and the shear stress what is the force acting along the direction Y, is divided by the area in Z. AZ actually represents that area normal to the Z axis.

So this will produce the same stress and this will produce the normal stress and posing the want element all the components of the stress is simply represented by this way or by the term of stress tension where the diagonal represents the normal stresses and off diagonal elements actually represents shear stress and following some sequence, following some order so that we will be able to understand how this order is following to look into the notation of the different stress tension or sign convention, like that.

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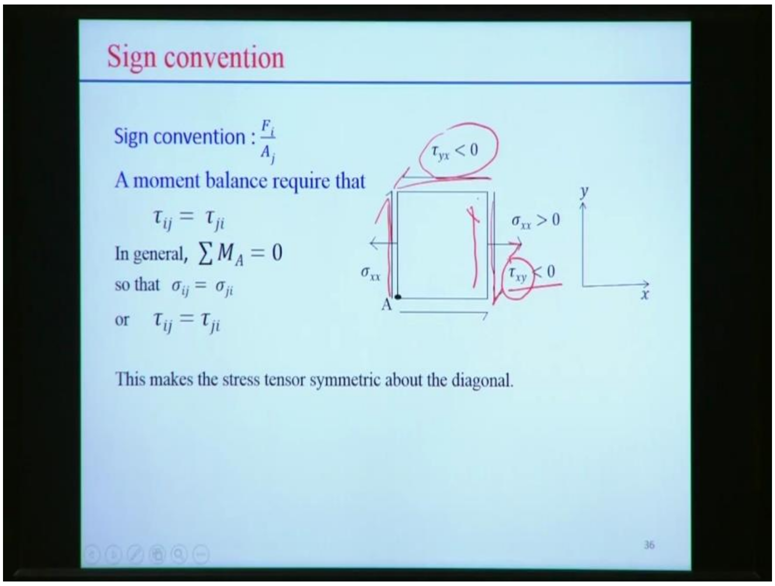

So sign convention, we try to focus on this say 2 dimensional case along the axis Y and Z, we see that the direction arrow which is along the direction X, this is represents the sigma XX that is the normal stress which is greater than 0. But shear stress when it is acting this direction or opposite side it is in that direction that shear stress actually here is the less than zero that means negative. So same the shear – shear tau xy, it is opposite to y, that mean negative y direction but right hand side, it is positive side of that X axis so because of the negative direction of the Y, that this tau xy actually in this represents the negative shear stress value.

Similarly when you consider this component this is opposite to the x axis, so out of x and y, one is the negative x axis because of that this component also represents the negative shear stress value. Similarly way we can define, if you look, if you want to define the shear stress positive shear (())(5:27) then you should act in this direction which should represent in this direction, that actually represents the shear stress which is positive in this case.

So depending upon the orientation of the axis we can adopt a several sign convention to define different shear stress value or normal stress value and we can so the whether it is negative or whether it is positive values. Now if we consider the moments with respect to A, that means general with respect to A moment equal to 0 for this element, so in that case we can say that tau ij equal to tau ji or we can say that sigma ij equal to sigma ji, that actually makes the stress tensor symmetric with respect to diagonal, so we will deal with all the stress analysis, basically with the symmetric stress matrix or symmetric strain matrix will be using further.

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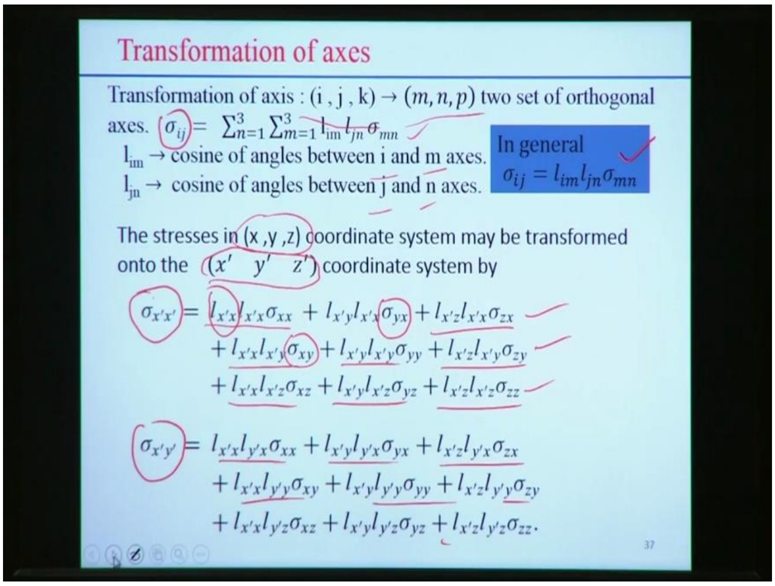

Now similarly to the 2 dimensional analysis, here it is also important to know the transformation of axes, suppose the state of the stress is define in one axis system, for example ijk system and we left to transform the state of the stress in another orthogonal set of axes for example m, n, p. So in that case sigma ij, the component of the stress can be represented like this… So m, im or Ljn, the d of cosine so all Lim specifically the direction cosine angles between i and m axes and Ljn is the direction cosine between the j and n axes so in general we can write the sigma ij equal to Lim Ljn sigma mn, this is the general representation of the stress with respect to different axes system.

So now stress represent in x y z coordinate system and we left to transform onto the x dot y dot and z dot coordinate system, so by using this notation we can say that x, sigma x dot x dot that is the one normal stress component with a new transform axis and that we represents that, that is equal to one component is the L x dot x L x dot x, so here L x dot x actually represents the direction cosines of angle of cosines between the axes x dot and x.

Similarly the sigma xy or xy component that can also be multiplied by the direction cosines L x dot y or L x dot x, then this is the way we can transform the stress or we can define the normal stress of shear stress component in a another new axes system. Here we find out that the stress tensor having total 9 components, so here 3 plus 3 plus 3, so total nine components can be transformed using the direction cosines and will be no, the direction cosines between the old axes and the new axes system and this is the system which represent or transformed thus different stress state with respect to the new define axes as compared to the old axes.

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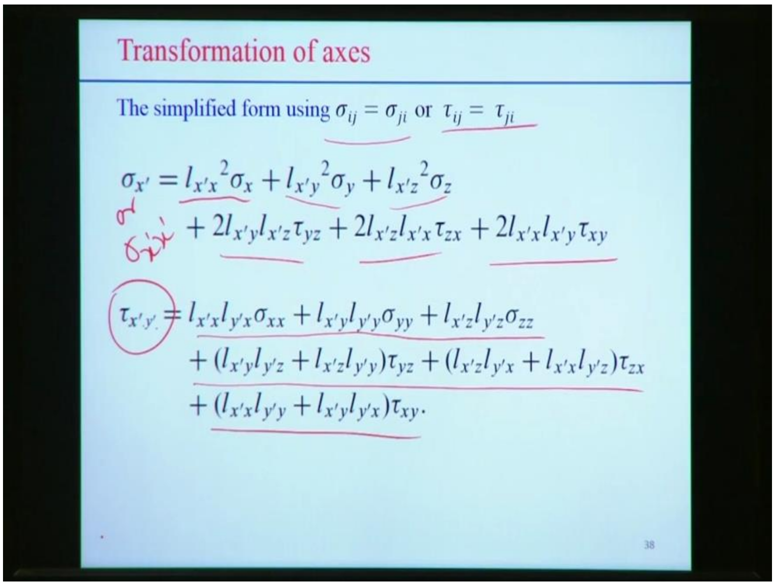

So similarly the shear stress components can also be transformed to the another axis system that means x here is the x dot y dot and z dot axis system by the similar Y. Now we can further simplify this expression looking into the symmetric measure of the stress tensor but we follow the sigma ij equal to sigma jr with the symmetric nature we can further modify this thing that means sigma x dot or you can say that sigma x dot or sigma x dot, x dot equal to this exist there are three normal stress components and another shear component and this transformation is having only 6 components and with this transformation we can define stress state and another set of axes.

Similarly when you try to represent the shear stress so similar we can further simply the expression but looking into the direction cosines and the stress state, different stress components values, you can define the shear stress value in the another new set of axes system. So this transformation of the stress from one axes system to another axis system using this simply formula, we can find out this thing.

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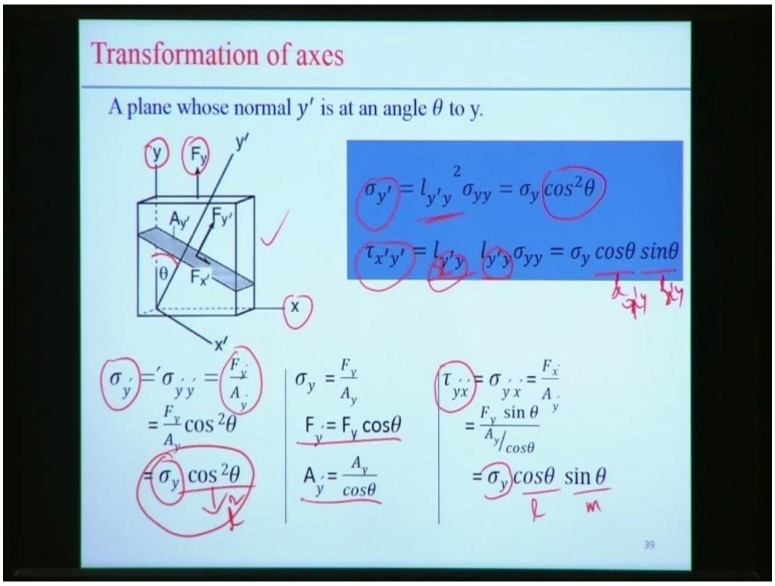

Now we can verify whether this transformation equations are in right direction or not, last class we have discussed that suppose theoretically we estimate that in a 2 dimension system there is a initially axis system is X and Y, we have defines the forces F Y is acting on that F Y and along the x direction which is FX. Now we try to define one set of plane goes normal, y dot is at an angle theta with respect to Y axis, so respect to the y axis the angle theta and that y dot actually represent the normal to this highlighted plane so from here you can find out, new normal stress sigma y dot or transform normal stress value to sigma y dot with respect to the sigma y which was defined in the x y coordinate system, here we can find out sigma y dot equal to simply F y dot by A y dot and A y dot is the component of the force with respect to the F y, that is F y cos theta and divided by A y.

So A y is the A y by cos theta. So by putting this we can find out sigma y dot equal to sigma y dot equal to sigma y cost square theta so by knowing that here we can see, this is the cos square theta, so this is the multiplication of the direction cosines, 2 directions cosines, if it is L and M with respect to different axis or with respect to the same axis then it becomes the square.

Similarly shear stress on the new transform axis system can define the similar way similar fashion, you can find out the sigma y cos theta sin theta, so if I define the direction cosines L and sin theta direction cosines M, just corresponding to L M sigma Y, so this is the way, that is the physical verification of this thing, how the transformation of axis helps to define the stress state and another set of axes system.

So if we try to use the formula what we have derive to find out this thing, so sigma y dot, actually the new stress, new transformed axis y dot, then we can find out the direction cosines square, sigma yy and from here direction cosine L with respect to ym y dot because they are making another theta that actually cos square theta comes to the picture. So here it becomes the sigma y cos square theta which is very much similar to the what we physically verify by looking into the orientation of the plane.

Similar way the shear stress can also be calculated with a new transform axis, the two different, directions cosines, we can find out that it becomes again the two directions cosines cos theta and sin theta. So it should be x dot y and y dot y. So it is not the same direction cosine, these are the two different direction cosines, with different values of the directions cosines, cos theta and sin theta, so this is L x dot y, sorry L y dot y, and this is L x dot Y. So these are the ways we can simply transform the stress state which was define to the one set of axes system with another set of axes system. So this is significant to further analysis of principal stresses, to find out the principal stress on a specific shear stress.

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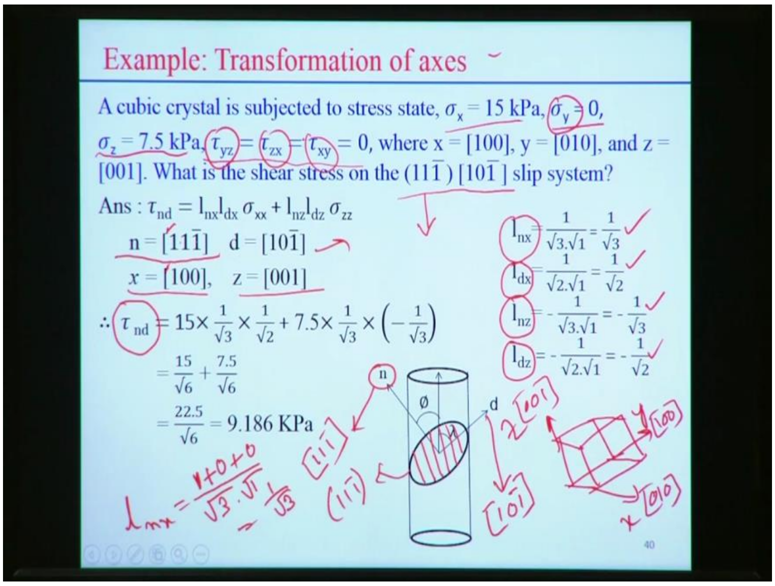

Now if you look into what example for that transformation of axis, you see that the example, we define the problem in such way that a cubic crystal is subjected to the stress state, sigma X equal to 15 kilopascal, for example sigma y equal to 0 and sigma z equal to 7.5 kilopascal so these are the stress state, normal stress to define at that x, y, z axis system but there is no shear stress acting on that, so we can simply say that tau yz equal to tau zx equal to tau xy equal to 0.

Now the x axis y axis and z axis are also define X 100, y 010 and z equal to 001. Actually this X Y and Z axis represents the axis of a cubic crystal. If you look in, represent the cubic crystal so you can define this is x axis so 100 direction, so you can define this is y axis, 010, and this is z axis and this is 001. So we need that (())(16:37), we can define the direction corresponding to X, corresponding to Y and corresponding to Z. So once we define Z now we need to find out what is the shear stress of the 111 bar 101 bar slip system.

So here we can see that 110, 111 bar actually define the plane, so that plane, for example this at the plane the slip will occur that is corresponding to 111 plane, now 101 bar slips that represent the directions for example, this is the direction D, this is corresponding to 101 direction. So first we defi the plane within that plane we define the direction the direction, at which direction the slip phenomena will occur.

Now, how to tackle this problem, so we can use our concept of the corresponding to the axis system so solve this problem. But here first we define that 2 different, first we define the plane that 111 and we can define on direction N, so this N direction is basically the same index, with this index, same index it is define normal to the plane. So here n is define like this 111 bar, direction d actually represents the slip will occur due to the shear loading, so d here represents the 101.

Now we have explicitly define the x axis, y axis and z axis but here we need to find out what is the shear stress, shear stress is acting along the direction d or specifically when you try to find out the shear stress tau nd that actually define the what specific stress step system, D actually defines the direction, which direction the stress is acting and that is over the plane nd.

Now we can find out that similar direction cosines, say for example Lnx to find out the tau nd, Lnx Ldx, Lnz, Ldz, these are the components are required, so to find out basically the direction cosine between the axis N and S, and corresponding n and z or between tnx or tnz, so it is a step for what, because we have defines the N and D, and X is already defined 100 axis, Z is defined 001. So from this n d x z system we can straight forward find out the direction cosines, so basically the dot product between these two enter, (())(20:03) as in terms of vector, n into x, so you can find out that Lnx, like this, if the middle index is Hkl then you can find out n, n into x so one multiply by one, plus one, plus 100 plus 1, minus 100 and root of 111 square that is 3 and root the power of 100 means, so that comes 1 by root 3.

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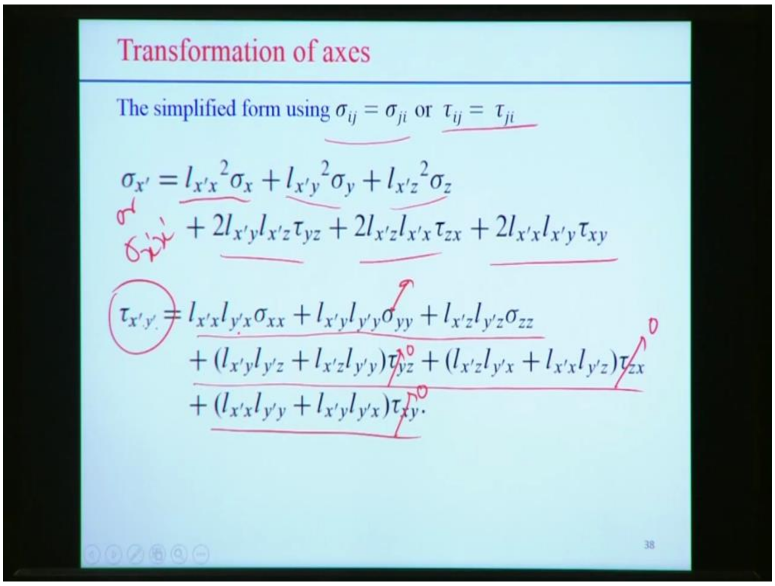

So that is correct, so similar fashion we can find out what is Ldx, what is Lnz and what is L dz. Now tau nd actually represents the component, here you can see the sigma equal to 0 and all the shear component equal to 0. So if we see the transformation of the axis system if we go back to that, that shear stress cannot be calculate their axes, 1, 2, 3, 4, 5, 6, components, but out of these 6 components sigma 0, all have 0, yz 0 xy equal to 0.

So during the using the formula of the transformation of one axis system to another, you will be getting only the, only I think two components.

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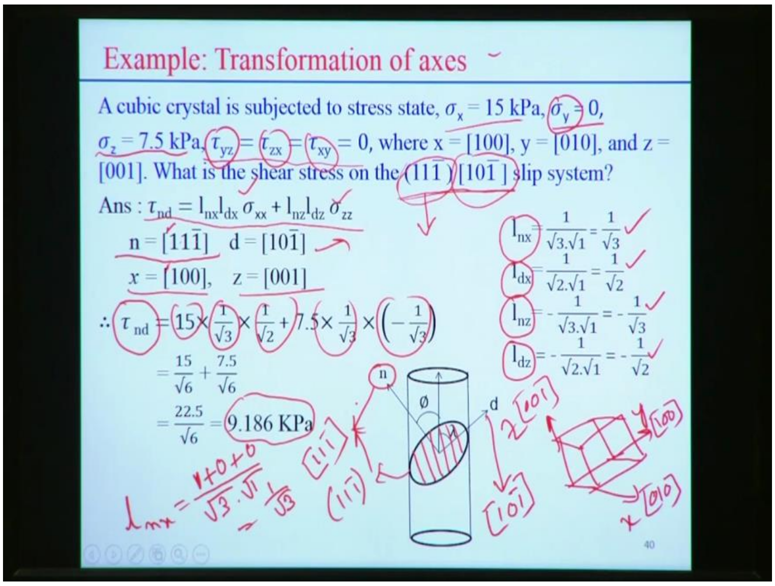

So that similar so that similar transformation formula use in and we are getting the tau nd is corresponding to this formula which is (())(21:54) elements sigma xx and sigma zz and corresponding which is multiply by corresponding direction cosines, so that direction cosines we can directly find out and by putting here we can find out the, here the total stress is equal to 9.186.

So these are the way to track that this kind of problem, third the stress state is given, one axis system, but we need to find out specific stress state in this some of the axis system, I think in this case we can directly use the transformation of the axes formula for the stress and we can find out the weak ends of the problem or we can tackle this kind of problem.

So few important things to point specific to this problem, first one is that while which is given the slip system we can define the slip system with respect 1 plus, we need to define one specific plane which plane the slip will occur and second thing is that which direction slip will occur. So first this indicates actually, clearly indicates that this is the representation of the plane and the second part actually represents the direction. When you try to draw the normal direction with respect to one specific plane that index will be the same. So this is corresponding to direction, if one part is the plane, so normal to that plane that is defined by the 11 minus 1 cell. So this way we can tackle the problem also.

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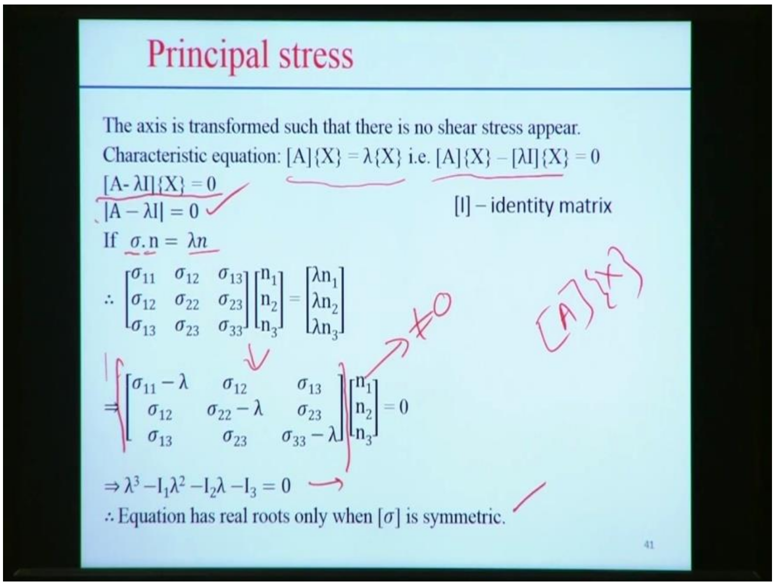

If there is a three dimensional stress state, then how we can find out the principal stresses, like 2 dimensional cases we have already discussed that they must exist some principal stresses in the sense the maximum, minimum stresses values but there doesn't exist any shear stress. But in 3 dimensional step stress we can find out the principal stress, first we need to suppose the stress state is defines as the matrix A.

And X is the variable here in this case, now we can define the characteristic equation such that AX equal lambda x, so same state variable, but we are multiplying some constant term lambda, further rearranging this equation we can find out this equation. So there must exists some principal stresses when the determination of this stuff is equal to 0.

Now similar way we can find out if sigma is the stress state and it is the variable, here we can find out the characteristic equation equal to 0, so this actually brings the determinant of this equation, so basically this actually determinant of the equation equal to 0, so this cannot be the variable, this cannot be 0. So when you do this manipulation to find out the principal stress then final we will be getting the one cubic equation like this.

So the roots of the equation actually represent the principal stress, so equation has the real root only when sigma is symmetric in nature, so there exist. Three real roots in this case we can consider, the roots of this equation as a principal stress value, so let us see how we can do the further analysis.

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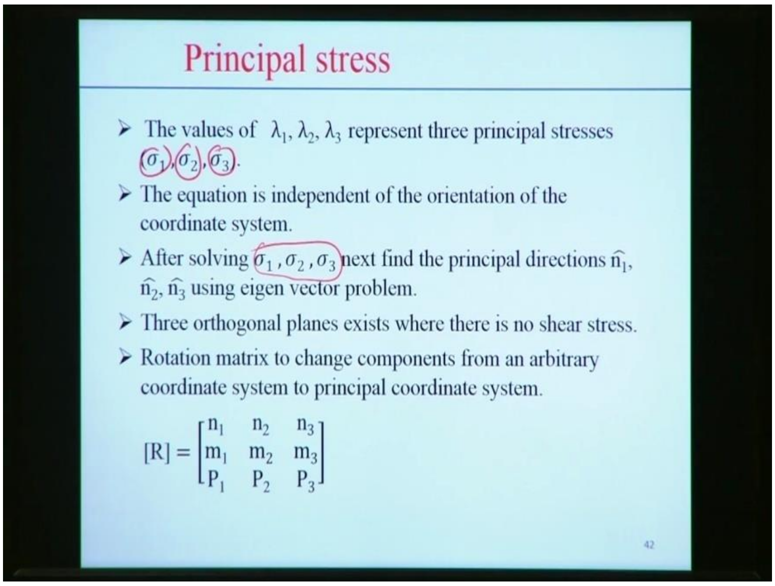

So the values of the lambda 1, lambda 2, lambda 3 is the roots of the cubic equation represent the three principal stresses and three principal stresses often designated like at sigma 1, sigma 2 and sigma 3. The equation is independent of the orientation of the coordinate system, so here actually we are using the concept of the orientation of the coordinate system and we has oriented the coordinate system in such a way that along that orientation the principal stresses, principal stress actually exist with that specific orientations.

So we don't know, the principal stresses sigma 1, sigma 2 and sigma 3 then we need to find out the principal direction in 1, 2 and 3, using again you have to problem, so we are not going into details how to find out this thing, the principal direction but we can highlight this thing that three orthogonal plane exist as there is no shear stresses. Sometimes we use the rotation matrix to change the components from 1, from arbitrary coordinate system to principal coordinate and then rotation matrix is defined like this.

So n1, n2, n3, m1, m2, m3, p1, p2, p3, actually these three actually define some orthogonal axis system and using this rotation matrix we can define the direction of the principal stresses.

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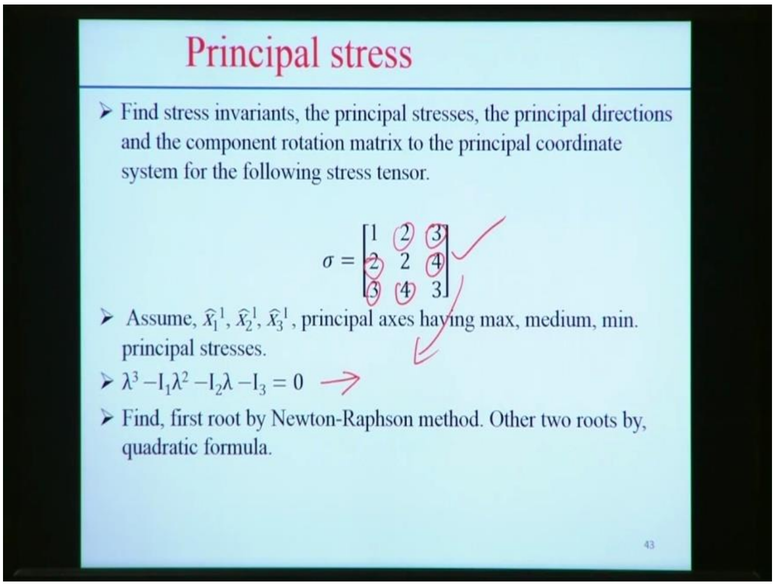

But for example we can find out the other points that principal stresses or principal direction cannot be find out considering the rotation matrix with respect to the principal coordinate system of the following tensor.

So suppose this stress is defined in this way, 123, 224, 343, so here the 2 and 2, here also 3 and 3, 4 and here 4, that means it represents the symmetric matrix. So first you find out the principal stresses corresponding to this stress tensor, we need to formulate the cubic equation first. So here actually i1, i2, and I3, is the stress invariants, you see that how this invariants can be represented in terms of different stress components.

So once the cubic index is formed from the basic matrix of stresses then this cubic index can also be solved, first root can also be find out by Newton-Raphson method. When we can find out the first root then our another 2 roots actually comes the quadratic equation and from directly solving the quadratic equation we can find out the other two roots also.

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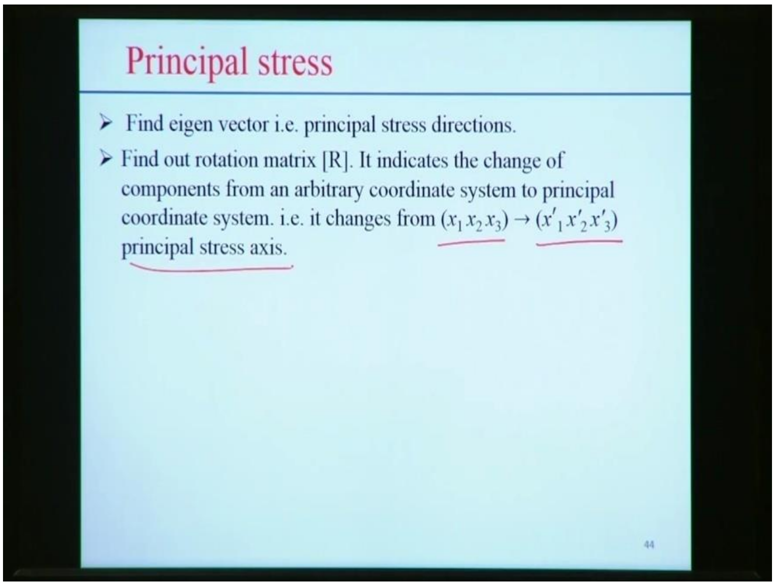

So then after finding out all the roots of the equation or ordinary principal stress from a specific stress tensor, they you need to solve the eigen vector to find out the principal stress direction. In this case it is easy to follow that to find out the rotation matrix and it indicates the change of the components, this rotation matrix from an arbitrary coordinate system to principal coordinate system, that means this is the arbitrary system to principal coordinates system. We can directly use the concept of the rotation matrix and to find out the direction but this is not with, this is beyond of the scope now.

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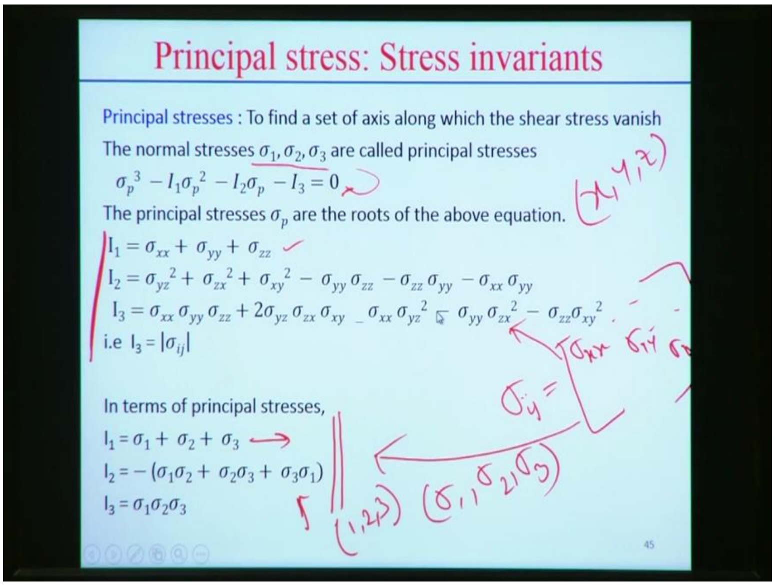

Now in terms of the stress invariants we can represent the principal stresses or we can modify the actually equations. So first we are going to try to find out the principal stresses that actually physically represent the set of axes system which is having no shear stress. So the normal stress is sigma 1, sigma 2 and sigma 3, actually call the principal stresses value, these are the roots of the cubic equation here so the stress invariants I1, I2, and I3 can be represented like this, so here I1 equal to sigma xx plus sigma Y1 plus sigma zz.

So here I2 equal to in terms of sigma yz zx, and sigma xx to yz and zz. So I3 also represent like that but remember that I3 is represented as the determinant of the sigma Iz. So actually here sigma Iz is equal to the… What was the define, along the X Y Z coordinate system so that is sigma xx, sigma yy and sigma zz and accordingly we can put the other components and that is the representation of the sigma yx… Determinant of that, actually it represents third invariant, and second invariant you can follow direct with the component of the stress value on the x y z product system.

Now this invariant can also be represented in terms of the principal stresses value, so the first invariant is the simply summation of the three principal stress value and second invariant also in terms of the three principal stress value, third invariant is simply the multiplication of this three principal stress. So this expression is the more simply expression to remember. I think it is the most simplified form in terms of the principle stresses, but this expression expressions are also little bit more number of terms are also involved here.

But this expression with respect to the original stress matrix, but this expression is the, when you find out the principal stress axis, so it is with respect to that it is defined. So we can say that 1, 2, and 3, this axes system plus stress invariants are represented and here the original x y z in terms of the x, y, z coordinate system, the invariant system, the invariants are represented here.

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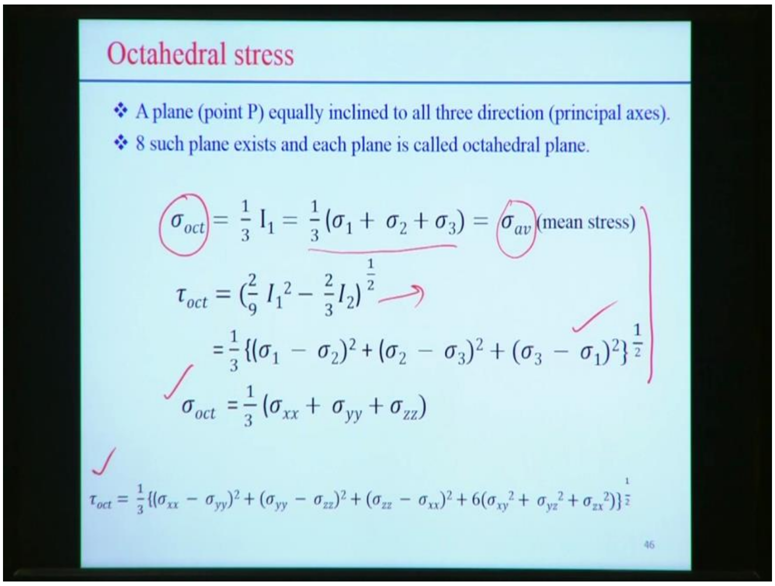

Now octahedral stress one of the main components or maybe this is more useful, specifically when you want to analyze the (())(32:49) plasticity, so in this case octahedral stress is more significant but with the basic idea what is the octahedral stress here, octahedral a plan equally inclined, so for example, point p equally inclined to all three directions with respect to the principal axes. So with reference to the principal axes if it is equally inclined to all three direction that when you actually define the octahedral plane.

So 8 such in a three dimensional coordinate system will be consider, then 8 such plane exists and each plane is called a octahedral plane. Now what is the normal stress on the octahedral plane that is defined the 1 third of first invariant, so 1 third of first invariant is simply 1 third of three principal stress or you can say sometime it is simply the average stresses of mean stress.

Similarly octahedral shear stress in terms of invariants is like a I1 and I2, so here we can find out in terms of the principal stresses this is the expression of the octahedral, on octahedral plane what is the shear stress value. Now these are the in terms of the principal stresses, but similarly we can actual stress tensor in terms of the x y z coordinate system we can represent the octahedral stress that is the normal stress, and this is the shear stress value in terms of the x y and z.

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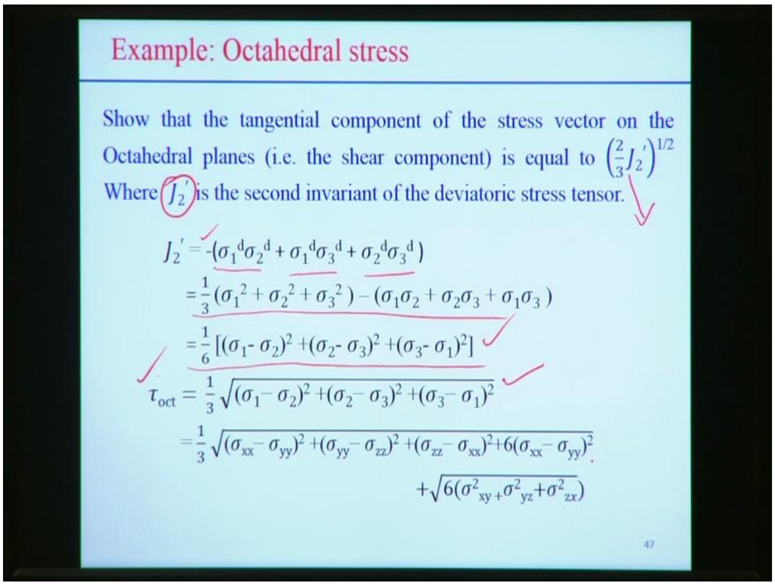

Now look into some problem then we need so that the tangential component of the stress vector on the octahedral plane, that means tangential component on the octahedral plan means the shear component here is equal to 2 third of J2 dot, where J2 dot is the second invariant of the deviatoric stress tensor. So here if we see that stress invariant is represented in terms of J2, so sometimes we use the in terms of I2 also and sometimes we use that stress invariant in terms of J also as well.

So where J2 dot is equal to the second invariant of the deviatoric stress component, I will come on later the part what is the deviatoric stress component but in each stress state can be decomposed into two parts. One is the hydrostatic component and another is the deviatoric component. So here dot actually represents the deviatoric stress component.

So if we look into that what is the second invariant in case of octahedral stress state, second invariant are represented like that simply negative of this principal stresses, that means sigma 1and sigma 2, sigma 2 sigma 3, and sigma 3 sigma 1, this is the represents of the second invariant. So similar thing we will try to follow here. Sigma 1 sigma 2, sigma 1 sigma 3, and sigma 2 sigma 3 but d actually represents the specifically, it is only deviatoric component, and negative sign also given.

So if we further manipulate this deviatoric stress component will be put that expression of the deviatoric stress component yet we can find out this expression counts is like this, 1 sixth of that is sigma 1minus sigma 2 square, plus sigma to minus sigma 3 square, plus sigma 3 minus sigma 1 square. So this is the expression of the second invariant of the deviatoric stress tensor. Now what is the shear stress of the octahedral plane that is the we have already derived this thing, that in terms of the x y and z axis that is the component of the octahedral shear stress.

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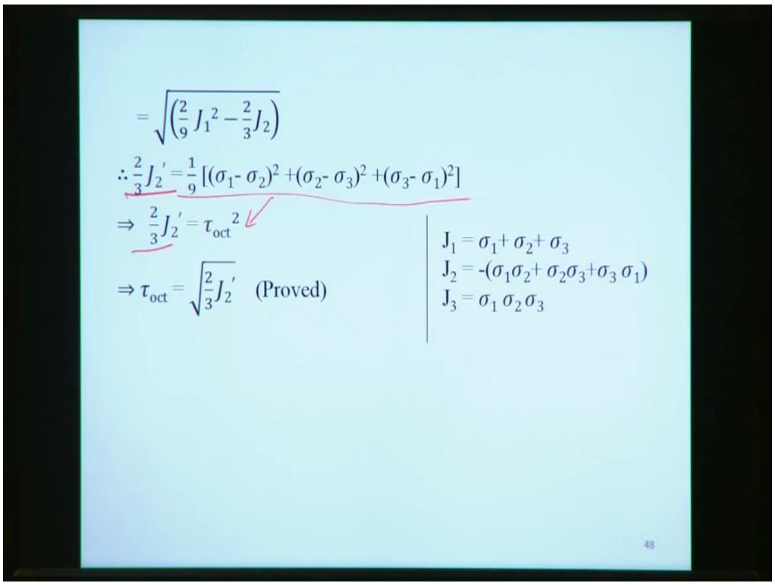

Now which is equal to the 2 by 9, first invariant and the 2 by 9 second invariant and then we can find out 2 third of the second invariant we can find out that we have already defined here at 2 third of the second invariant is also, this is the deviatoric part, this is the deviator, 2 third of second invariant is, this is equal to the tau octahedral square so here you can find out tau octahedral stress is 2 third of second invariant. So we use this formula J1, J2 and J3 that means the (())(37:41) is stress.

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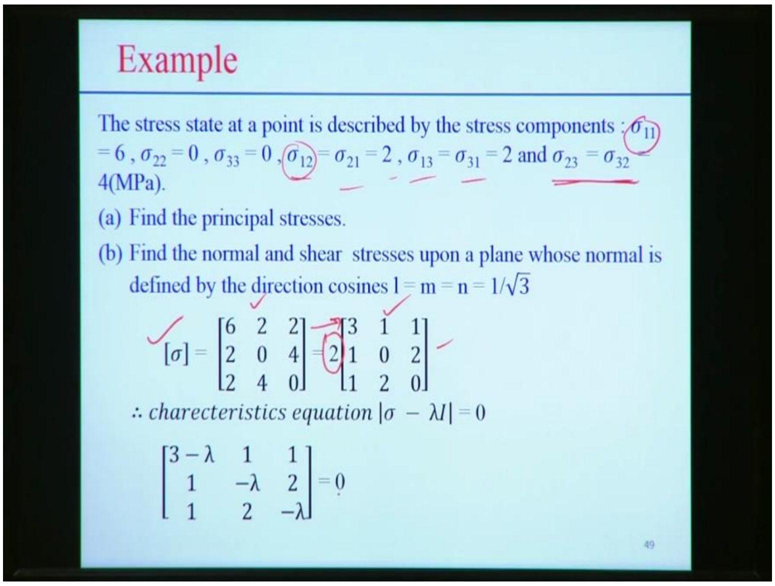

Now look into a example to make it clear, how we can find out the different principal stresses. So for example if we define the stress state one specific point, the sigma 11, sigma 22, sigma 33, 12, 21, sigma 13, sigma 31, sigma 23, sigma 32, all are given. Now you need to find out the principal stresses. Now what in the state of the stress, the state of the stress is defined like that, putting the simply component following some rotation here, here if you see the nature of the stress tensor then it is symmetric because here the shear component 12 21 equal to same and 13 311 is same and 23 32 is same. So we define as the stress state.

Now if you see that this multiplied to, if we make it common for all the components here who can represent from here to here, keeping the, 2 as a multiply component here. So here the multiplication, so it is very careful to use this one, now you need to find out the characteristics equation based on this stress tensor, that you can following the similar what we have discussed, that sigma, all the diagonal component, the constant term deviation, and minus lambda here, minus lambda, first one is the 3 minus lambda and giving the other component same, this is the equation.

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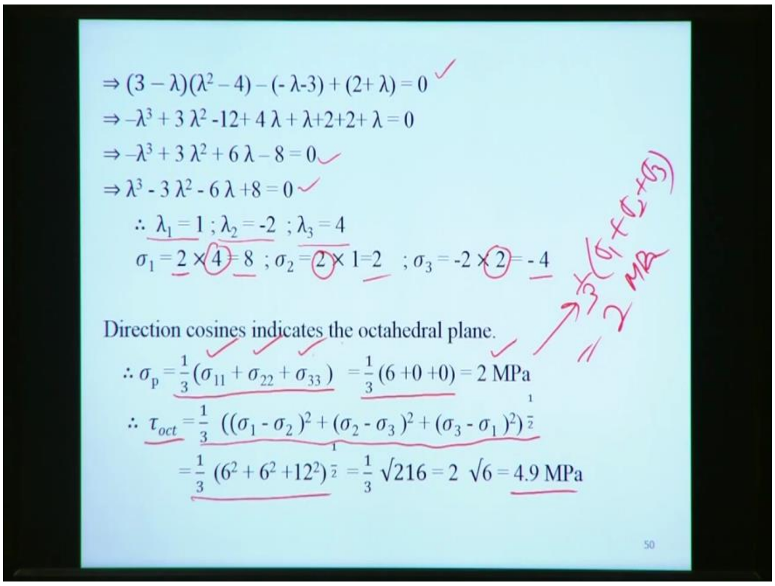

So now we put the determinant of this equation that ultimately it is coming like this, so this is the cubic equation is formed here. Now direct investigate into the cubic equation we can easily find out the lambda 1 equal to 1, lambda 2 equal to minus 2 and lambda 3 equal to 3. These are the three roots of this equation, but this roots of the equation of the stress tensor here which is x exclude the multiplying factor 2, so here actually principal stresses, when you try to find out the 4 is the 1 root, but we multiply the multiplying factor 2 here, similarly sigma 2 and similarly for the sigma 3, then we have final the actual principal stresses here. So here maximum values of the principal stress 8, and minimum is minus 4 and middle one is 2.

Now we need to find out the normal and shear stress upon a plane whose normal is define by the direction cosines. So we need to find out the normal stress and the shear stress component when the direction question is given. Now this plane with respect to the principal stress axis, so with respect to the principal stress axes, this direction cosines actually indicates the, it is a octahedral plane. So on that plane we need to find out what is the normal stress and shear stress value.

So within the octahedral plane this is the expression of the normal stress value, this is average value, we can find out that actual, with respect to the 1, 2, 3 axes, it is 2 megapascal. Then actually we can find out this is from the matrix here, so we use this plus this, diagonal the original matrix, this 2 megapascal. But the same thing can also be calculate by looking into their principal stress value, sigma 1 plus sigma 2 plus sigma 3. You can find out the same values.

Now octahedral shear stress we use in terms of the principal stress is here, and if we put it here we can find out the 4.9 megapascal. So this other way we can find out the different stress state, the octahedral and we can find out the principal stresses for the actual stress state.

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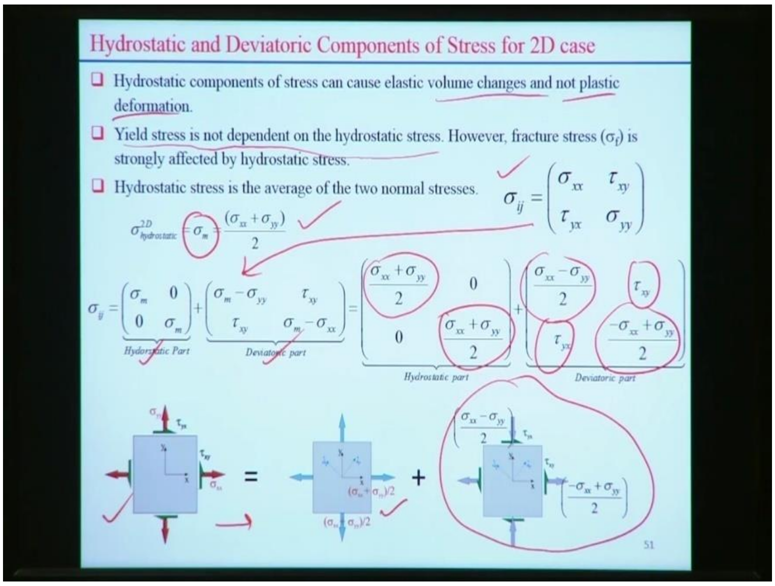

Now as we already mentioned that the stress state can be decompose into two component one is the hydrostatic component another is the deviatoric component. And for 2 dimensional cases now hydrostatic components of the stress actually cause the volume changes and not in plastic deformation. So hydrostatic component actually changes the volume without change of the (())(42:32) but may not be involved in the plastic deformation.

But other way the yield stress is not dependent on this hydrostatic component but fracture stress actually strongly affected by the hydrostatic state of the stress. So hydrostatic stress is the average of the two normal stresses. So all this phenomena actually reverse followed in case of metallic materials. Now suppose this is the stress state, sigma xx, sigma yy, tau xy and tau yx, this is the two dimensional state on the xy axis system.

Now hydrostatic stress component is the simply average of the two normal stress component, now if is this stress state, you can divide into two components, one is the hydrostatic part, another is the deviatoric component, in hydrostatic part we assume this is the hydrostatic stress component for example sigma m, sigma m, so all the hydrostatic components are there and another is the deviatoric components are there making the balance.

So here you can find out sigma x, sigma y is this towards the hydrostatic components and here is the deviatoric components, all the 4. So if we see, graphically we try to represent this is the initial stress state, we can define into two parts, one is the hydrostatic component, here we see it is subjected to only the equal amount of the stresses with the x and y, both direction. Here the stresses is different, two normal stresses are different, at the same time it is subjected to some amount of the shear stress, this is corresponding to the deviatoric stress component.

So any stress state can be divided into this two component to make it the simple, simplify the calculation for the effect, individual effect of the hydrostatic stress or deviatoric stress component for the further analysis, different plasticity theory.

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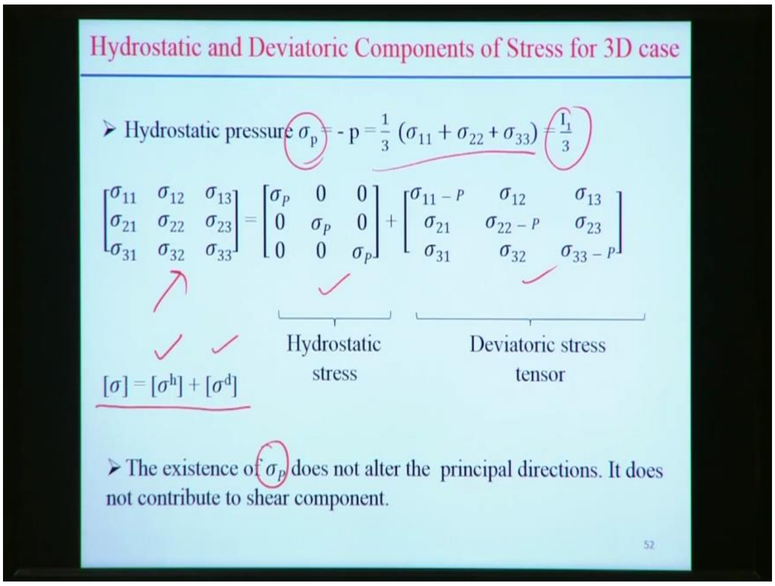

Similarly in case of three dimensional state stress we can decompose at general stress state into two components, one is the hydrostatic state another is the deviatoric component, so hydrostatic component is sometime same that is the pressure, defines as a pressure, so that is simply average of this thing, three normal stresses and which is one third of the first stress invariant. And now if you see that is the sigma, is the consistent of hydrostatic part, another is the deviatoric stress component.

But the existence of the sigma P, so when you try to separate out the hydrostatic component that does not alter the principle direction, so principal stress direction remains the same. It is not, It is independent of the amount of the hydrostatic stress values, but it does not contribute to the shear component so that we have already discussed that in pure hydrostatic only the normal stress component but the absent of the shear stress component, but the deviatoric, the normal component as well as the same component are there.

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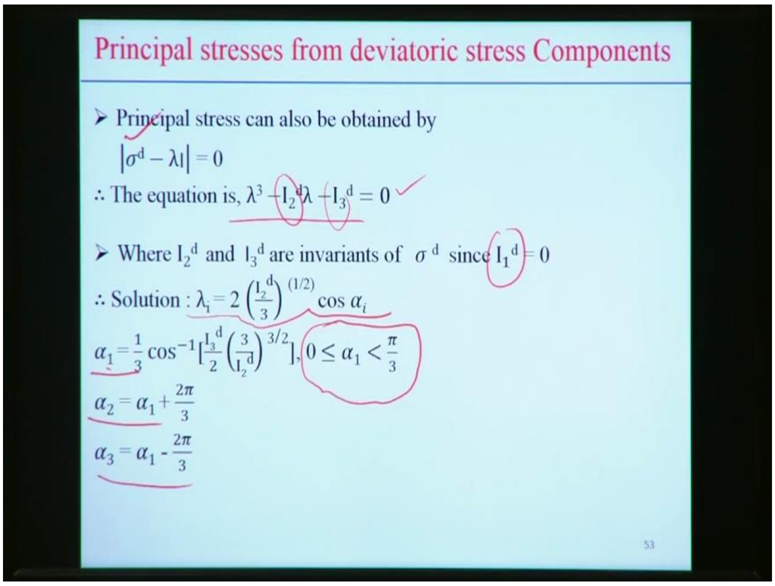

Now by looking into the hydrostatic stress state or when you try to divide the deviatoric stress component and hydrostatic stress component from the general state of the stress then by using the deviatoric stress component we can find out the principal stresses in similarly way. Now if we separate only the deviatoric component and if we find out the characteristic bases on the deviatoric component we can find out this equation.

But if we see that this deviatoric component we can find out there are two stress invariants although this is a cubic equation but two stress invariants I2 and I3 is there, because I1, first stress invariant become 0 in this case, then from this cubic equation can also be straight forward or can be solved by following suppose lambda is the root of the equation then we can find out, directly you can say that lambda I equal to two times of second deviatoric invariant third to the power ½ and cos alpha i.

So alpha 1 and alpha 2, alpha 3 are also defined here, but first we try to find out the alpha 1, this formulation can also be used for further analysis, if alpha 1 lies between 0 to pie by 3. If it is out of this range then we cannot use this formula to find out the roots, in that case from the conventional way we need to find out the roots of the equation.

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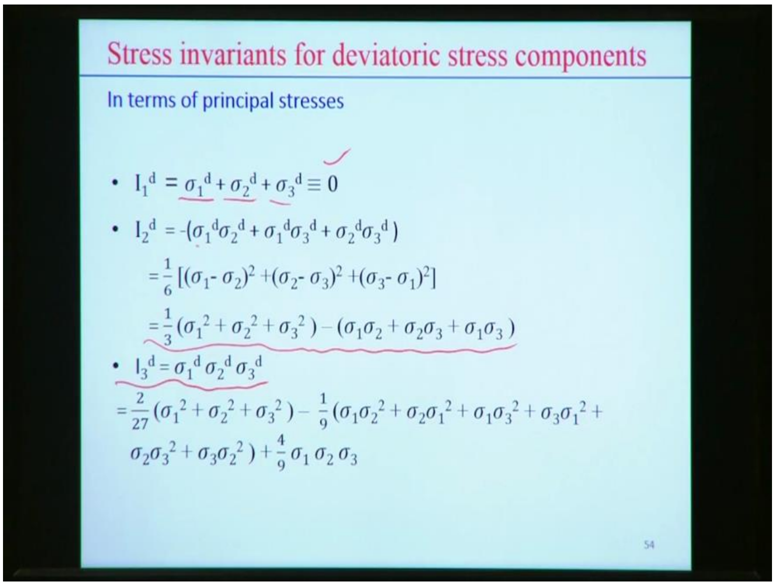

Now what should be the stress invariants when we consider only the deviatoric stress component, so if we see the deviatoric stress component equal to sigma, express the three principal stresses of the deviatoric components, it becomes 0. Then second invariant of the deviatoric component it becomes in terms of the principal stresses is like this, and third invariant also like this. So see we follow the similar formula to even further deviatoric stress components to find out the different invariant and finally these are the all expression of the deviatoric stress components in terms of principal stresses.

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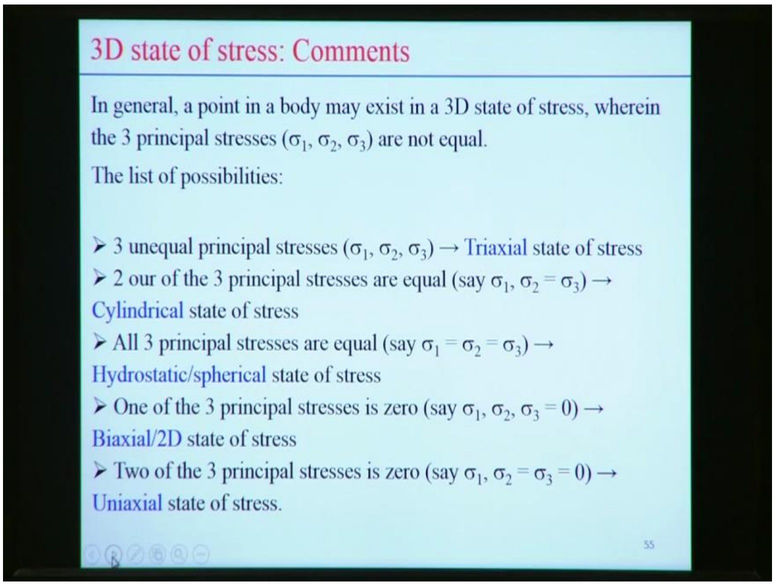

Now we have discussed the different types of 3 dimensional stress state and how we can find out the principal stresses, but the format need to adjust here based on the 3 dimensional stress state and where the three principal stresses are not equal assume that thing, so there maybe the different possibilities that first is the three are not equal principal stresses may exist the actual case or practically. When thus such situation exists so sigma 1, sigma 2 and sigma 3 not equal to with each other then we can Triaxial state of the stress.

Second case is the when two out of the three principal stresses are equal, for example sigma 2 equal to sigma 3 but not equal to sigma 1 then we can say the Cylindrical state of the stress. In third case 3 principal stress are equal, in that stress state we can say that Hydrostatic state or spherical state of the stress. So Hydrostatic state stress we have already discussed. Now in such cases one of the three principal stresses is 0, for example sigma 3 equal to 0 then we can say that Biaxial or 2D state of the stress.

Finally two out of 3 stresses is zero say only one stress is the, principal stress is the 10 then we can Uniaxial state of the stress. So these are the different types of the state stress to practically apply or allies during the analysis of a stress during the actual problem.

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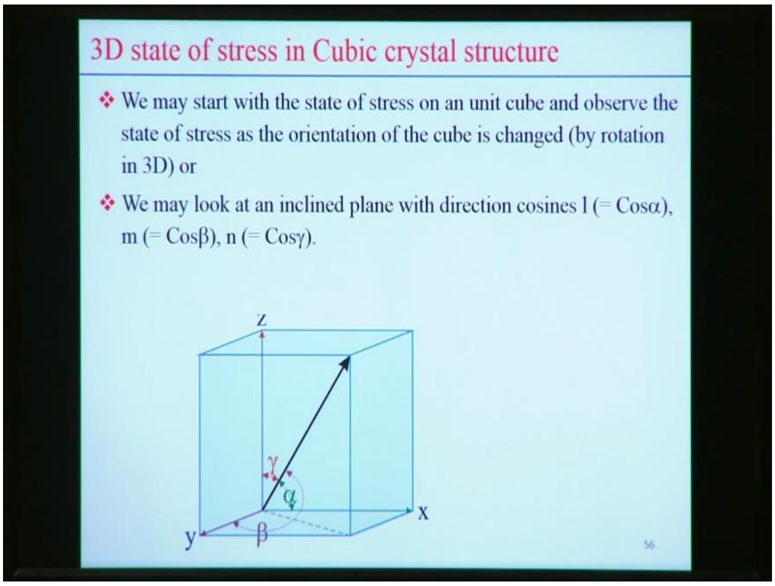

Now we come back to the what is the different type of stress state exists in case of cubic crystal structure, maybe we can start with the stress state on an unit cube and observe that the state of the orientation of the cube is changed by rotation in the 3D or we can take the view that will be a look in the inclined plane with direction cosine L, M and N.

So that direction cosines L is define Cos alpha, this alpha is the angle in one specific direction and with respect to that, respect to that direction, basically the diagonal direction and what is the angle alpha and similarly cost beta direction cost is m, come gamma direction cosine is n. So stress state can be explained simply by observation in the cubic structure or can be looked into that analogy knowing the different direction cosines and we can further analyze these things.

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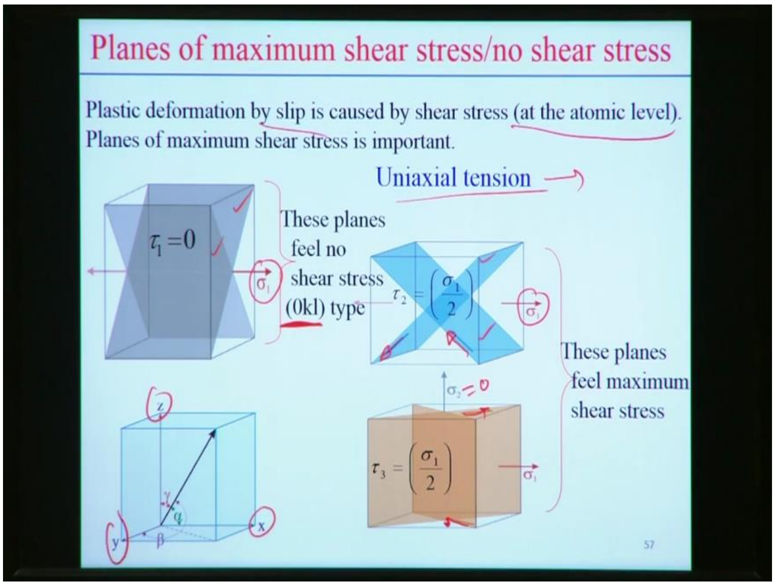

So let us investigate, physical investigate what are the different stress state actually exist in the cubic structure, so here is the most important, plastic deformation actually happens by the slip process at that atomic level, so within the cubic crystal structure it is more significant or important to know what is the maximum or what is the stress of the shear stress within the unit cell.

So let us first in the, under the loading condition let us look into the first Uniaxial tension, so that means when it is subjected to some Uniaxial tension what will be the stress state, if we say that this is the unit cell or unit cubic and it is subjected to Uniaxial tensile and you need to produce the stress sigma 1, that is the only principal stress is acting around this direction sigma 1.

Then you can find out the two different planes exist it is highlighted here, so one plane is this, another plane is there, in the type of 0kl, on that plane the shear stress becomes zero, now if we investigate some other plane working with the state of the shear stress there, if we look into that then another plane, this is the plane and this is the another plane.

So all this plane actually define as a reference the coordinate system X, Y, Z, we can find out that if we apply one principal stresses along on this phase or along this direction, principal stresses is basically acting along x direction then this, with respect to this direction the plane highlighted here at another plane or there exist the maximum shear stress, tau 2 which is up of the normal stress value.

Similarly other orientation of the plane, also sigma 2, but here sigma 2 is equal to 0, and if it is subjected to sigma 1 only Uniaxial stress state then the here also tau 3 that means here also maximum stress can be produce sigma 1 by 2, the different plane is on that plane that is also highlighted then the plane why it is subjected to the maximum stress state, so all this plane can also be defined in terms of the (())(53:54).

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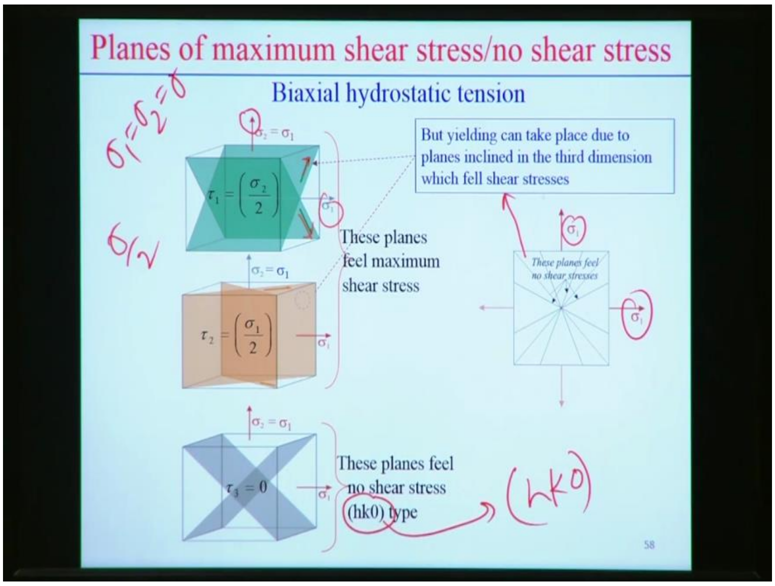

Similarly planes of the maximum shear stress of no shear stress when it is subjected to the Biaxial hydrostatic tension. So if we look into the Biaxial tension say sigma 1 here, and here it is this axis the another principal stress exists sigma 2 but here sigma 1 equal to sigma 2, so with this situation the maximum shear stress actually produce the sigma 2 by 2 or I can say that sigma by 2, sigma 1 equal to sigma 2 equal to sigma, then this maximum shear stress exist on to corresponding plane which is highlighted here.

Similarly another two plane that is also sigma by 2, shear stress two plane that is also is sigma by 2, shear stress two plane that is also exist by Biaxial stress and this fit actually maximum shear stress but some other plane there exists no shear stress so that no shear stress is the simply (())(55:03) is the type of hk0, that type of plane with the unit cell that is subjected to no shear stress.

But if we overall investigate these things then when it is subjected to sigma 1, both the equal Biaxial stress state or both side x and y axis having the similar amount of the stress then we can find out the line this plane feel no, there are several planes exist not having any shear stress, no shear stress. But some other plane which is subjected to amount of the maximum shear stress value, but when everything can be take plane so slip will initiate due the planes inclined in the third dimension which will shear stress.

Although it is explained that there is a certain plane exist, there is no shear stress subjected to the shear stress but using this, because this is completely subjected to the state of the say Biaxial stress, but yielding may occur if there exist some shear stresses in the third direction.

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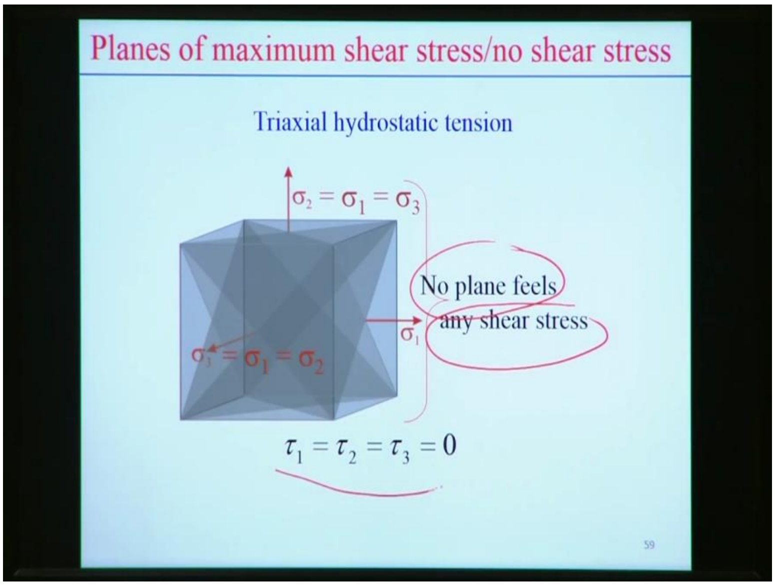

Similarly you can investigate in the case of the Triaxial hydrostatic tension. So in this case the sigma 1 equal to sigma 2 equal to sigma 3, all three axis the stress is equal and then it is typically subjected to the Triaxial hydrostatic stress state, so no planes feels any shear stress in this case, that is the case of hydrostatic tension so shear stress will be 0 with this stress state.

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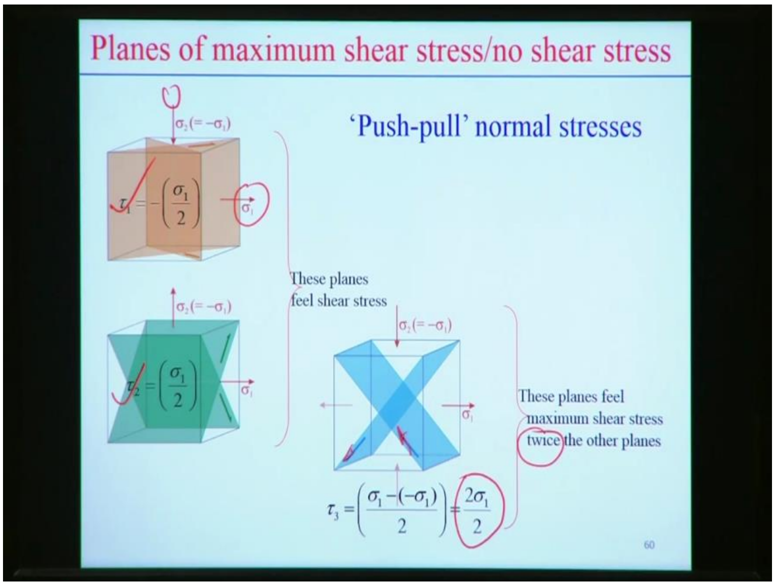

This is another case, for example push-pull normal stresses, that means tensile stress and another side is the compressive stress, with this situation specific plane which is subjected to but thus equal to minus, so here the two state of the stress but they are equally (())(57:18), but the direction is different, one is the tensile and another is the compressive. So in this case the maximum shear stress will be the by 2.

And another cases we can find out the maximum stress is the sigma by 2, one case is the shear stress negative another case is we can positive according the convention so the planes are different here also, so all these planes feel shear stress which is the magnitude of the half of the normal stress value, but in certain planes also exists within the unit cell, here is (())(57:54) so that planes is actually subjected to the maximum shear stress twice that of the other with as compared to the other stresses, so twice sigma y, sigma 1 by 2.

So that means this is the, twice of the what is subjected in this case, tau 1and tau 2, tau 3 is basically 2 times of the shear stress as compared to the other stress. So this is the typical stress state exist with the application of the different load with the cubic cyrstal structure.

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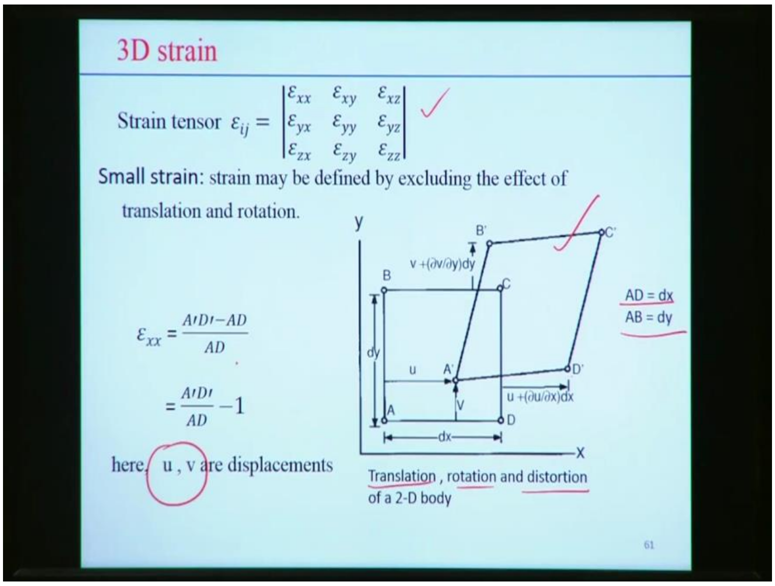

Now we will shift to the next topic that is called 3 dimensional strain and strictly focus on the small strain, so like stress tensor we can find out the strain tensor in the similar way, with respect to the axis x, y and z. But during the deformation actually it is subject to the translation, rotation as well as the distortion, specifically this figure indicates for the 2 dimensional body.

But small strain actually define by excluding the effect of the translation and rotation of the small strain, only distortion. So if we look into this picture, it say displacement with x and y axis along x and y axis, this is displacement u and v. Now suppose A, b, c both are initial position and after application of the role this is the final position a dot, b dot, c dot and d dot.

So this final position and the initial position if we compare the final position actually subjected to both translation, rotation as well as distortion. Now how we can find out the distortion in this case in terms of the displacement u and v here. Suppose ad equal to dx, ab equal to that elements, ab equal to dy.

Now when you (())(60:02) the strain along x axis so Epsilon xx is the what is the a dot, d dot minus initial value ad with respect to the original ad so this is the expression of the strain in this case.

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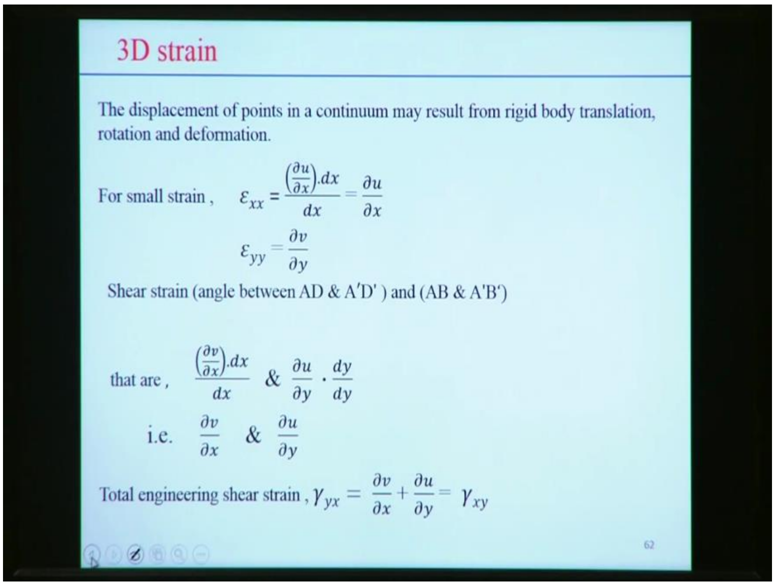

Now the displacement of the point in continuum may result from the rigina body translation, rotation and deformation in actual case but when you try to focus on the small strain tensor then we can estimate difference…

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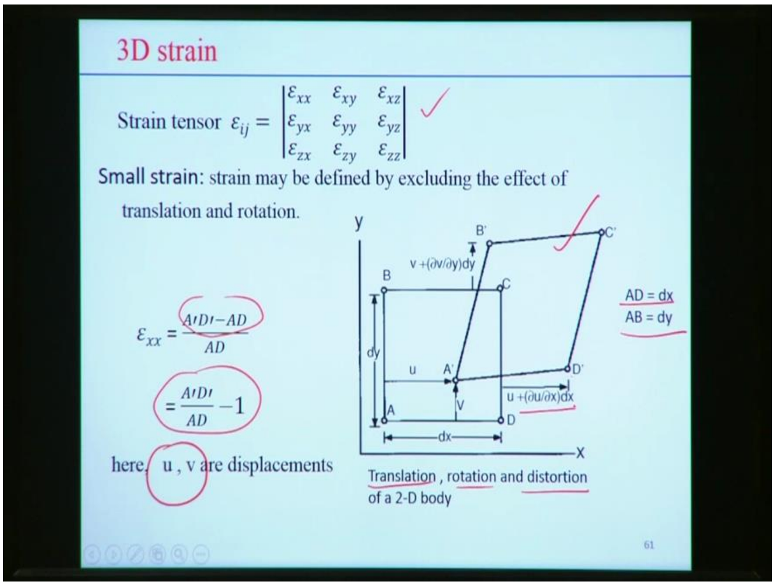

This a dot, d dot ad in terms of the del u by del x, with respect to dx, this is the initial, it is the final and what was the initial A.

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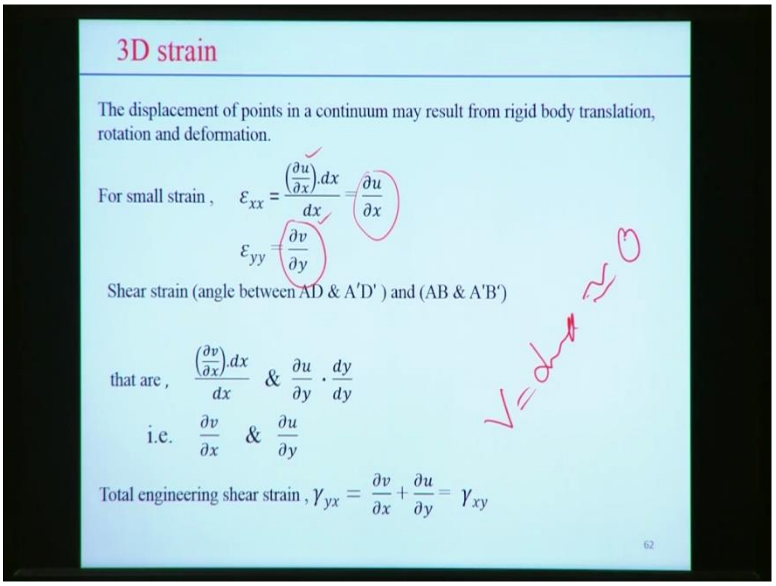

So corresponding to that, that is the difference and with respect to the initial level, so del u by del x actually represents the strain here. Similarly Epsilon yy that means y direction the normal strain can define del v by del y, so this is the partial derivative of displacement d with respect to x and y. Now we try to represent the strain, sheer strain basically in terms of the angular form, so in small distortion case the shear strain can be define, can be approximated, simply the angle.

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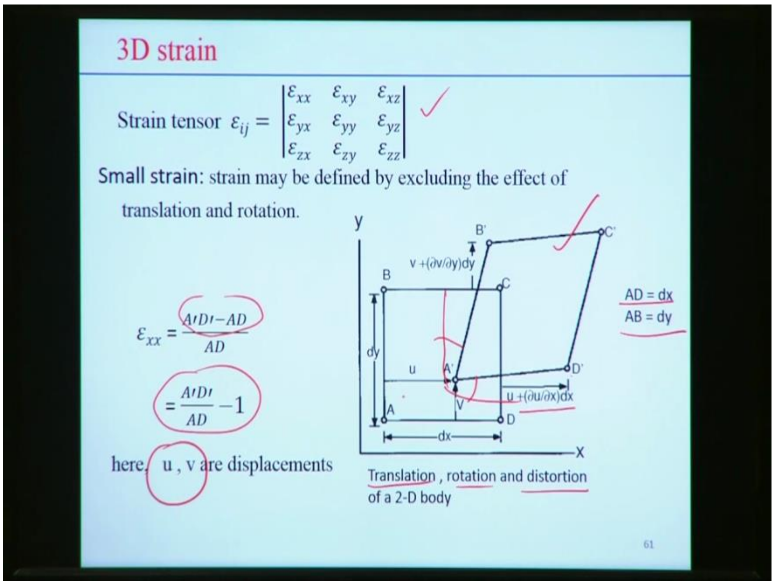

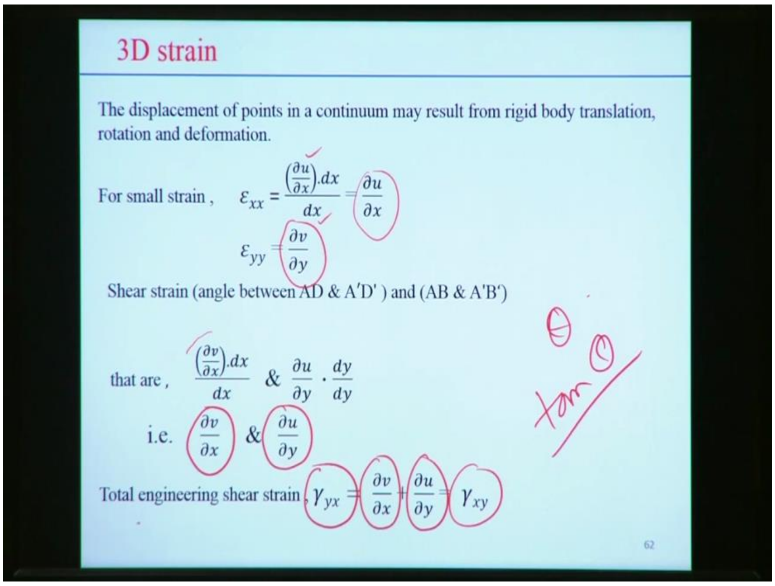

So shear strain actually represents this part, the total angular form here. So the angle between ad and a dot d dot, and ab and a dot b dot, that angle this angle actually represents the shear strain so from here you can find out that basically that angle is theta we try to find out the. If angle is theta we try to find out the tan theta here and from that tan theta this dx, and we can find out del v by del x and another component is the del u by del y. So total angular deformation, we can say that comes in the shear stress which is the del v by del x plus del u by del y and due to the symmetric measure we can find out gamma yx equal to xy.

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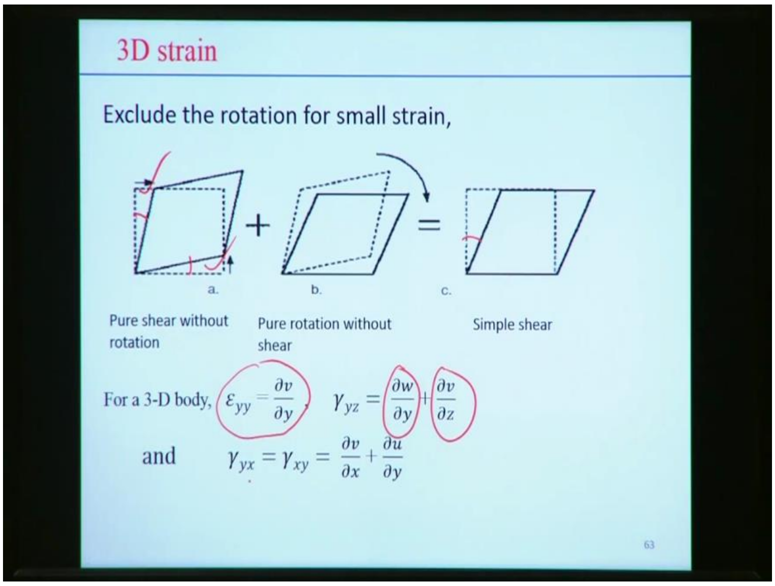

So this at the elementary shear strain we can find out m if we see, that the pure, if you see the vertical representation of the shear that if you see with application of the load, with small 2 dimensional element this is the pure shear happen without any rotation so simply the initial position with the dotted line with application of the load, it states at cell. The solid line actually state the final state that is subjected to Po shear without any rotation.

Now with a deform cell if we simply rotate then it takes this cell that is called the simple shear so that simple shear this angular form, actually consists of this two both, both part is there. So here we can define the quality difference of the normal stress, normal strain is like that for the shear strain is the, this component and that component which comes from two different part and finally we represent the shear stress by two component.

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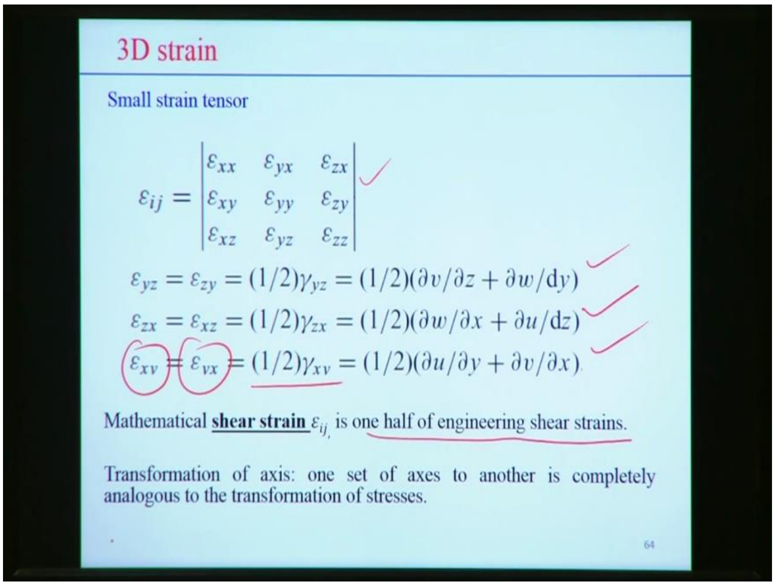

Now small strain tensor can be represented the similar way what we did for the stress tension and this is the typical expression of the shear strain component. Mathematically the shear strain Epsilon ij is the half of the engineering shear strains. So that is why we have written the Epsilon xy or yx or whatever this is half of the shear strain and we can represent this as the components of the shear strain. Now transformation of the axis what we did in case of the stress analysis is similar to the, in this case it is also similar to the translation for the stress analysis form.

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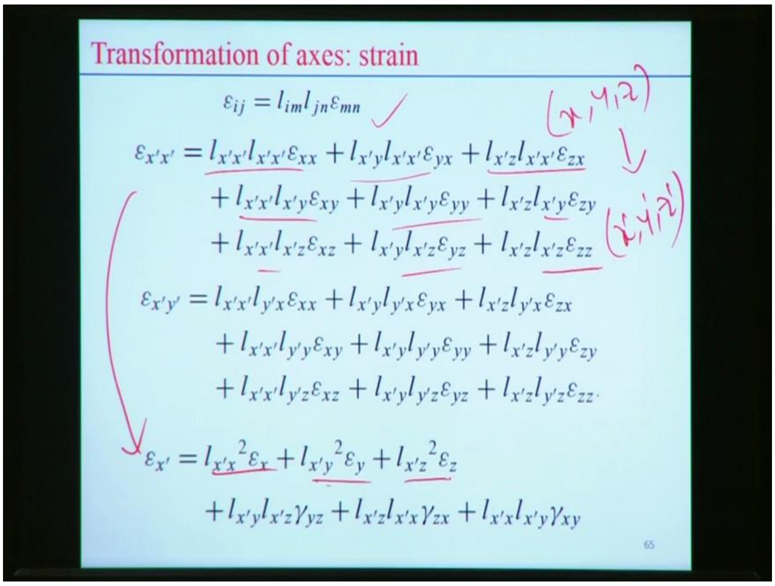

We can use simple formula transformation of the axis like that Epsilon ij equal to to limljn and Epsilon mn, so we can find out the l, l represent direction cosine and here we are trying to find out the transformation with respect to x, y, z that is correspondent to x dot y dot and z dot, that axis system is the state of the strain. Similarly using the direction cosines we can find out 9 components and that again 9 component can also be consider as the 6 component within the symmetric measure of the strain component as well.

Similarly shear strain can also be calculated from the 9 component to forward to the 6 component similar to the stress analysis part. So now transformation on the axis in the strain specifically here the analysis is focused on the small strain but the large strain, (())(66:06) on the large strain tensor, the large strain are not tensor and cannot be transformed from one axis system to another simply by tensor transformation.

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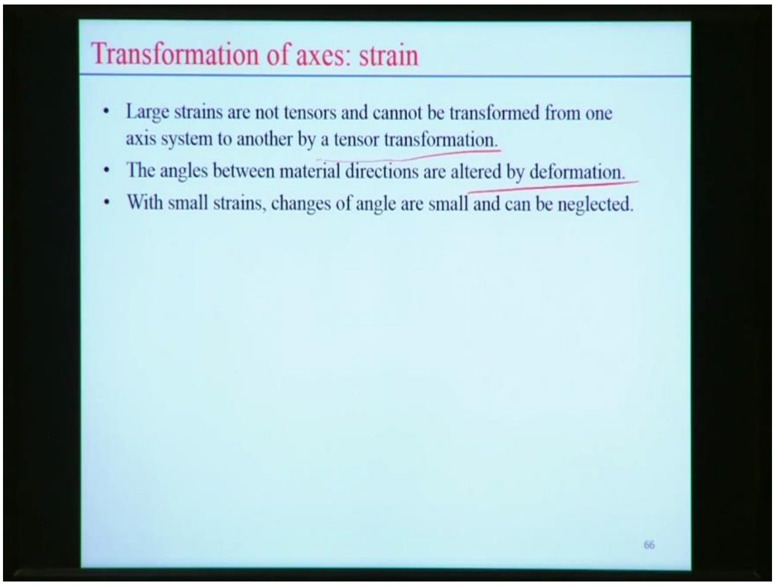

The angle between the material directions are altered by deformation that is also through because this large strain tensor we cannot transform using the simple transformation because in this case the angles between the material direction actually altered by the deformation, but with small strain change of angles can be neglected and we can use this transformation of the axis from one axis system to the another axis system, there is a small strain tensor exist.

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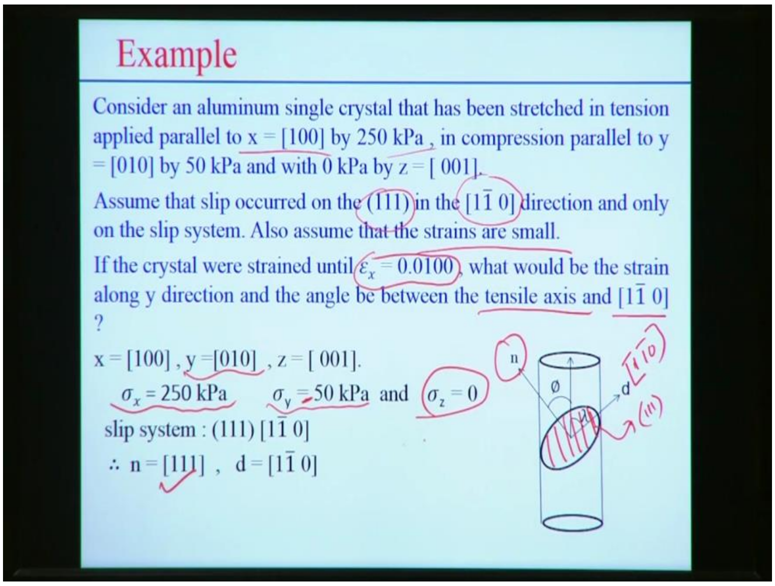

Now we look into one example to make the clarification or application of this transformation from one axis to another axis and to apply in the problem itself, so similar kind of problem we have discussed but this is the further extension of that… For example consider aluminum with single crystal that has been stretched in tension applied parallel to the x axis 100 by 250 kiloPascal.

So sigma x is basically 250 kiloPascal in compression parallel to y axis, so this is the y axis and it is subjected to the 50 kiloPascal but it is a compression with 0 kiloPascal, so sigma z equal to 0. Now slip system so I think sigma y can be written as minus of 50 kiloPascal. Now with the slip occurs on this plane 111 and 1 bar 1 and 0 this direction, for example we define this slip plane, this corresponding to the plane 111 and direction d with slip direction occur 1 bar 1 0.

Also assume that the strains are small so in the case if the crystals were strained Epsilon x is 0.01 then what maybe the strain along y direction and the angle between the tensile axis and 1 1 bar 0 we need to find out. So first we will define this plane, the normal direction to this plane, n is basically 111 index, and d 110 so that z axis is also defined. So all axis system is define here, further direction also define here.

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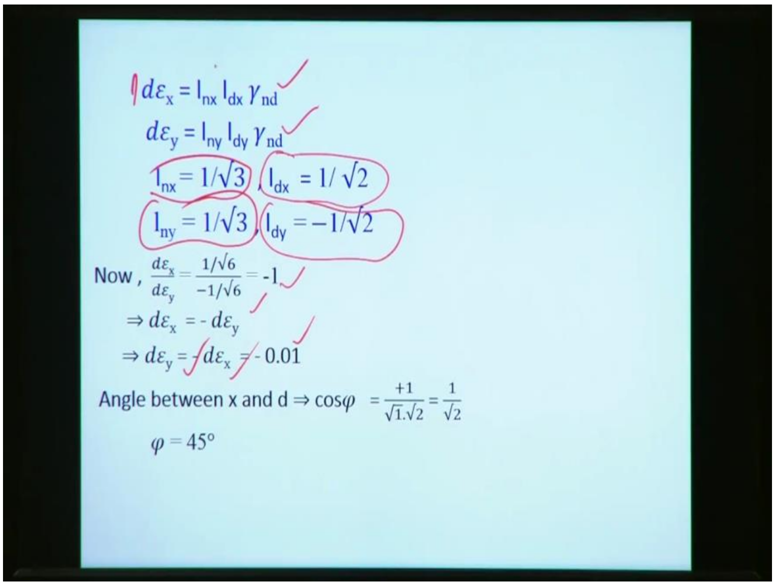

Now we use the transformation of the axis from one axis to another axis, we can find out, so here d Epsilon x is the consist of the nd and d Epsilon y the two direction cosines and the strain with respect to n and d. Now here we can find out the direction cosines similar to l nx and l dx, similarly in l ny and l dy. All direction cosines we can find out. Now we can find out the ratio of the strain it is minus 1, so d Epsilon y by d Epsilon x almost the ratio is the same.

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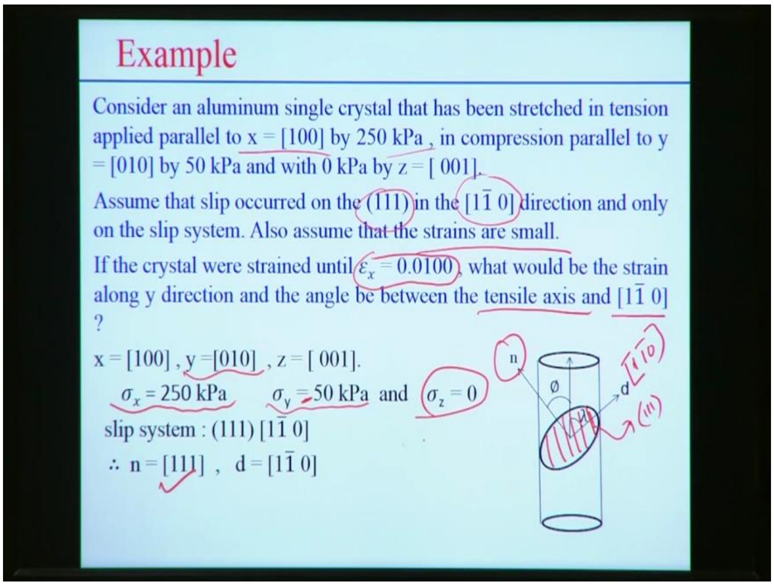

So similarly with this stress state we can simply find out the range of the stress is same as the range of the strain.

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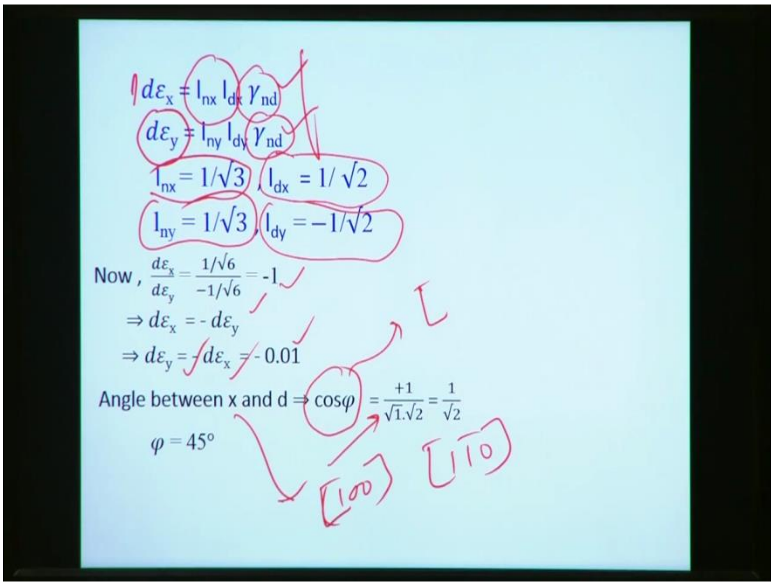

But this analysis we can find out the data of the applied stress because we assume that normal strain along x axis is equal to the direction cosines and with respect to the shear strain with respect to the nd, so similarly with respect to y is the same amount of the shear stress along the direction d on the ratio so that is why the direction cosines are same so it is the range of the strain with y axis is also the same.

Now angle between the x and d can be consider as the Phi so that means x and d, x equal to 100 and d will probably define…We have written the tensile axis and 110, so x axis is the tensile axis here, at 11bar0, so 11bar0 if we put into this thing, we can find out the angle between 1 by root 2 that means Phi equal to 45 degree. So these are the typical problem so simply using the knowledge of the transformation of axis, we can apply this problem to solve what to analyze the stress state and specifically happening within the crystals itself.

Single crystals or maybe we can use it in the poly-crystal to solve the strain and plastic deformation or slip mechanism in poly-crystal and (())(71:54) as well. So hopefully it was understandable the analysis of the strain and actually the purpose was to analyze all these strains, how to apply the knowledge of the 3 dimensional or 2 dimensional stress strain, specific to these subjects that means in case of different crystal structure or plastic behavior of the different crystals in this case.

So you enjoyed this analysis of the stress state and we will simplify to it in case of 2 dimensional as well as the 3 dimensional so next time I will stick to the next module that was, that is basically the different theory of the plasticity that we used various specific to the continuum scale or normal materials what not very specific to the individual crystals as well. So thank you, thank you very much for your kind attention.