Crystals, Symmetry and Tensors
Professor Rajesh Prasad
Department of Materials Science and Engineering
Indian Institute of Technology, Delhi
Lecture - 28 a
Magnitude of a second rank tensor property in a given direction

Let us now discuss what is the meaning of magnitude of a second rank tensor property in a given direction.

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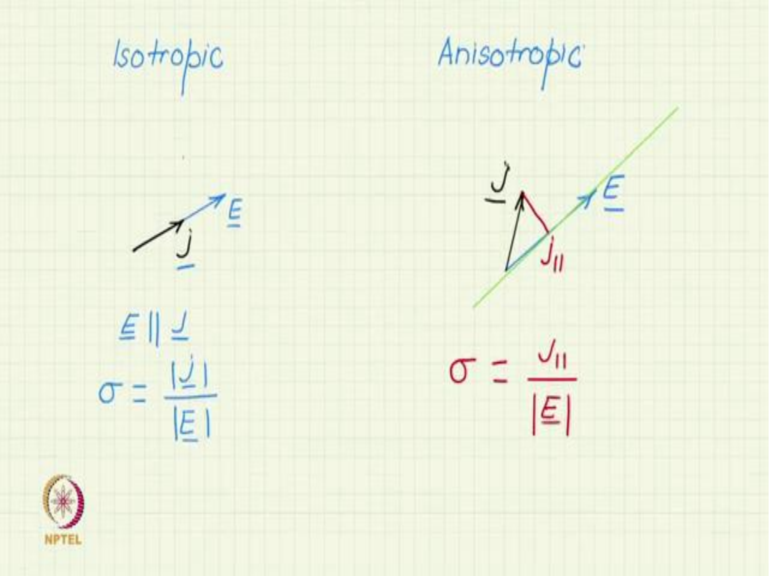

Because once the second rank tensor is involved, you know that let us consider the isotropic material and let us continue with our example of electrical conductivity. So, if you have an isotropic material, you apply an electric field in a given direction. The current density is also generated in the same direction. The direction of the electric field and direction of the j are parallel. So, in this case, there is no issue, and we define the electrical conductivity sigma simply as magnitude of j divided by magnitude of E. Both are in the same direction.

So, the vector nature here is really not so important. They can be treated even as the scalers. But as soon as you come into the anisotropic domain, we have seen that although I apply an electric field in certain direction, j may not appear in that direction, j may appear in some other direction. So, the direction of the current density vector is not the same as the electric field. So, how do I define, if I want to define the electrical conductivity in a given direction, which direction should I choose?

The electric field direction or the current density direction. So, by definition in this case, we take the direction of the electric field as the reference direction. So, suppose we want to find the conductivity in any given direction, we apply an electric field in that direction. The current density generated may not be in that direction, but then we take the projection, we take the projection of j, let us call that j parallel in the direction of E.

And we define the electrical conductivity in the given direction as J parallel divided by the magnitude of E. That is component of the current density in the direction, magnitude of the component of the current density in the direction of the electric field divide by the magnitude of the electric field is the conductivity in the direction of the applied electric field.

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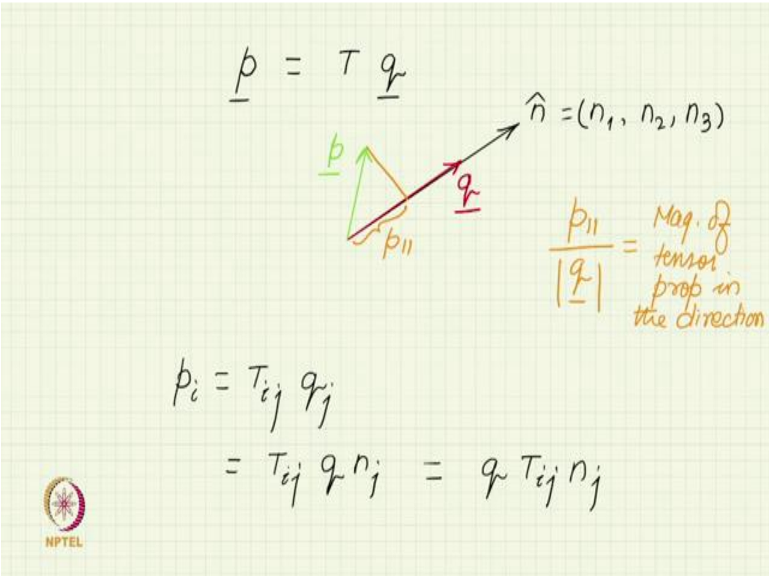

So, this is true in general for any second rank tensor, because all second rank tensor always relate some vector p to some other vector q. So, this means that if I want to find the magnitude of the tensor property in our direction of interest, we have to apply q in that direction. So, the black is the direction of interest, we apply q in that direction. p will be generated in some other direction based on the tensor. And then we take the projection of p, that is p parallel and define the tensor property in the black direction, direction of our interest as p parallel by the magnitude of q is the magnitude of tensor property in the direction of interest.

Now, let the unit vector along this direction be n cap with components n1, n2 and n3, so we can express the property in terms of this unit vector and the tensor. So, we have this, the vector p in terms of components is p i and that as we have seen from the tensor equation is T i j, q j. So, q j is the applied vector and p i is a resulting vector.

Now, q j can be written as the magnitude q times n j, where n j are the components of the unit vector. Or we can say that they are direction cosines of the direction of interest. So, we can write this as q times T i j, n j. Now, as we are seeing that we have to take the component of the vector p along n to find the property along direction n, we have to take the component of p along n.

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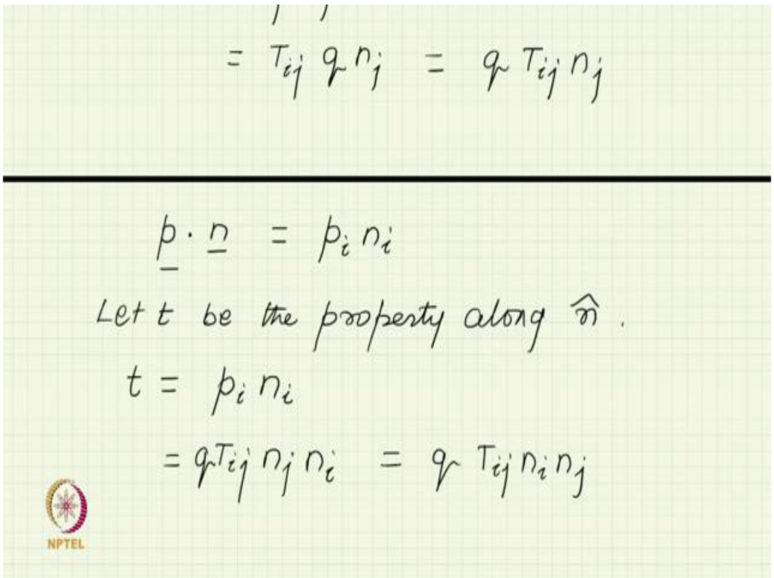

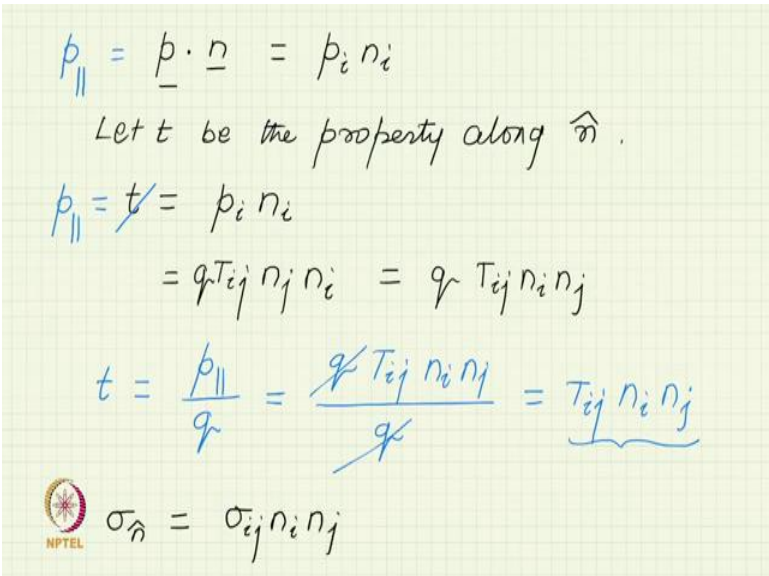

So, component of any vector along a direction is the dot product with the unit vector. And this can be written as in the indicial notation and in an orthonormal basis, we can write this as p i, n i. So, the property T, property in the direction, so T is the tensor, but now I am using T for the scalar value of the property along that direction. Or shall I use, maybe, let me use a slightly different symbol, let me say that t. Let t be the property along n.

So, that means we have to take the parallel component and that is p i, n i. So, t is p i, n i, but p i from what we have derived above is T i j, q times T i j, n j, n i. Or I can rewrite it as q T i j, n i, n j. Sorry, this t is not the property, so let me make the correction. T is the component, so this is t is the component of p. So, what I have written here is the parallel component of p. Let me write it as p parallel, so this is p parallel.

Now, by definition, t is p parallel divided by the magnitude of the applied vector q. So, the resulting vector, the component of resulting vector along the direction of interest divide by the magnitude of the applied vector. So, if we write this now, by q and cancelling q, gives us the final formula, T i j, n i, n j. So, property in any direction is just this expression, T i j, n i, nj.

This of course has a double sum both over i and j, so there will be nine terms in this summation. So, in the specific case, for example, electrical conductivity, electrical conductivity in the direction n, let us say electrical conductivity along the direction n is the conductivity tensor sigma i j times n i, n j. Thank you.