Crystals, Symmetry and Tensors
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Lecture - 27 f
Coordinate Transformation of tensors of different rank

We have seen how coordinate transformation works for vectors and we also saw how coordinate transformation works for a second rank tensor.

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So, now we want to extend this idea not so much by derivation but by analogy to coordinate transformation of tensors of different rank.

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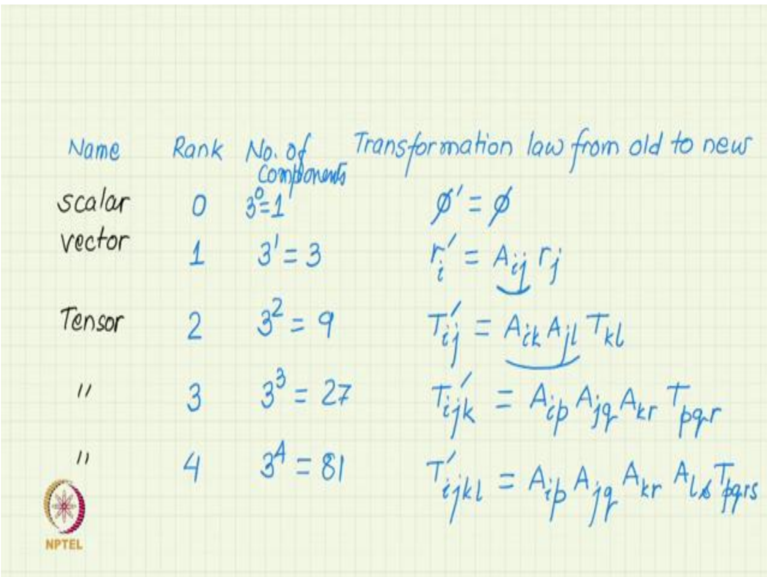

So, to do that, let us first look at a scalar and vectors, they can also be thought of as tensors. So, a scalar is a tensor of rank 0. If I can add one more column for number of components. So, a scalar has one component, 3 to the power 0, is equal to 1 component. And what is the transformation law for a scalar? Since a scalar does not depend upon the coordinate axes, a scalar will remain in variant. So, coordinate transformation has no effect on the scalar, so if we write the scalar, let us say as sum number phi. So, phi prime is equal to phi.

Vector is a tensor of rank 1. So, it has 3 components, 3 to the power 1 equal to 3 components and we saw that for vector we have r prime i is equal to A i j, r j, where A i j is the coordinate transformation matrix. Now, for second rank tensor, we saw that it has 9 components, 3 to the power 2.

And in our example, we used sigma, the electrical conductivity sigma i j, but let me write it in general as a tensor T. So, T i j is equal to A i k, A j l, T k l. So, you can see that the scalar, which was the zeroth rank tensor, did not require any A in its transformation. The vector, which is first ranked tensor required A one time. Whereas T i j, which is a tensor of second rank, is requiring the product of two A's, A i k, A j l.

By analogy, we can write the transformation law for third rank tensor which will have 27 terms. I give the same symbol T, but now I write three subscript for the third rank tensor and three A. So, A, let me write A i p, A j q, A k r. Sorry, here I missed the prime in second rank also, left hand sides are all primed. They are the new components. And T prime p q r. Sorry, on the right-hand side, not T prime.

And now finally for fourth rank tensor, 81 terms. So, T prime i j k l will be A i p, A j q, A k r, A l s, T p q r s. So, you can see as the rank of the tensor is increasing, the number of subscript is also accordingly increasing. And for transformation you require more numbers of A's in the expression. Of course, fourth is not the highest rank tensor, you can have tensors of higher rank also, but we stop the discussion here and for higher ranks also the same analogy works, and you can write transformation law for tensor of any rank. Thank you.