Crystals, Symmetry and Tensors
Professor Rajesh Prasad
Department of Materials Science and Engineering
Indian Institute of Technology, Delhi
Lecture 84
Index Notation

We will discuss an Index Notation which also is related to Einstein's summation convention. In this video, these two notations and summation convention is very very important in handling of tensor quantities.

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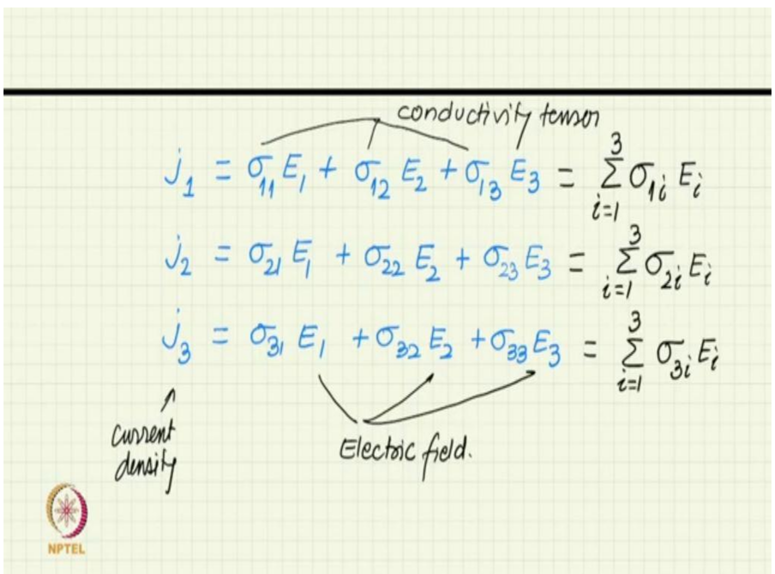

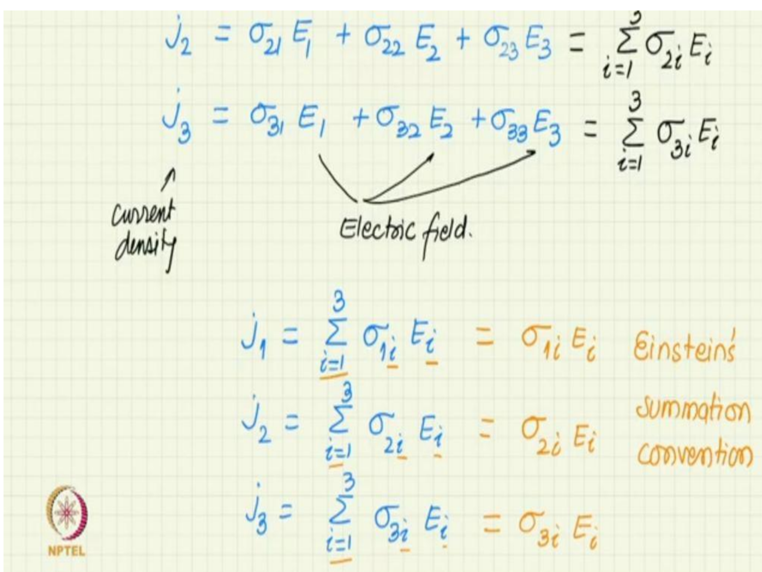

So, we saw while discussing the introduction to tensor that electrical conductivity was written as a tensor; and the nine components of the electrical conductivity sigma11 to sigma33 relates the three components of the current density vector. 3 components of the current density vector to 3 components of the electric field vector; these are the three components of the electric field vector. So, each component of the current density is related to the 3 components of the electric field; and that gives 3 components of the conductivity tensor. These quantities are the conductivity tensor.

Now, we can write this equation in the summation convention using sigma as you all know; so, I can write the first equation as. So, I can write this as sigma1i Ei, i going from 1 to 3. Similarly, j2 if sigma2i Ei, i going from 1 to 3; and J3 sigma3i Ei, i going from 1 to 3. So, this notation is a little bit more compact and writing them out in full; summation saves us writing all the terms. Now, Einstein observed and introduced a new notation for further simplifying in this. If you look at the right-hand side term, you find that the summation is over I; and i is the subscript which is repeating in the right-hand side term.

One is a fixed term fixed index, where i is a repeating index; and the summation is on the repeating index. So, in all three cases, the summation is over the repeating index. So, the simplification which Einstein introduced was simply to drop the summation sign, and write it as sigma 1i Ei. And assume that whenever the index is repeated, summation is implied; so, it becomes even easier to write. So, this is what is Einstein’s summation convention, sometimes humorously referred to as Einstein’s greatest contribution; so, it makes writing the equations easier. And further, all the three equations if you see have the same form j1 is sigma 1i Ei, j2 is sigma 2i Ei, and j3 is sigma 3i Ei.

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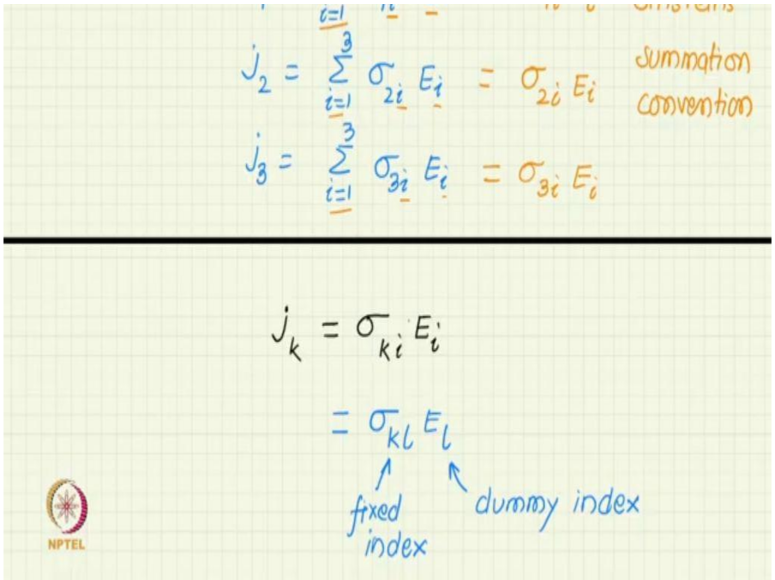

So, the three equations can jointly be written as using another index jk is equal to sigma ki Ei, where both i and j, i and k are now running from 1 to 3. But, since on the right-hand side, i is repeated summation is only over i, k; we do not do the summation. But, since, as k varies from 1 to 3, we reproduced the three equations j1, j2 and j3.

So, sometimes because you can also see an interesting thing that since summation is over the repeated index, it does not matter what repeated index I use. So, even if I write this as, let us say kL EL, where I use another index L. It does not make any difference in the equation; because i going from 1 to 3 gives us the same three terms on right hand side, as L going from 1 to 3.

So, L being repeated is again indicating summation over L; and the variation of L is from 1 to 3, so you get the same expression. So, that is why these indices the repeated indices are sometimes called dummy index; you can replace them by any letter. But, k is not dummy, and if I make k to some other subscript, then the equation will become inconsistent; because k appears on the left-hand side, but not on the right-hand side. So, this is the fixed index; and same fixed index should appear in the equation on both left-hand right-hand side. But, it does not matter what dummy index appears on the right-hand side of this equation. It can be replaced by any other subscript. Thank you very much.