3D Space Groups II: Trigonal / Rhombohedral / Hexagonal confusion

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Lecture 22 b

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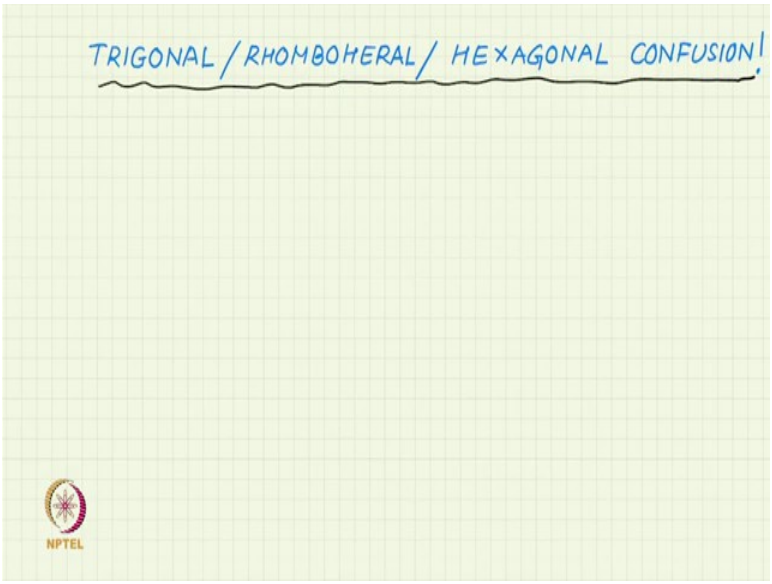

In this video, I want to address a very widespread confusion regarding the uses of word trigonal rhombohedral and hexagonal, that is why I have titled it trigonal rhombohedral hexagonal confusion. So, the confusion is because these words sometimes are used in different meanings in by different authors and there are several such instances in the literature. However, International tables or International Union of crystallography through their publication international tables have tried to clarify the situation or try to recommend a situation.

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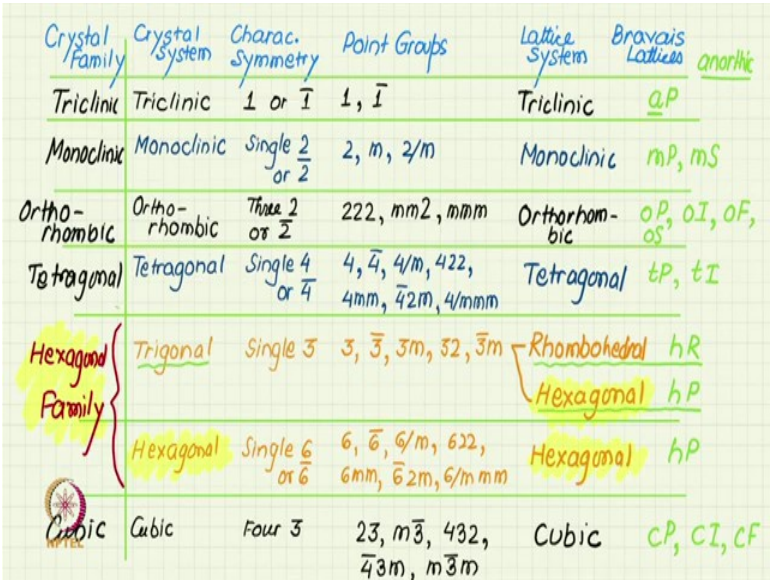

So, recommend a certain use of these words. Let us look at this table we are familiar with this table from earlier videos which I have given on this topic. So, essentially, we are defining this Crystal system and there are 7 of them, 7 Crystal systems from triclinic to cubic and the essential definition of the crystal system is through characteristic symmetry, that is each Crystal system should represent certain characteristic symmetry.

So, triclinic for example has only 1 or 1 bar symmetry. So, a crystal which has 1 which is no symmetry or 1 bar and inversion Symmetry and that is all no other symmetry of higher order. So, that kind of Crystal will be called a triclinic crystal or will belong to triclinic Crystal system. Similarly, tetragonal should have 4 or 4 bar cubic has 4 3 4 axis as we all know. So, these are the characteristic symmetry or defining symmetry we can say for the Crystal system.

Of course, there are only 32-point groups and based on these defining symmetry different point groups belong to different Crystal system. So, 1 way of looking at Crystal system is that it is the classification of these 32-point groups. So, again, the point groups 1 and 1 bar belong to the triclinic group point groups 222, mm2, and mmm will be orthorhombic these 7-point groups 6 6 bar 6 by m 622 6 mm 6 bar 2m and 6 by mm, it is a hexagonal and tetragonal are very rich in point groups each of them have, both of them have 7 point groups each.

So, these 7-point groups belong to the hexagonal Crystal system. In each of these point, group either you have a 6-fold axis or a 6-bar axis, so far, so good. So, crystal says 7 Crystal systems, 7 different classifications of 32-point groups into 7 types. However, there are, if we

now look start looking at the lattice that is where little bit of confusion begins and let us try to now write the lattice system for each of them.

Lattice system is not very commonly used but it is there in the international table and we can we should try to use it. So, triclinic there is no issue in using lattice system for triclinic, monoclinic because in these cases lattice system and Crystal system are exactly the same. So, there is a 1 to 1 correspondence between them. So, these 6 Crystal systems correspond to 6 lattice system, there is no issue.

However, in the trigonal system in the trigonal Crystal system there is an issue and that and it will over start overlapping with hexagonal and that is why I have kept them in a different golden colour. So, trigonal system, trigonal Crystal system has 2 different lattice systems associated with it and that is rhombohedral. So, first difference you can see that a trigonal system, trigonal Crystal system is the only Crystal system whose lattice system does not have exactly the same name as the crystal system.

So, Crystal system is named trigonal but the lattice system is rhombohedral, but that is not all, trigonal has 2 lattice system associated with it the, other lattice system is now just to add to confusion is called a hexagonal system. So, trigonal Crystal system has 2 lattice systems rhombohedral and hexagonal and similarly, you can see now that the hexagonal lattice system belongs to 2 Crystal system it belongs to hexagonal as well as it belongs to trigonal.

So, then it could have been better, you may say that a better terminology could have been used and all but these things are historical and we are now stuck with it and this is the recommendation also of international tables. So, let us now, let me now write the Bravais lattices in each system. So, triclinic system, the Bravais lattice is only primitive and the letter symbol used is a here, not T, because T will probably T is used for tetragonal. So, a is for anorthic, this is the only system in which none of the 3 axis need to be orthogonal.

So, and only a Primitive lattice is there, primitive Bravais lattice is there, that is AP. Monoclinic, you can have monoclinic primitive or you can have unscented monoclinic which I am writing as ms, s is standing for side centred, then orthorhombic there are 4 lattices, this is the maximum number of lattices. So, orthodontic p, orthorhombic I, body centred, orthorhombic f, face centred and orthorhombic s, side centred tetragonal has 2, the Primitive tetragonal p and body centred tetragonal I.

Rhombohedral trigonal lattice has 2 trigonal Crystal has 2 lattice system rhombohedral and hexagonal and both are having the letter symbol h. So, h R and h P, hexagonal rhombohedral and hexagonal primitive. So, this is what leads to confusion, you see that the system is trigonal but it can still have a lattice which is hexagonal P or it can have rhombohedral lattice which again is labelled hexagonal R.

Hexagonal lattice, hexagonal system has hexagonal lattice h P and cubic Crystal as you know very well has 3 Bravais lattices cubic P, cubic I, and cubic F, primitive, body centred, and face centred. So, you can now, I have this First Column which I have not yet labelled, I have not given the names here, the crystal family, Crystal family again has one-to-one correspondence with the crystal system, for most of the system except for trigonal and hexagonal.

So, if I write the family triclinic system belongs to triclinic family, monoclinic system belongs to monoclinic family, orthodromic system belongs to orthorhombic family, tetragonal to tetragonal family, in cubic to cubic. But as you can see the Bravais lattice hexagonal P belongs both to trigonal and hexagonal system and the hexagonal symbol H is used for trigonal lattices also. So, with this in mind, these 2 Crystal systems were decided to put in one family and that family was named as hexagonal family.

So, you can see that the hexagonal word is being used in many different senses, to be precise in 3 different senses. Hexagonal can be used for hexagonal family, hexagonal can be used for hexagonal Crystal system in which case it means these symmetries or hexagonal can be used for hexagonal lattice system in which case it can be lattice of either hexagonal Crystal or even for a trigonal Crystal.

So, the word hexagonal has many different meanings. And also, note that although you have trigonal Crystal system there is no trigonal lattice system at least everyone or trigonal lattice there is no primitive trigonal lattice. Trigonal Crystal system, if it has a primitive lattice there will be a hexagonal primitive lattice or a primitive lattice in hexagonal are lattice.

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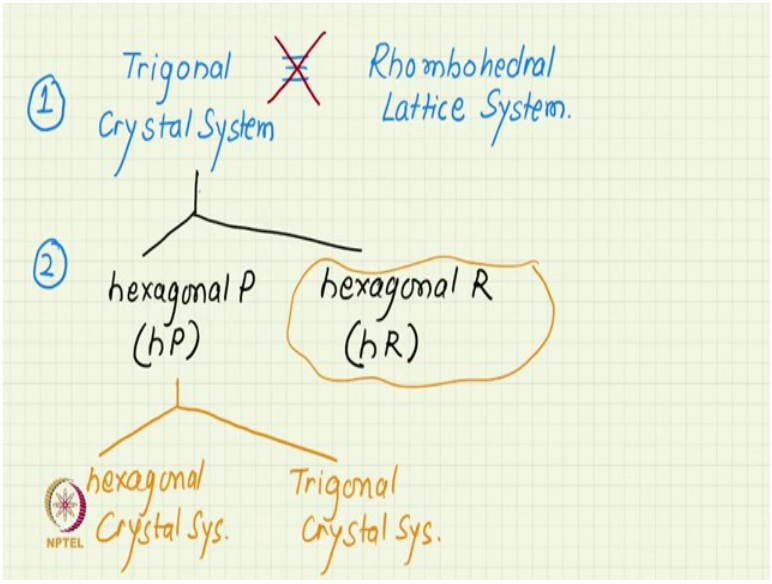

Let me try to summarize some take home message. So, one take-home message is the trigonal and rhombohedral, trigonal and rhombohedral are not equivalent or interchangeable words this is not equivalent. Trigonal is used for trigonal Crystal system, so there is a crystal system named trigonal, rhombohedral is used for rhombohedron lattice system, there is no trigonal rhombohedral Crystal system there is a rhombohedral lattice system. So, this is one point to keep in mind, the second point to keep in mind that within the trigonal Crystal system, there are actually 2 lattices.

Not one but 2 lattices, one of them is hexagonal P or h P and another 1 is hexagonal R or h R. So, 2 lattices belong to the trigonal Crystal system. And if you look at the hexagonal P lattice system, this also as we have seen belongs not only to hexagonal but also to trigonal Crystal system. So, it belongs to hexagonal Crystal system as well as to trigonal Crystal system.

So, I think we have, I have been able to confuse you sufficiently by this time. So, it is time to stop here, but in one of the future videos I will take up I have not yet explained exactly what is this hexagonal R lattice which is usually many times not shown in simpler or Elementary textbooks. So, we will go into the details of hexagonal P and hexagonal R lattice for the trigonal Crystal system in another video. Thank you very much.