Crystals, Symmetry and Tensors
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Lecture 21 c
3D Point Groups XVI: Group-Subgroup Relations

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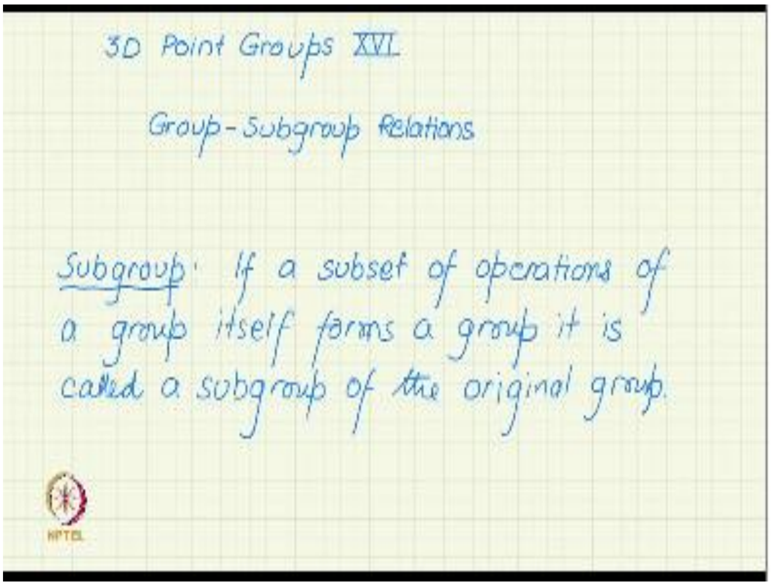

In this video we will discuss group subgroup relations. So, I have defined subgroup here you can look at this definition. If a subset of operations of a group itself forms a group, it is called a subgroup of the original group.

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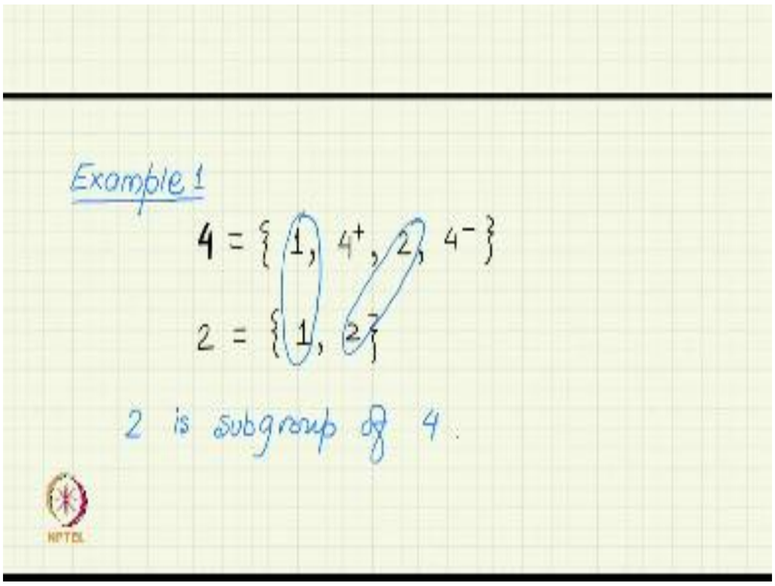

We will look at an example so I take a very simple example. Let us say the group 4 so, group 4 consists of as you know 4 operations the identity counter clockwise rotation by 90 degrees a rotation by 180 degrees to and a clockwise rotation by 90 degrees which is 4 minus.

Similarly, if you look at the group 2, group 2 has only two operations 1 and 2. But both these operations of group 2 are also present in group 4. So, the identity is present in group 4 and the 180-degree rotation 2 is present in the group 4. So, we will say 2 group 2 is subgroup of 4.

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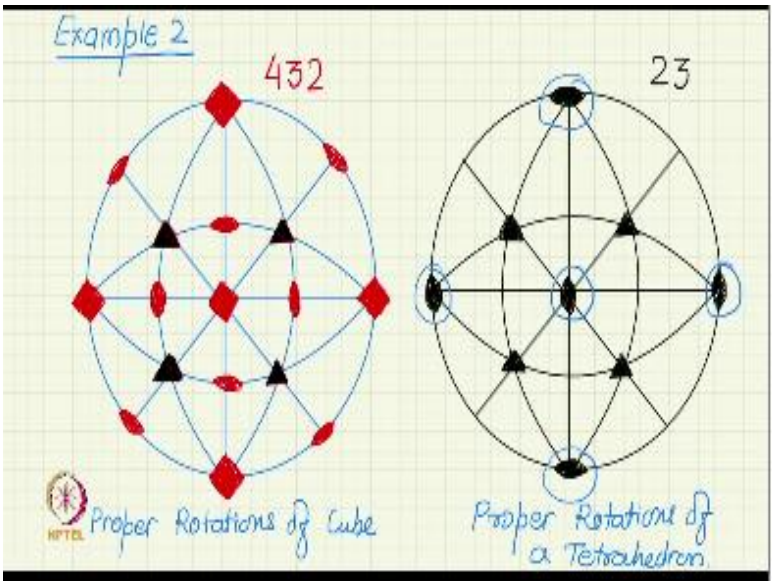

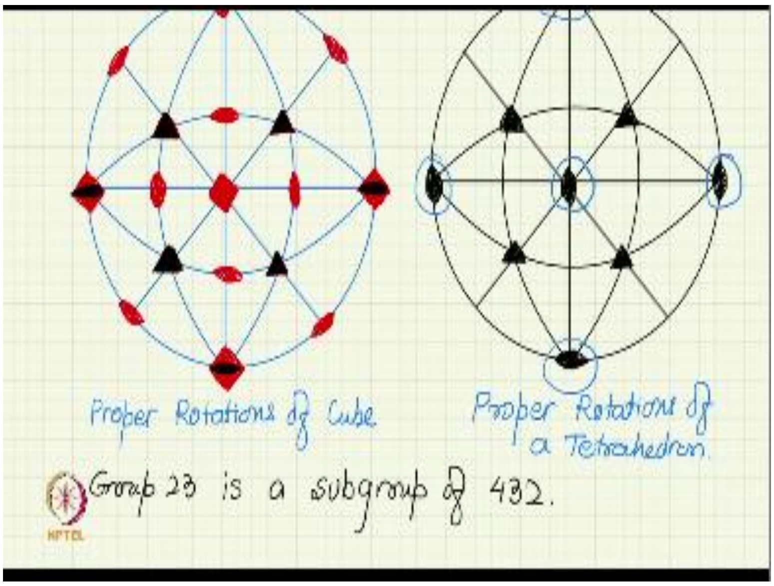

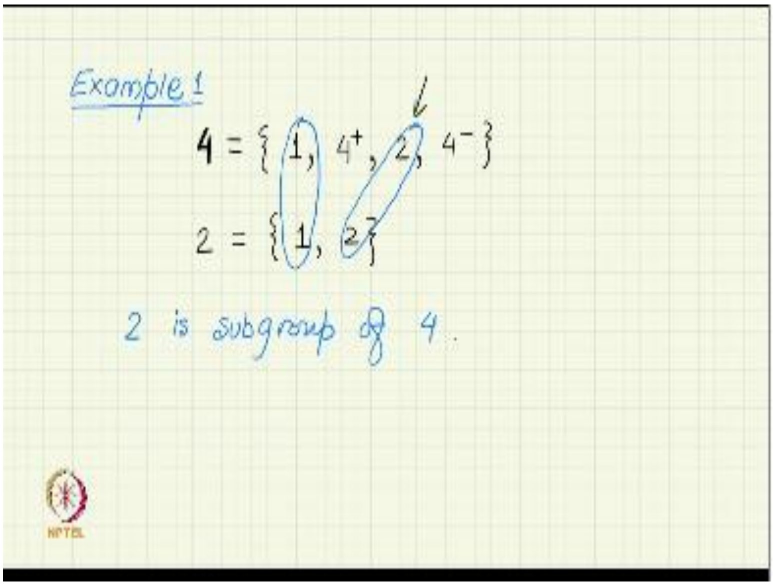

Let us take a little bit more involved example and we go to the cubic groups both of these 432 and 23 are cubic group. Recall that 432 is the group of proper rotations of cube of rotations of cube and these are proper rotations of a tetrahedron.

Now, if you compare these stereograms you can see that the operations of 23 are all included in 432. So, if I colour the 432 operations also in black to emphasize this relation. So, for example this 3 fold along the body diagonal is also presenting 23.

All the 43 folds are present 23. But 23 also has the 2 folds for example, these 2 folds. But there are 3 orthogonal 2 folds. So, this is one 2 fold this is another 2 fold and this is the central 2 fold. These are 3 mutually orthogonal 2 folds and if you see these 3 folds are present also in this 432 the 432 central 4 fold. I have drawn mistakenly as 2 fold. So, let me fix that so originally it should be a 4 fold for 432 I did not notice that. But now these 2 folds of 23 are exactly aligned with the 4 folds of 432. And as you have seen here that the 4 fold axis also contain the 2-fold operation.

So, these symmetry operations of 2-fold are embedded in 4 fold which I see by so by embedding these 2 folds for comparison do not confuse it with 4 fold. Rotoinverse and axis which has similar symbol I am only trying to show that the 4 fold is containing the 2-fold which is there in 23. So, you can then conclude that group 23 is also a subgroup of 432

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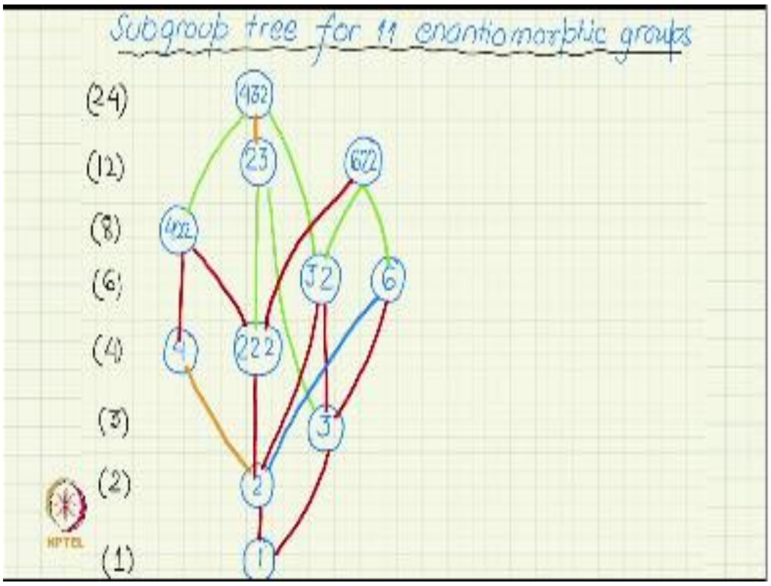

Now one exercise which is important in group theory of symmetry of crystals is to try to look at the group subgroup relation of a set of groups. So, you can do it for the entire 32 point groups but that becomes a much more involved drawing.

So, let me try with 11 enantiomorphic groups which do not have improper rotations that is they do not have mirror planes or center of inversion or any roto inversion axis. So, now here let me show the group subgroup relations by using lines and we have already shown that 432 contains 23.

So, they are related by group subgroups which I joined with the line and we also saw that 2 is a subgroup of 4. So, these two we have seen as an example. But we draw more lines to connect other groups subgroups relationship.

So, 432 contains 422 and 32. Then if you come to 23, 23 contains 222 and 3 622 subgroups is not included in 432 obviously because it has a 6 fold axis which is not there in 432. But 622 will contain both 326 and also interestingly it will contain this 222.

Now 222 if you look at 32, 32 contains obviously 3 as well as 2, 222 includes 2 and 422 also contains 4 as well as 222. And finally 2 contains 1 and 3 contains 1 and 6 of course 6 will contain 3 and 6 will also contain 2. So, this completes our group subgroup relationship for the enantiomorphus group.

Of course there are many as you know we have an 11 groups only there are 21 more groups which will have improper operations if you try to as it is a good exercise to try to relate them as group subgroup relationship but I leave that leave that as an exercise I do not want to try that here on video because the diagram becomes very messy I will say with many lines. But it is a good exercise to do.