Crystals, Symmetry and Tensors
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Lecture 16a
10-points groups associated with 5 lattices

We are now on 2-dimensional plane groups, we have not yet reached the plane group, what we have discussed is 10 2-dimentional point groups.

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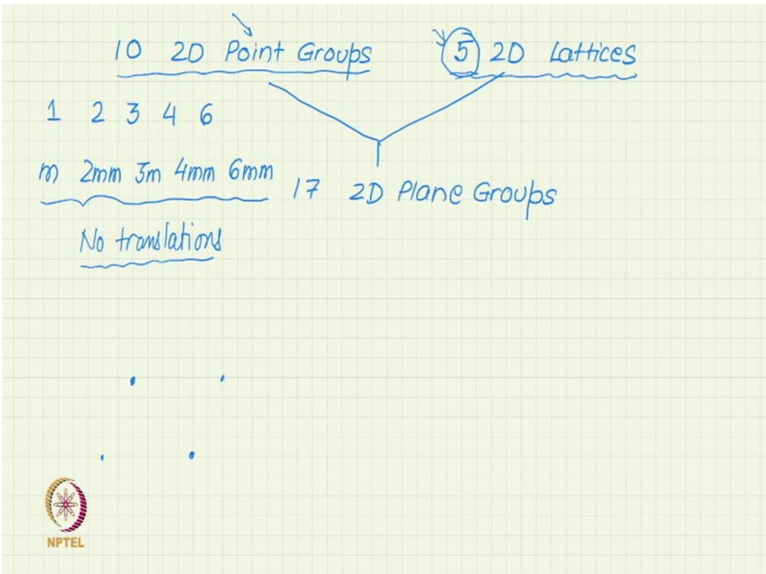

So, that is that should be all clear to you by now, that in 2 dimensions the only point groups that is only symmetry groups which will leave one point unchanged will be 1 2 3 4 or 6 and the corresponding versions with mirror so, m 2mm 3m 4mm and 6mm. You should now be familiar with the reason why we write double m's in the case of 2 4 and 6 and a single m in the case of 3 and 1, 3m also has 3 mirrors, but those 3 mirrors are equivalent to each other by the 3-fold rotation, 4 mm has actually 4 mirrors, but not all 4 mirrors are equivalent to each other by 4-fold rotation.

So, two of them are equivalent to by 4-fold rotation another two are also equivalent by 4-fold rotation, but one set of one pair of 2-fold is not equivalent to another pair of 2-fold by 4-fold rotation. So, we want to show those pairs. So, each m in 4 mm case each m is actually representing 2 mirrors, which are equivalent by 4-fold rotation, but another m represents another

2 mirrors which are not equivalent to the first set of 2 mirrors. So, we have seen these things in the previous class, we will again revisit them later.

After that we were so, since it is point group, so, one point is left unchanged. So, automatically translations are not allowed. So, in all these combinations the mirror plane and the rotation we are looking at symmetries which will leave some points unchanged and in the case of 2mm 3m and so, on where you both have rotation axis or rotation point and mirror they should all pass through the same point to leave that point unchanged.

So, translations are not allowed, there is no translations here, because translations you know translations cannot leave any point unchanged, it will always shift. But when we are talking of periodic patterns, translations become important and it is translations which gives you the periodicity. So, then we have to look at how many different kinds of translations are there and that is characterized by lattices, because lattice actually gives you the set of translations through which this periodic pattern will come into self-coincidence.

So, that we claimed that there are five types, we met three of them in the previous lecture. So, the way we were doing it was that we were trying to see, so, these five-types are also again by the symmetry, the symmetry which these lattices will have. So, if lattice has no symmetry and we saw that it is not possible for a lattice to have no symmetry at all, because every lattice has the 2-fold symmetry.

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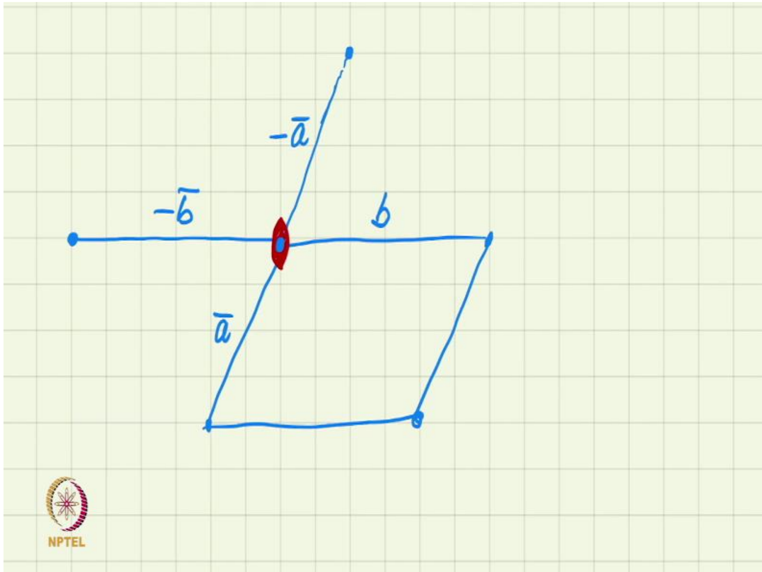

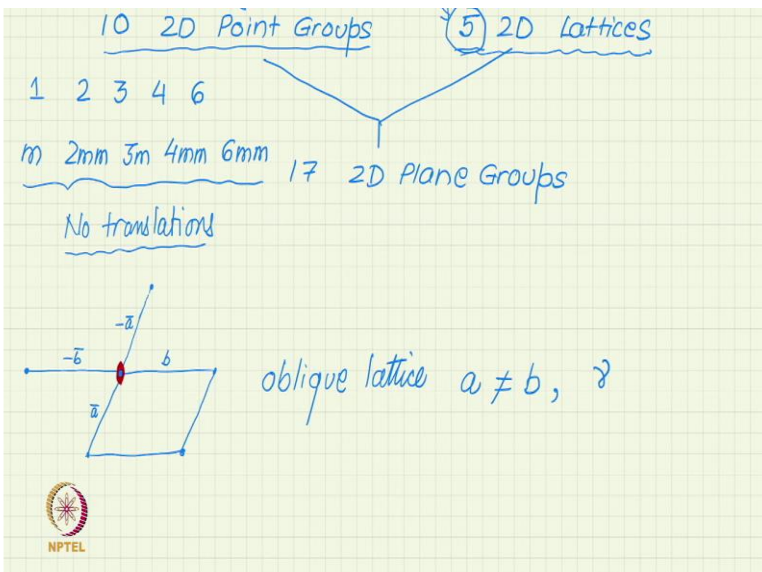

The 2-fold symmetry we call for every lattice translation. So, if we call this a translation a then minus a will also be a lattice translation which will take you to some other lattice point. Similarly, if you have this as b, then minus b will be also a lattice translation which takes you to another point. So, the each lattice point itself becomes an axis of 2-fold. So, this is any way guarantee. So, this is the minimum symmetry which any lattice can have and we call that this kind of lattice, the oblique lattice.

So, there is no relation between a and b. So, the lengths of a length of a and b may not be equal and there is no relation no particular restriction on angle gamma, gamma can be any arbitrary angle. So, even with non-equality of a and b and even with arbitrary gamma the 2-fold anyway will emerge. So, that is your minimum symmetry lattice and has been named oblique lattice.

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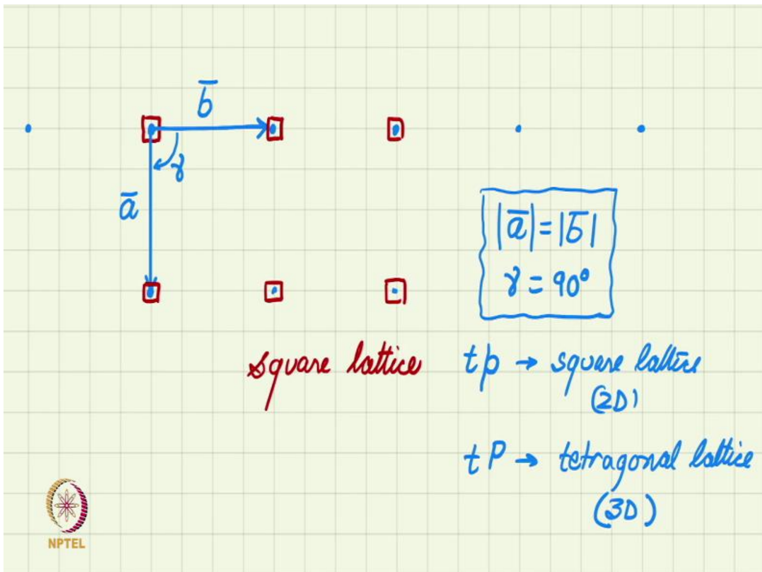

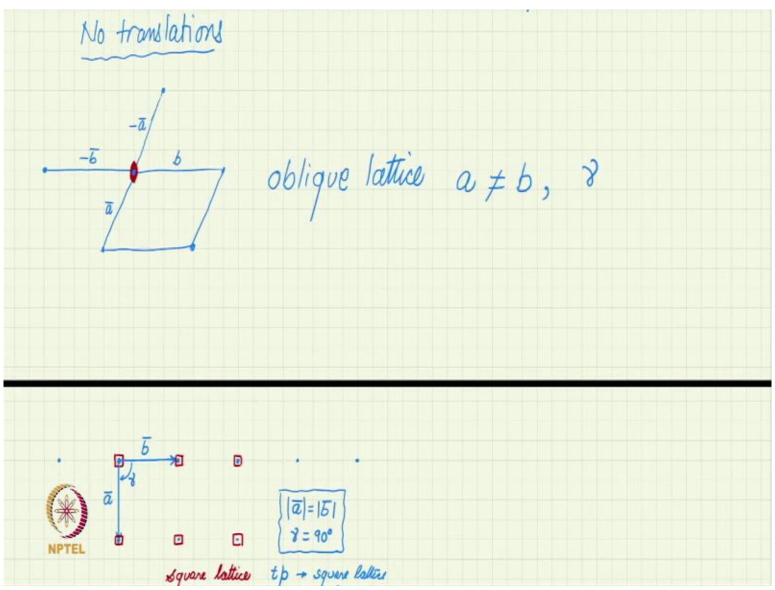

Instead of 2-fold if you have let us say some other symmetry, if you want to endow your lattice with some other symmetry, some other interesting symmetry. Let us see, so, apart from 2-fold what are the symmetry you can give to a lattice. So, let us give a lattice a 4-fold, so let us begin with the lattice point and let us make that lattice point and axis over 4-fold axis. Now, of course, since you have a 2-dimensional pattern of lattice point we are talking about 2d lattices. So, there will not be just one lattice point there will be other lattice points.

So, as soon as you have one more lattice point you know that by translation you will have entire row of lattice points both in the forward and backward direction, but that will give you still a 1-dimensional lattice, it is giving you a 1-dimensional lattice. If you want to have a 2-dimensional lattice, you will have to have lattice points in other directions also. But now, since you are insist you can have it in any direction, but since you are insisting that the lattice point is a 4-fold axis of symmetry then whatever this lattice translation is there, this will rotate by 90 degree to generate another equivalent lattice point there.

Unless and until you do that, this 4-fold symmetry will not be there. So, the 4-fold symmetry itself forces that you have another lattice point at 90-degree, to a given lattice vector and at the same distance because this since this is a rotation of this vector, the two vectors will have the same length. So, although we call one of them let us say a and another b, we now know that a and b cannot be arbitrary, if I want 4-fold a and b should be of the same length. And the angle between a and b is gamma, gamma also is now no more arbitrary, gamma has to be 90-degree.

So, these two constraints are there on the lattice parameter if we wish to have a 4-fold axis. And of course, all lattice points since one lattice point is a 4-fold axis and all lattice points are equivalent we know, so all these lattice points will have 4-fold axis of symmetry. So, obviously you have now what is called a square lattice however, the symbol is chosen as tetragonal, tetragonal P. Tetragonal is a 3-dimensional lattice where a equals b not equal to c, here there is no question of c, is still in the international convention may be to relate it with the 3 dimensional lattices.

This will be one plane of 3d tetragonal lattice, the basal plane of tetragonal lattice. So, the 2d lattice also is given the name, EP. And I am making a mistake here. So, let me correct myself because that is what I told you last time also that, that is what distinguishes the notation from 2d to 3d. In 2d we will use a small p in 3d we will use the capital P. So, tp will be a square lattice whereas, T capital P will be at actually a tetragonal lattice in 3d.

So, it is not just equality of the lattice parameter and the angle of 90 degree, we see that if you have this equality and if the angle is 90-degree, the lattice is actually endowed with a different symmetry. So oblique lattice where it was having only 2-fold. Now, with this constraint on the lattice parameter, it has become a 4-fold axis, so, it has a different symmetry and that is the reason of that is the reason why we want to call it or count it as a different lattice.

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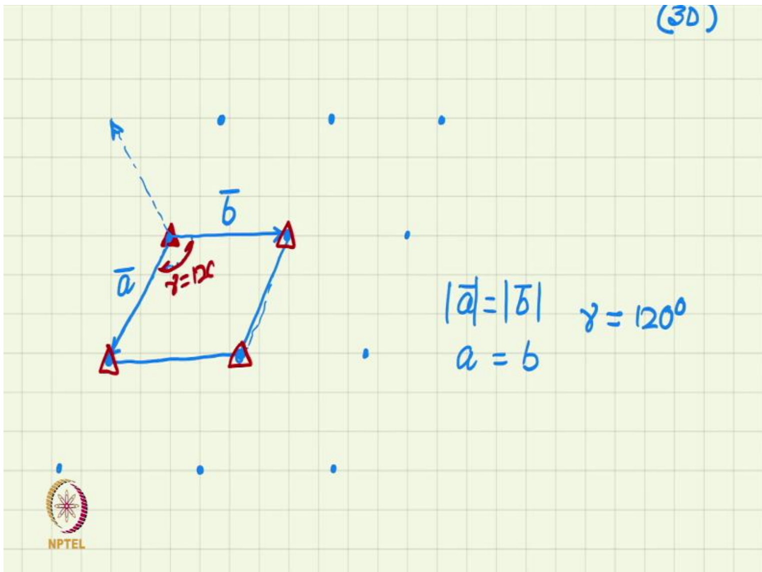

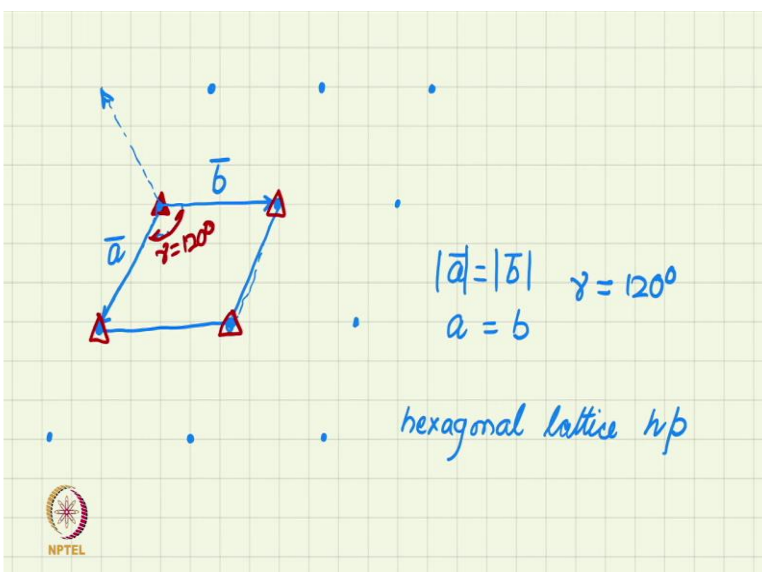

So, 4-fold we have given now, what else you want to give 3-fold, why not 3-fold. Suppose, you have 3-fold, do the same analysis have a lattice point on the 3-fold and have any other arbitrary lattice point. Now, this distance can be arbitrary. But, now, our insistence or our requirement that I have a 3-fold axis in my lattice or in my pattern, I know that this translation has to be rotated by 120-degree to generate a 3-fold.

So, in other directions again by 120-degree. So, again you see, the 3-fold is also demanding that a is a and b are equal, means you can write it as modulus of vector or you commonly write simply without the vector sign a equals b, but now, gamma is equal to 120 degrees and of course, you can generate once you have two lattice points which are not collinear you can generate the entire lattice simply by vector additions you will keep getting other lattice points.

So, this will be your unit cell all lattice points are identical. So, all lattice points will be 3-fold, of course, more symmetries will get generated, we will look at that, but currently we are simply stopping here. So, this we could have called a triangular lattice, but we call it is still a hexagonal lattice, we will see the reason soon sorry, I also need practice to remain in 2 dimensions, so hp, you can consider 6-fold or we have considered 2-fold 3-fold 4-fold, the only axis now remaining rotation axis is the 6-fold. So, you can try to construct a 6-fold lattice also.

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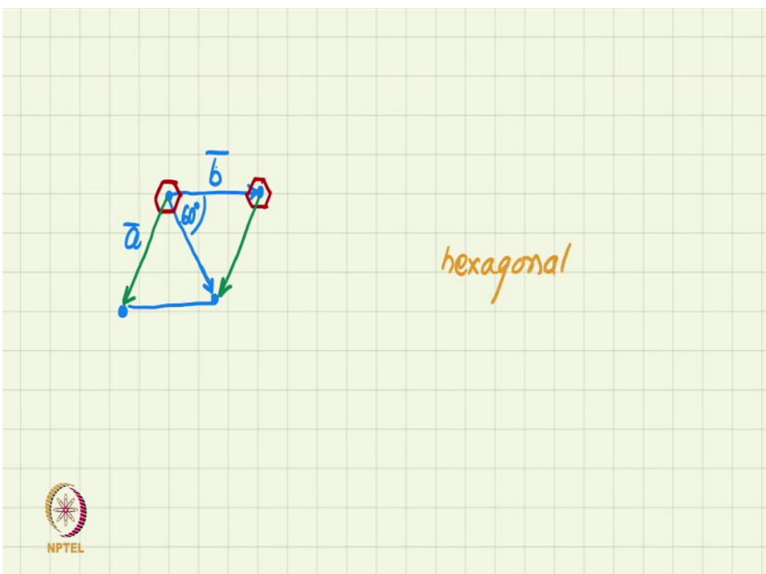
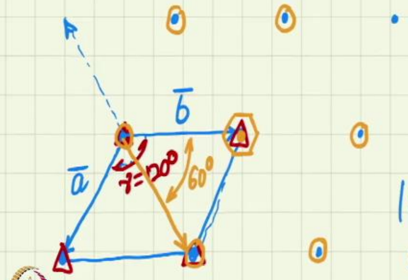


(2D)

tP → tetragonal lat

(3D)



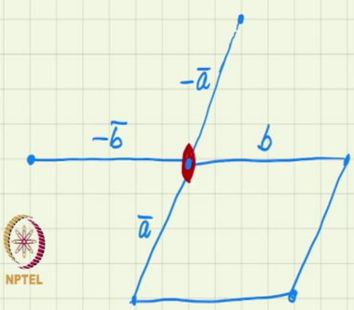
$|\bar{a}| = |\bar{b}|$ $\quad \gamma = 120^0$

$a = b$

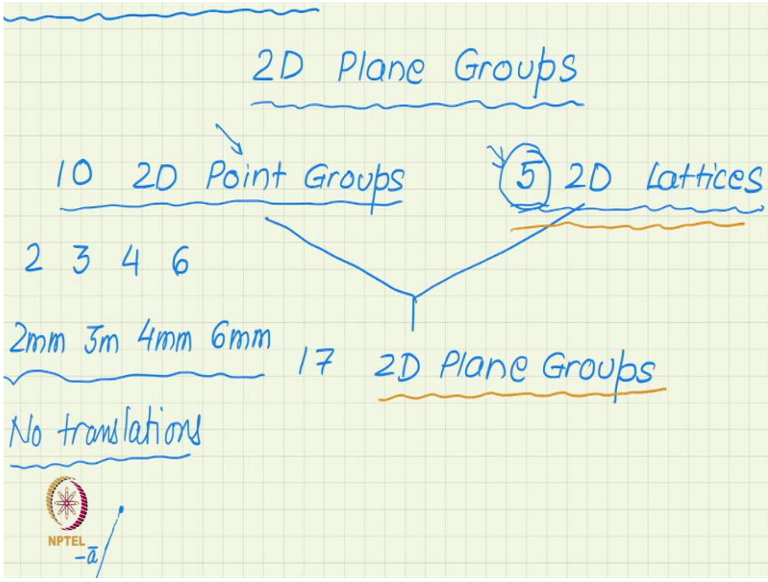
hexagonal lattice hp

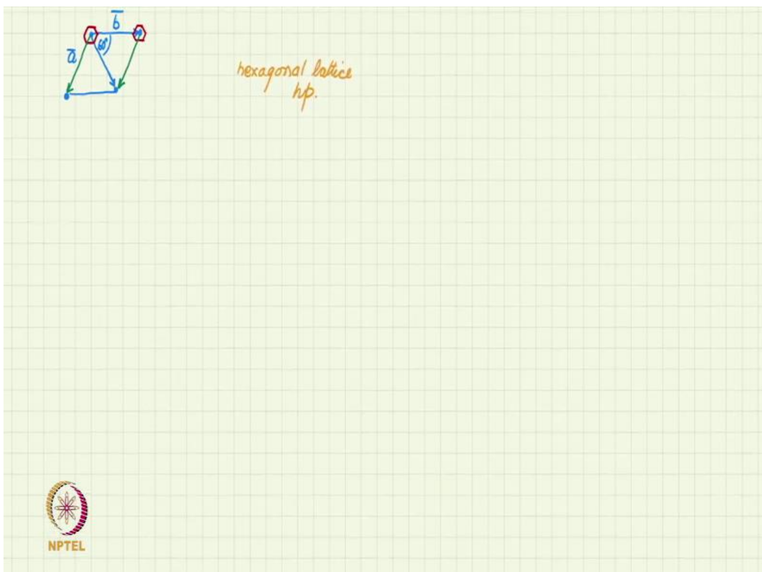
m 2mm 3m 4mm 6mm 17 2D Plan

No translations



oblique lattice





So, 6-fold will require a rotation by 60-degrees. Now, you will see that you have also generated joining any two lattice points you get a lattice translation. So, you have also generated let us say this green, green lattice translation and if you start the green lattice translation from your original point and then you get another lattice point here.

So, if you now call this green one as a and this is b, you see actually there is no difference between the lattice which you created by a 60-degree rotation and a lattice which you created by 120-degree rotation because here also when we created the lattice you did not notice or we did not highlight maybe you will notice that there is a vector at 60-degree.

So, a lattice generated by a 120-degree rotation that is a 3-fold and a lattice generated by 60-degree rotation which is the 6-fold are actually the same lattice and that was the reason why the triangular lattice while also called a hexagonal lattice, because actually it is the same and this also we call a hexagonal lattice. So, 6-fold and 3-fold gives you the same lattice. So, although we were calling it 3-fold from the point of view of lattice if you see that will become the all the points are actually 6-fold, because there are six vectors around any lattice point.

So, if you if you focus on any lattice point you can see that lattice has six neighbors surrounding it. So, it is really not a 3-fold axis. It is a 6-fold axis. But does that mean that you cannot have a pattern with 3-fold axis, because if you are trying to create a 3-fold lattice, you are getting actually a 6-fold lattice, same thing same issue was there in this oblique case also, can you not have a pattern without any 2-fold rotational symmetry because, when we tried to create a lattice without 2-fold symmetry, we found that we are bound to find 2-fold rotation. So, that is where the difference between lattice and the plane group comes.

In the plane lattice yes, in a plane lattice, you can never avoid 2-fold axis, in a plane lattice, you can never have 3-fold, we will always have 6-fold, but you can have patterns with 3-fold symmetry if you start putting objects only with 3-fold symmetry at these lattice points. So, that is the motif. So, 3-fold axis in a pattern will come from the motif not from the lattice. Similarly, if you put in an oblique lattice, a pattern without 2-fold symmetry, a motif without 2-fold symmetry then you will destroy the 2-fold symmetry of the lattice and you will get a pattern without 2-fold symmetry. So, those things we call as different plane groups based on the same lattice.

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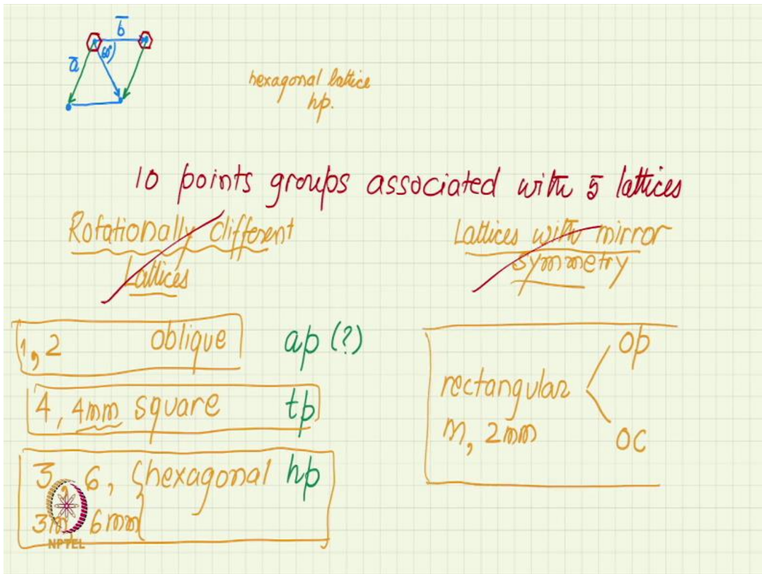

So, rotationally you saw that in terms of rotation, we will look at rotationally different lattices. So, we have oblique then we have square and we have hexagonal. Oblique, I did not give you the symbol, So, oblique also should have a symbol. I am a little doubtful about it, but let us know for the moment I think we can check this a probably it has a symbol a, so ap tp and hp, question mark for a little bit of doubt I have.

Then we saw last time in these cases we are not talking thinking about mirrors. So, mirror gives us two rectangular lattices. So, we saw that there was a primitive rectangle and rectangular lattices were given the symbol orthorhombic. So, orthorhombic primitive and orthorhombic centered, op and oc.

So, the we have completed these five lattices only these are the possible symmetries which are possible, because we saw that only 2 3 4 and 6 these are the symmetries, rotational symmetries. So, 3 and 6 gave you the same hexagonal symmetry, 4 gave you the tetragonal and 2 gave you oblique. Similarly, if you have the mirror, then you can have m or you can have 2mm b will come under rectangular. If you have 4mm, so, that will come in a square. Well, this classification way of writing is not looking good to me, because I said rotationally different lattice and this is lattices with mirror but 4mm is also with mirror.

So, probably my heading is not justifying what I am writing, so let me cross out the heading for the moment, just look at what look at you just let us distribute the 10-point groups. So, the headings should be 10-point groups associated with 5 lattices I think that seems more reasonable. So, 4mm will go to square then 3m and 6mm, these will come to hexagonal. So, I think we have now all, one is oblique, so, that will go there. So, we have all the 10-point groups now, so, 10-point groups and 5 lattices. Hexagonal lattice has 4 different point groups supports 4 different point groups. The square supports 2, oblique supports 2 and rectangular also supports 2.