Crystals, Symmetry and Tensors
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Lecture 15c
Difference kinds of 2D lattices

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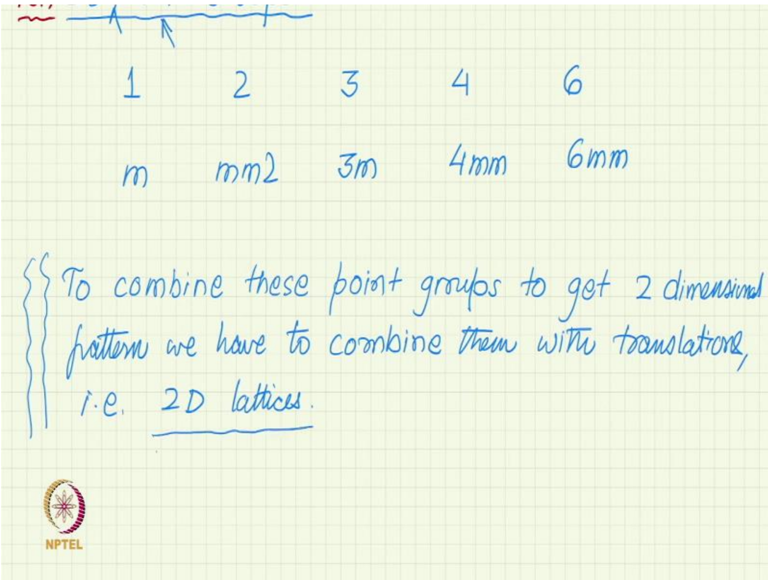

So, we have now 10 point groups, we want to combine them with translations and translations are represented by lattices. So, we want to see how many different kinds of 2D lattices are there with which we can combine these 10 point groups.

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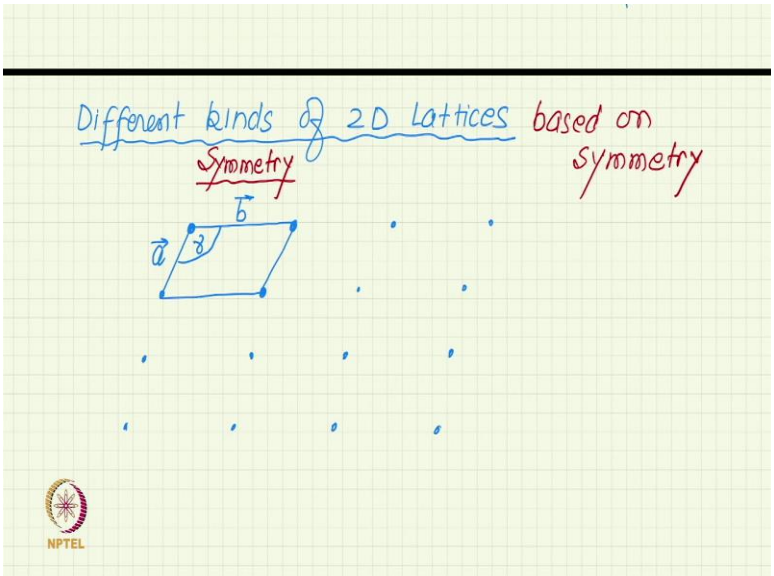

When we say different kinds of 2D lattices of course, you have seen that 2d lattices you can select a parallelogram as your unit cell with two basic vectors a and b an angle between them gamma and once this is given once the unit cell is given, you can keep adding more unit cells or more lattice points and you will get your entire 2 dimensional lattice point 2 dimensional lattice.

So, if we think that each a b and Gamma gives me a lattice, there are infinitely many different lattice because there is no limit of the way in which you can vary a b and gamma you can choose any a, any b and any non zero a now only requirement on these 3 quantities or they should not they should not be 0. If gamma is 0, then you do not have a 2 dimensional you have a 1 dimensional direction and if a or b is 0 again you lose the periodicity in that direction.

So, once you have 3 non-zero numbers a b and gamma you get a lattice and infinitely many variations are possible, but when we say different kinds, again, our kind is based on symmetry. So, the focus is symmetry. So, how many different kinds of symmetries for lattice we can have? Or how many different kinds of lattices based on symmetry. So, instead of focusing on symmetry of a finite object, which was the point group, we are now looking at in finite lattice and we want to look at the symmetry of these lattices themselves.

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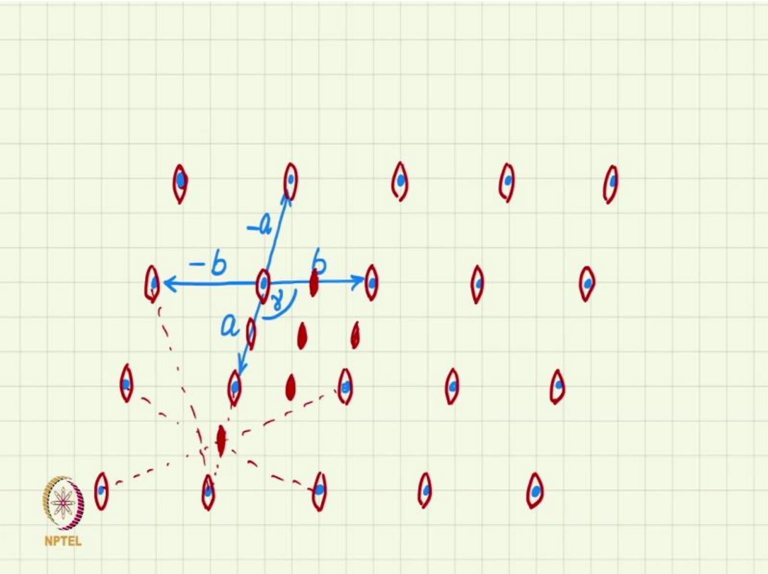

So, let us characterize them on the basis of symmetry. So, I have just drawn some sort of an arbitrary 2d lattice if a and b are not related and gamma is arbitrary, then what symmetry do I get? Identity will be expected symmetry. But translation is forcing a kind of symmetry, translation is forcing a kind of symmetry, you can see that if you have a translation b. Do not do have a translation minus b.

And if you have a translation a do not do have a translation minus a. What is this telling about the symmetry at the lattice point. So, each lattice point automatically becomes a 2-fold. Even in the most general case, I am not filling my 2-fold axis, because I want to preserve the identity of the lattice point which will be lost. I am using open 2-fold notation here. Of course, you know that if there are 2-folds combined with the translation a, what does it do?

A 2-fold with the translation a generates another 2-fold between the between the 2, it generates a 2-fold a by 2. So, you can see also that you do not have to rotate 180 degree about the lattice point you can even rotate 180 degree in the midpoint of 2 lattice points and everything will come into self-coincidence. That is also a 2-fold axis because if I rotate about this 2-fold, let us say then it will take these 2 lattice points into each other.

It will take these 2 lattice points into each other. It will take these 2 lattice points into each other it will take this lattice point into sub lattice point down below. So, the midpoints of the lattice points are also 2-folds. So, same thing is true for b also and a plus b is also a translation vector the diagonal vector. So, in that direction also you will generate it, 2-fold at half a plus half b, that will be in the center of the unit self. So, even the most arbitrary 2 dimensional lattice will be decorated with these 2-folds it will have all these 2-folds. So, this kind of lattice we can say least symmetric lattice.

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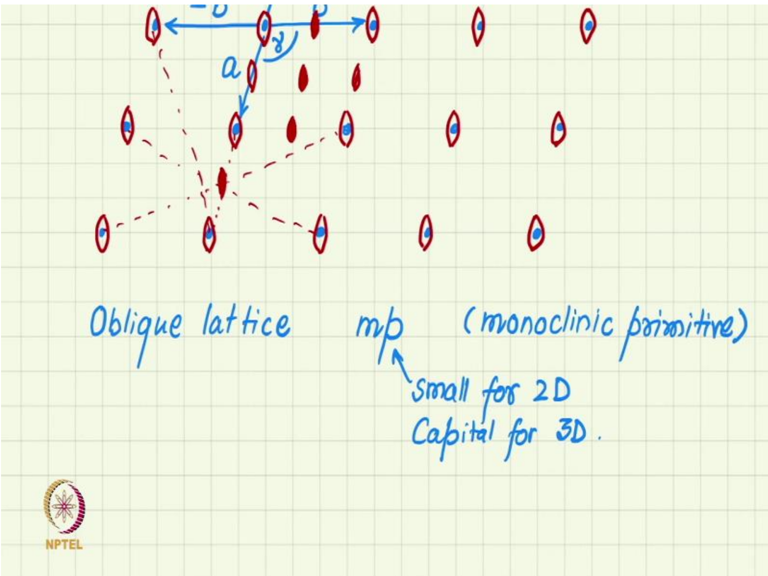

So, we call this the oblique lattice, it is least symmetric but it is still has 2-folds and the symbol for this is mp, m comes from the 3 dimensional counterpart of such lattice that is monoclinic. So, the International notation tries to somehow bring a connection with 3d when giving the notation the name wise it is not called monoclinic name wise it is called oblique lattice to indicate that it is 2d, but notation wise m comes from monoclinic in 3d.

Only thing is that if monoclinic primitive lattice in 3d was there, then p will be written with capital. So, that is the distinction one makes. So, p has to be small for 2d. Now, when we are looking at the lattice, we are forgetting about the point group. So, then we will combine the lattice and the point group to generate the whole symmetry of the pattern which will be called the plane group. So, first we are doing part work and then we will combine the whole thing.

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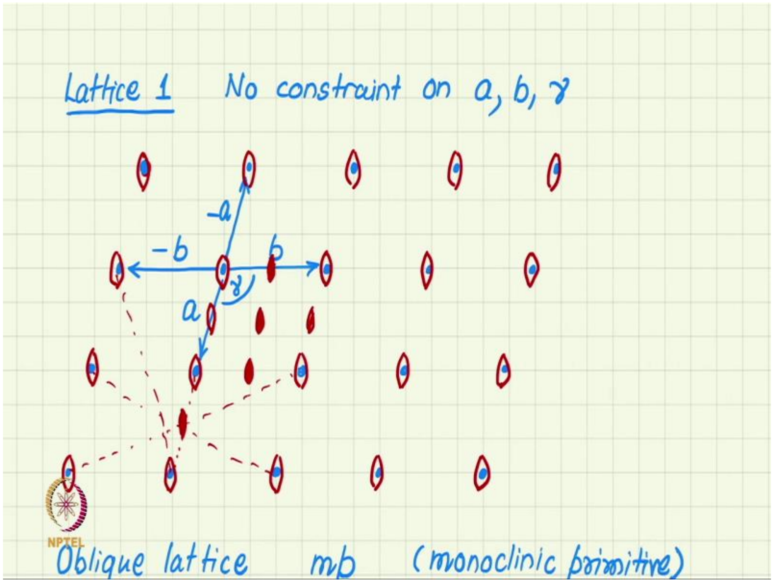

Let us call this lattice 1. Is there any relation between a b and gamma there is no relation or no constraint they can have any value? So, we were trying to classify by symmetry. And the least symmetry, as somebody said was no symmetry. So, we were trying to create a lattice with point group 1 with no symmetry. But we saw that is not possible we by default, we have this 2-fold symmetry and we called it oblique lattice.

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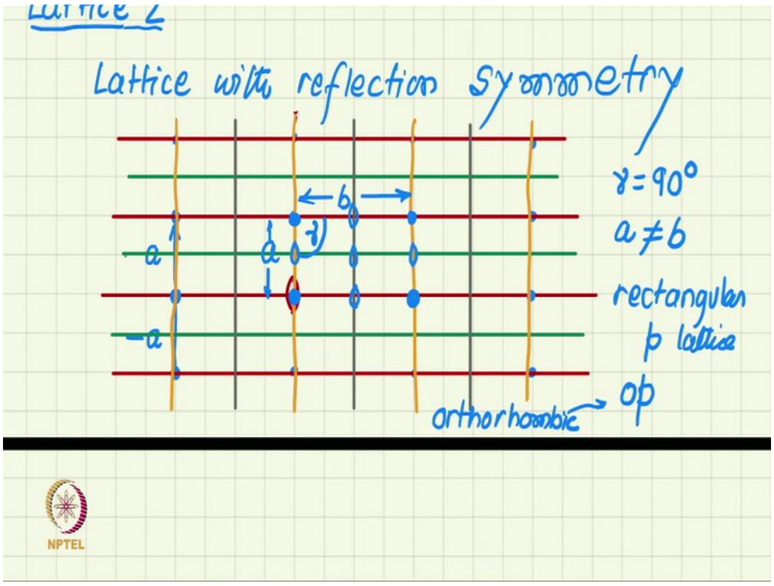

So, the next symmetry the 2-fold we will not consider the next symmetry we can consider 3 4 and 6 or m and 2 mm and so, on. So, let us consider the next symmetry as the mirror plane because that is an interest that puts some interesting constraints on my lattice, because, although this has 2-fold if a b and gamma are arbitrary, you are not seeing anywhere any mirror plane there are no mirror planes in this.

So, this oblique lattice is lacking mirror planes you cannot draw any plane in which you can reflect and get the lattice point into self-coincidence. So, if I have a lattice with reflection so, let us have a mirror plane as soon as I have mirror plane there are certain constraints on the lattice points which will be there. So, for example, if I put a lattice point on my mirror plane, there is no problem, one periodicity direction can be taken as in the direction of the mirror plane itself.

So, because each lattice point is then getting reflected into itself as far as the points on the line is there so, mirror plane is satisfied, the second set of lattice points can be considered in a direction perpendicular to the mirror plane because I want to preserve this mirror plane and when I know that if any vector a I take then lattice itself forces minus a and if a is perpendicular to a mirror, mirror also forces minus a. So, this a and minus a from the lattice and a and minus a by the mirror it is satisfied if a is perpendicular to the mirror.

So, I can have a possible lattice like this and this lattice will have mirror planes of course, all lattice points are equivalent. So, if one set of lattice points were having this horizontal mirror other lattice points also will have this horizontal mirror. So, you buy one and get several free because all lattice points have to be identical and I have said that from my one offset of lattice point mirror is passing. So, all of these are mirror.

But then when we considered freeze group there again we saw that if a mirror is combined with the translation a perpendicular to the mirror then mirrors get generated that a by 2 also and here also you are seeing that although I started with the red mirror I have green mirrors also because these are also acting as mirrors. So, now, I have. So, you can see I have a beautiful lattice which has 1 set of mirror planes.

But since I put it as perpendicular it has does it have another set of mirror planes also because, 2-folds were any way there any lattice will have to 2-fold and 2-fold we have seen a mirror passing through 2-fold creates a mirror at 90 degree to it. So, now, we are seeing that these 2-fold and mirrors are also getting generated. So, you have mirrors like this also. So, I use yet another color maybe and these mirrors also will generate a mirror halfway between them.

So, if you now see these were the lattice points. So, you have a very rikt system of mirrors and 2-fold all the intersection points are now 2-fold all the lines are mirror you can call this as your a you can call this as your b you can call this gamma. The constraint on gamma now is gamma has to be 90 degree because we took the second row of lattice points 90 degree to the mirror and 1 row parallel to the mirror.

So, gamma gets constrained to 90 degree a and b are not constrained and they do not have to be equal either. So, this is a rectangular p lattice symbolize op again o coming from 3d orthorhombic. So, symbol in the symbol the name for the 3d lattice comes but 2d lattice has the name as rectangular lattice, but symbol as op.

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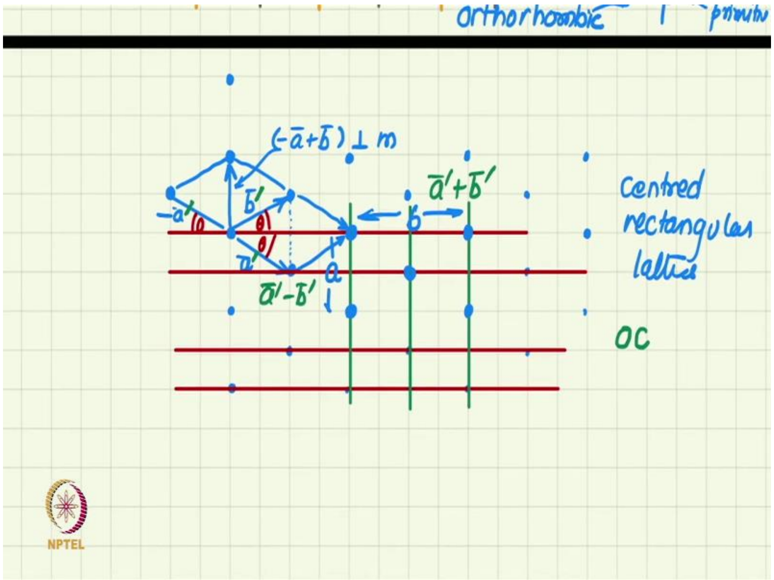

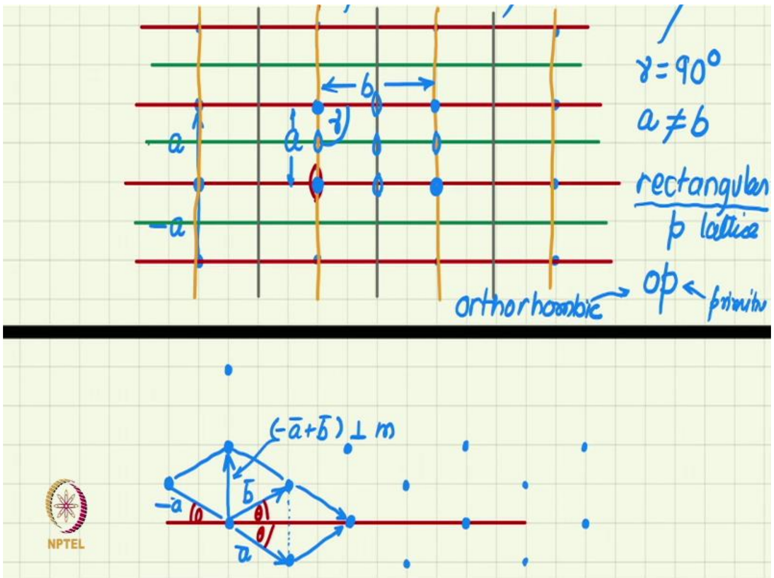

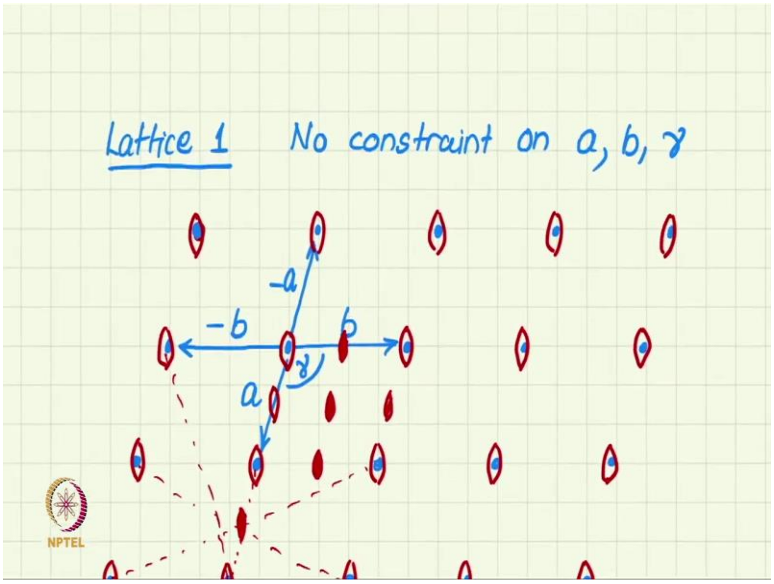

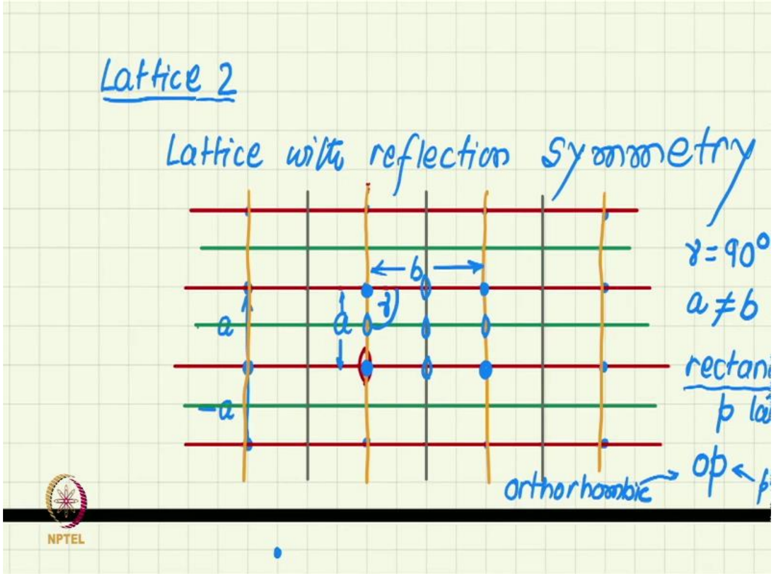

Is there any other way of preserve the mirror the constraint which I was putting that let me have a row of lattice points perpendicular to the mirror, if I do not have that luxury and if I say that let us have a lattice point which is at an angle to the mirror plane like this but I want to preserve the mirror plane? So, I have to reflect this lattice point. So, I will get another lattice point there.

And if I have 2 lattice points, 2 lattice translations combination of them will also give me a lattice point. So, if I combine these two. I again find that. So, I cannot avoid if a lattice has a mirror it has to have several lattice points along the mirror line whole row of lattice points along the mirror line. So, that first assumption was not that so arbitrary that I took lattice points along the mirror line even if I do not want to take a lattice point along the mirror line the mirror symmetry as well as the lattice periodicity will generate a set of points along the mirror. That is part of the symmetry game.

Similarly, I have negative of this vector also and then if I combine these two let us label some of them let us label 1, 2 and then this is minus 2 and 1 and minus 2 again can combine to give you a lattice point there and what is the orientation of that lattice translation if you combine 1 and 2 is reflection of 1. So, this theta has to be equal to this theta which would be equal to this theta. So, if you combine 1 and 2, 1 and minus 2 the vector which you are getting what are you getting maybe instead of 1 or minus 2 I should label them as a b or something.

Suppose this is a, this b this is minus a. So, this vector is minus a plus b. So, I am asking what is the orientation of minus a plus b with respect to the mirror line, minus a plus b will be perpendicular to the mirror line. So, you can see that the other assumption also in the first case to have a set of lattice points perpendicular to the mirror was not really arbitrary, even if you start with an arbitrary lattice point which is not inclined, you will generate in the n a set of lattice point perpendicular to the mirror and a set of lattice point parallel to the mirror.

But now, if you look at this lattice the lattice which you have created it is slightly different it is not primitive if you think in terms of rectangle it is having centers also of the rectangle this way of creating this way of creating gave me a primitive and that is why the key was their primitive but this way of creating it again gave me parallel and perpendicular rows of lattice point, but it is giving me this middle lattice points also.

And since all lattice points are equivalent, again you will have mirror planes passing through all these lattice points and similarly, you will have mirrored planes passing through perpendicular set also. So, now, you can take this as your a, this as your b and call this centered rectangular lattice. No, not necessarily a and b are arbitrary is not a square it is a rectangle.

Student: Sir there angle will be equal these thetas.

Professor: Theta angle is equal because a is reflection of b. So, reflections should not a reflection of b in Sorry, I am using 2 a and b. So, let me use prime on these let us say a prime b prime. So, a prime is a reflection of b prime. So, reflection will make the angle equal then minus a prime is just opposite of a prime.

Student: a prime and b prime will have length also.

Professor: sorry,

Student: a prime and b prime are equal.

Professor: Haa a prime and b prime are equal. So, that is why it is important to keep this notation different a and b, do not are not related a and b are not related because b is finally a prime plus b prime and a is finally b prime minus a prime or a prime minus b prime, if I am drawing that downwards. So, in the prime symbols they are equal because a prime is a reflection of b prime.

So, reflection does not change length. So, they will be equal in length, but when you combine them since one of them is a plus b and another is a minus b then this new a and b which are the sides of the rectangle, they are not equal. So, they are not constrained unless and until a and b themselves had some relation which will make this equal. So, that will be accidental or coincidental.

So, this we will call orthorhombic c in terms of symbol or centered rectangular lattice. So, we generated 3 lattices, one was oblique, one is primitive rectangle and one is centered rectangle there are actually 5 lattices. So, 2 more lattices we have to create which we will do in the next class.

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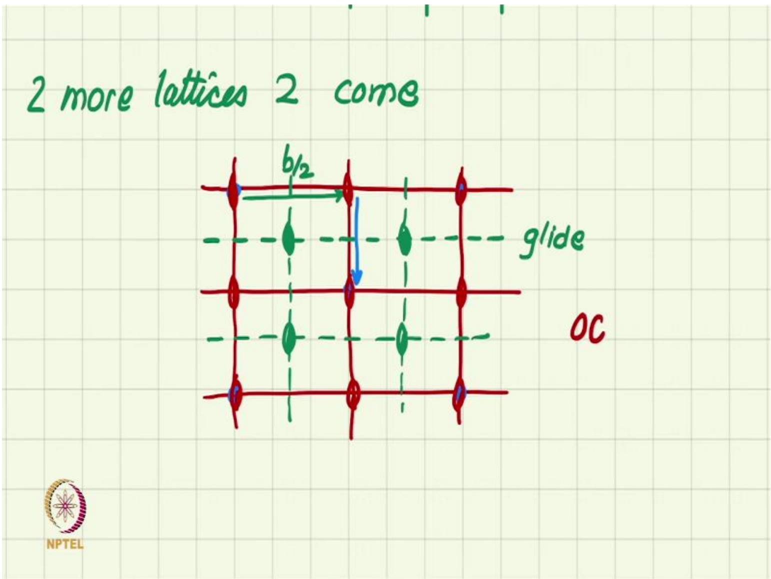

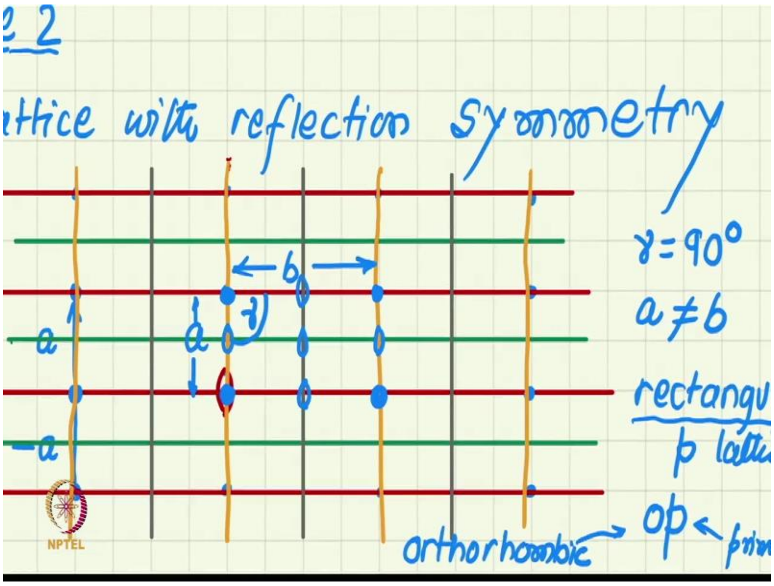

Professor: Lattices should have.

Student: Second lattice should have more (())(24:35).

Professor: Yeah, it is heavy. So, we have not looked at it carefully, let me draw that so just the unit itself for quickness and simplicity. So, you have mirror here, you have mirror here, one more have mirror here, one mirror here, one mirror here, one mirror here, and these are the lattice points thank you for reading that because I was missing that point when I said the decision you write, if it has the same symmetry then it is the same lattice even if it is the centering point.

So, centering point should not create a new lattice, it is a symmetry which should create the new lattice. So, why this we are considering with the center as a new lattice that now if you see. So, these lines are glide lines, because, you can see that you can translate this mirror sorry translate this lattice point by b by 2 and then reflect in this dashed line. So, you will reach this lattice point the center lattice point.

Similarly, there is a glide here and there is glide this way also. And intersection of glides are also mirror by 2-fold just like intersection of mirrors are 2-fold intersection of glides are also 2-fold. So, now if you compare you will see that this is really having much more symmetry element. So, OC has more symmetry than op.

Which was having only the mirror has 2 sets of perpendicular mirror and 2-fold, it has 2 sets of perpendicular mirror and 2-fold also 2 sets of or 1 set of perpendicular glides also with intersection as 2-fold, new 2-folds coming and that is coming because we have the centering lattice points, absence of centering lattice point removed the glide and the green 2-fold.

But if the central lattice point is there, then you have the glides and the green 2-folds also. So, in terms of symmetry also this centered lattice centered rectangular lattice is a different lattice from the non-centered one. So, thank you very much.