Crystals, Symmetry and Tensors
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Lecture 7C
Stereographic Projection - II

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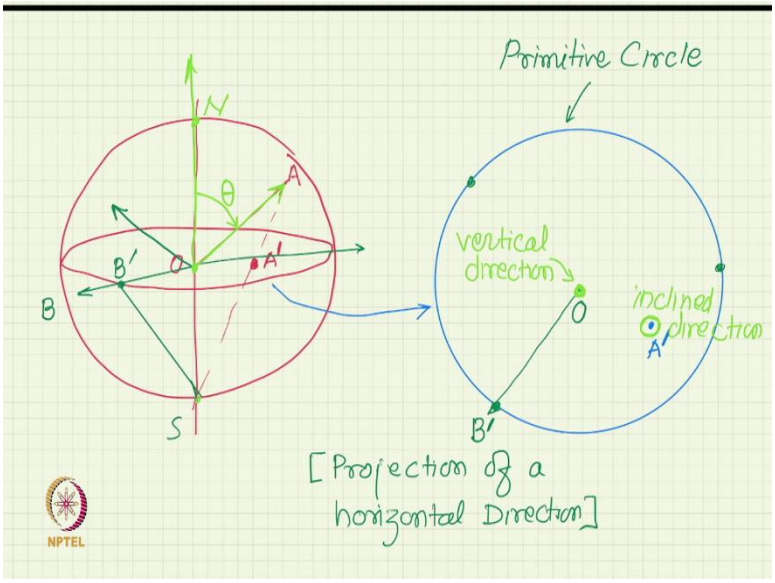

Now horizontal direction we know is lying in the plane. So, it is at 90 degree to the vertical. Vertical direction we know, is along the vertical so it is perpendicular to the plane of the projection. Inclined direction is not specified fully by just saying incline. You have to give an angle of inclination of this direction with respect to vertical or with respect to horizontal. So, this A prime, suppose I know this projection point A prime. Can I say that the direction OA what is the angle theta, the angle of inclination of OA with vertical. So, that is a nice interesting geometrical exercise which we will do now.

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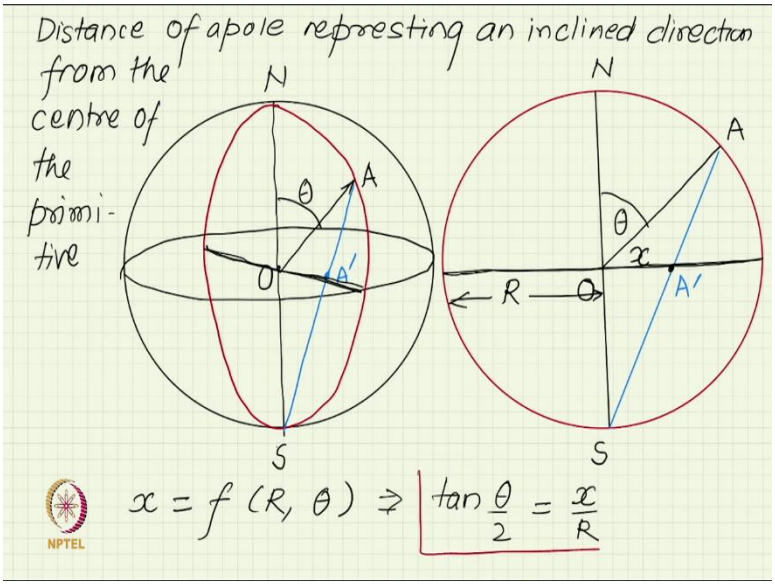

Can you see the geometry? So, we have the direction OA, we know that we will project it by joining A. A is where the direction hits the sphere. So, we know that we will project the direction OA into the projection plane by joining it with S. So, I join SA so it gets projected into A prime.

The direction ON and direction OA, two inclined direction, two different directions will define a unique plane and that unique plane also will cut the sphere into another circle. That is the red circle, red circle in both the diagram. In the first diagram since it is perspective it is appearing as a distorted circle. In the second diagram we are looking down on the circle, so it is a perfect circle.

But otherwise, the labels are exactly the same in the two diagram so S and N are the same points SN. OA is the same direction inclined at an angle theta to the vertical or to ON and SA is the line which will project and A prime is where it cuts the primitive circle in this left figure. It will cut the corresponding diameter shown here at A prime. So, this is a nice construction from which now you can easily find a relation between the angle theta and the distance of the projection point A prime means essentially we are asking that yes, an inclined direction got projected into A prime, how far it is from OA.

I know, if the distance of a prime was 0 from the center of the circle, then what was the angle of inclination? 0 degree. If it was equal to the radius of the circle, what was the angle of inclination? 90 degree. So, 0 and 90 is the obvious thing which we have already understood from this.

Now if it is not 0 or not R. Let us say half R it is exactly in the middle of the radius, then what is the angle of inclination of the corresponding direction of which this point is the projection. So, that is what we are trying to solve here. So, distance of projection, distance of pole, this is pole… A prime is pole. So, distance of a pole representing an inclined direction from the center of the primitive. This is what we are trying to establish.

Now the geometry is in front of your eye, you can easily do this. All we want to find out what is this distance X in terms of this radius R and of course the angle theta because as you vary theta X will vary. Theta is a function of X, you can easily see. So, what is that? Please take some time to solve this geometry. Find X in terms of X as a function of R and theta from the geometry shown here. Sorry, what is that? 10 theta over 2 into X bar very good …very good.

So, you must have looked at the triangle SO A prime. So, if you look at the triangle first of all you should look at the arc NA and recall your geometry ma'am in school. When she was still telling you this theorem and you are thinking that it is totally useless because you did not know that it will come in the stereographic projection.

The theorem was that on any arc, if there is if the arc subtends an angle some angle at the center which is theta it will say subtend the same arc will subtend half that angle at the circumference. So, NOA is the angle subtended by the arc NA at the center. NSA is the angle subtended by the same arc on the circumference.

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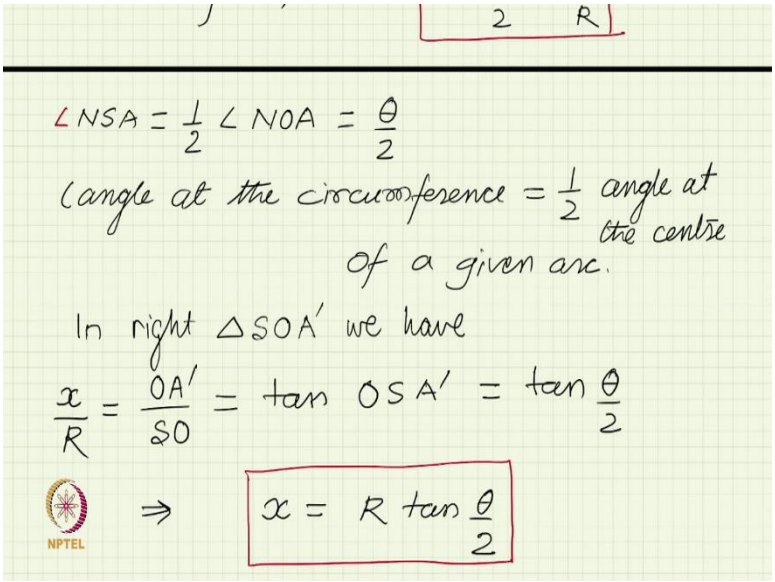

So, the angle NSA…. so is equal to half angle NOA is equal to theta by 2. Angle at the circumference to half angle at the center of a given arc. So, please thank her on the coming

Teachers Day. Now we use this result so this angle becomes theta by 2 and then the right angle triangle SOA prime, we have OA prime by SO is tan of angle OS A prime. OS A prime is same as NSA. So, that we have found theta by 2 and OA prime is the distance which we wanted, SO is the radius.

So, we get this nice relation that X is equal to or tan theta by 2. You can verify for the two special cases with which we are now very comfortable. The vertical direction theta is 0. Tan theta by 2 is 0, X is 0. So, for vertical direction the distance from the center is 0. So, it will be at the center, the horizontal direction theta is 90 degree, so theta by 2 is 45 tan 45 is 1, X is equal to R.

Other directions it will go by this formula and you can see this is not a linear formula. So, you should not mistakenly think that if you are at distance R by 2 you are at 45 degree. If you are at distance 0, you are at 0 degree. If you are a distance R, you are at 90 degree but if you are at a distance R by 2, you have to solve this trigonometric equation to get your value of theta.

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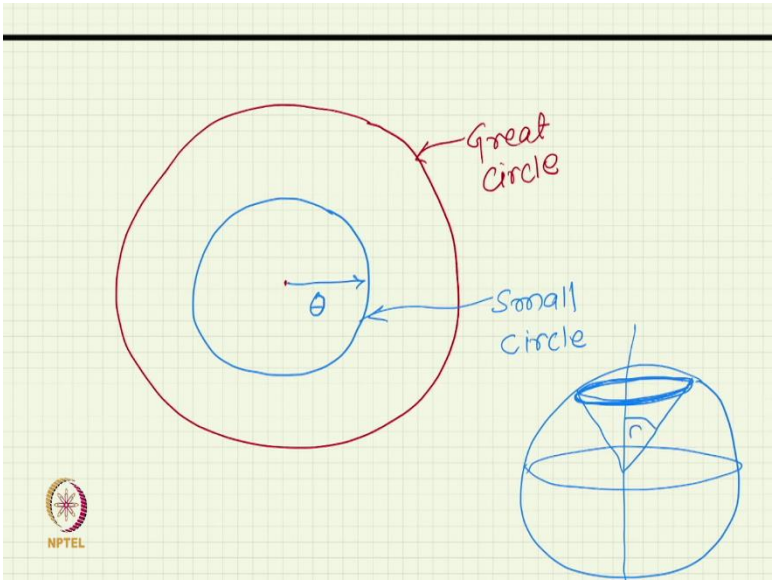

Yes, very good very good observation by Subha Jyoti. So, what he is saying, that suppose we make… there is a primitive circle around the center and all points on the primitive are at 90 degree to the vertical but what about if I draw some other circle around the primitive? What is it representing? The cone of equally inclined direction to the vertical. So, cone of directions inclined at an angle. So, in terms of distance it is some constant distance.

In terms of angle also it is some constant angle from the vertical. So, these are so why we were calling some circles the great circle because the red circle the primitive circle was a great circle. You can see now you have a smaller circle. So, the blue one will be called a small circle. Any circle other than the great circle is known as a small circle because if you think in terms of again…, if you think in terms of 3D how will it look like? In 3D it will look like a cone of equally inclined direction, so that is cutting in this circle, is not it? And this circle is not the largest circle on the sphere. This is a smaller circle.

So, any circle which is equal to the largest circle on the sphere are called the great circle. They are actually greatest circle but the phrase used is great circle and any circle which is smaller than the great circle are known as a small circle and a small circle centered at the origin or centered at the center of the primitive are projection of directions which are equally inclined to the vertical or you can say projection of direction lying on a cone of semi Apex angle theta because this angle will be theta. So, that is a good observation.

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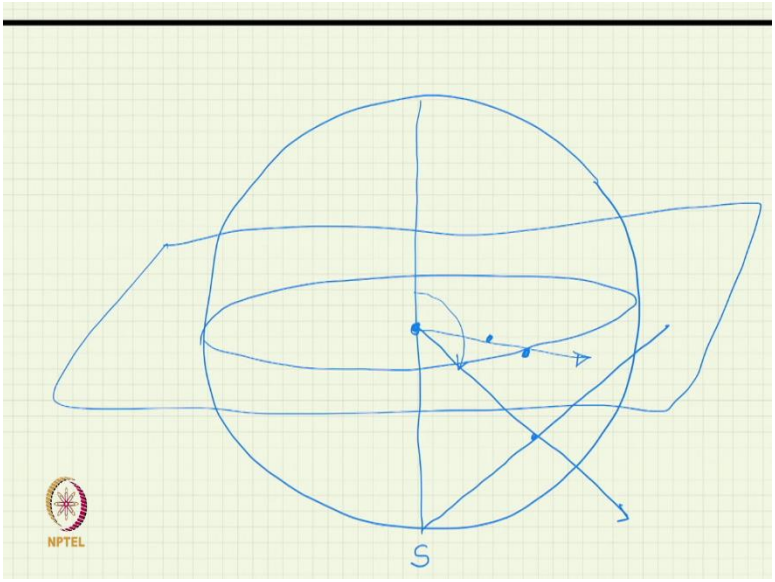

And now how do we handle a direction which is going down. So, there can be a direction like this. So, if this is our primitive circle, a vertical direction projected at the center. An inclined direction came out and projected somewhere inside the circle and a horizontal direction, a 90 degree direction projected exactly on the circle.

Now what will happen, if the angle of inclination is actually more than 90 degrees that it is going now not in the northern hemisphere, it is hitting the sphere in the southern hemisphere. If you join this point with the South pole, you can convince yourself that you will be outside the primitive. So, this point will hit the plane of projection somewhere outside.

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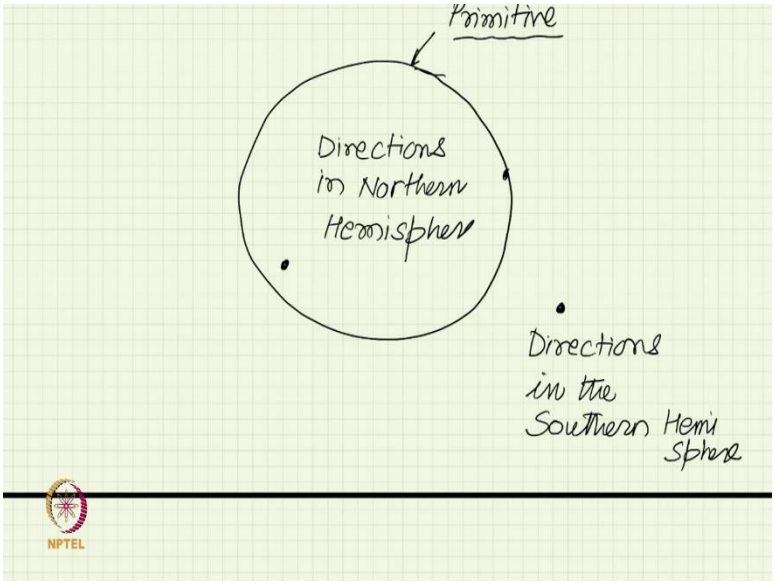

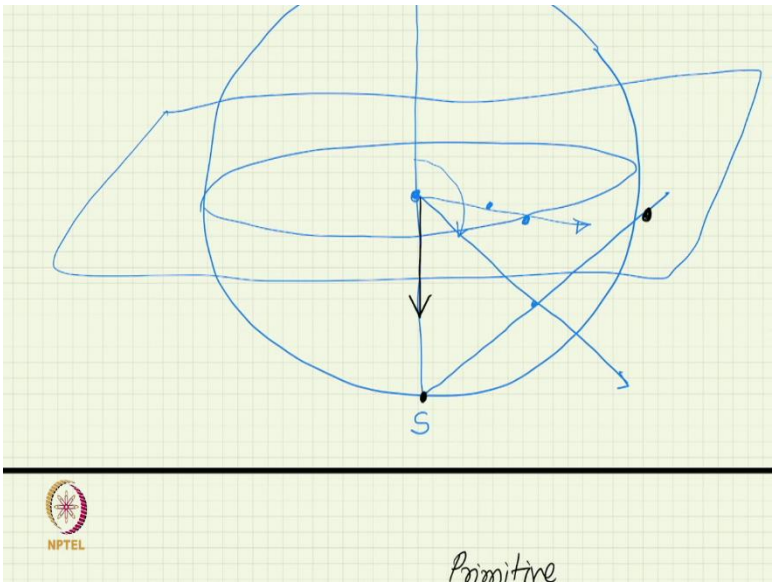

So, that is another point we can make. This is a primitive point inside the primitive add directions in the northern hemisphere. So, any point which is there is in the direction northern hemisphere. Point on the primitive is a horizontal direction you can say point on the equator. Point outside a point like this are directions in the southern hemisphere. Your tan theta by 2 formula will work for this also.

So, they will go out of their primitive. Now useful convention so that becomes little messy that things are going out of control, things are going out of the primitive. So, sometimes we do not like this although mathematically or in terms of projection, it is all fine and its angle of inclination from the vertical can be found by X is equal to R tan theta by two. You will find that now X is greater than R because theta by 2 for 90 degree was tan 45 which was 1.

If you have an angle greater than 90 degree, then theta by 2 is greater than 45. So, you will have more than 1. Tan theta by 2 will be more than 1 so it will be outside the sphere. Ultimately… so a direction pointing to the South pole is towards infinity. So, point at infinity. So, South pole really cannot be uniquely projected. So, North pole or the direction going vertically up has a unique representation the center of the circle but a direction going down does not have a unique representation. It will be somewhere some point on infinity can be anyway.

So, you can say a singular point or failure where we will not like to apply our projection. So, we will not try to project the South pole itself with respect to the South pole. In fact, the distance also see it becomes again you are joining now if the direction was going down it hits the sphere on the S and we decided to join S with the point on that. So, you are joining S with S, so you are actually stuck there you are not even going to infinity.

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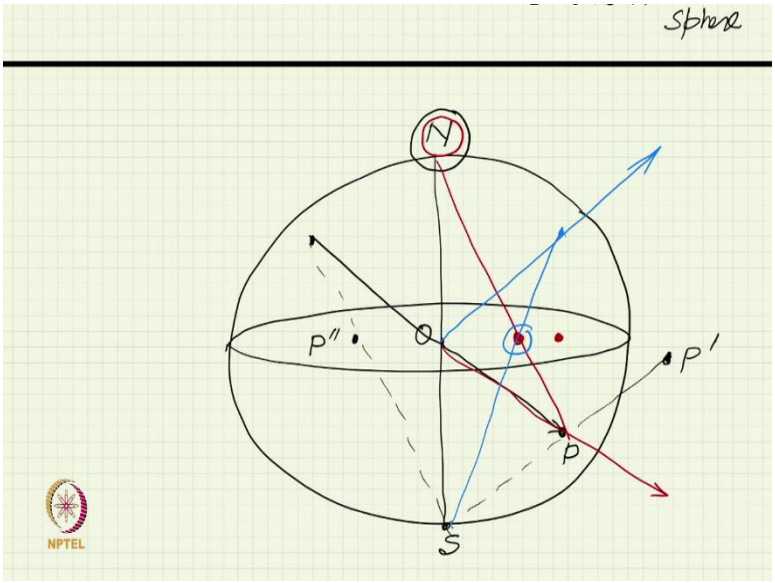

So, sometimes another way of solving this problem of southern hemisphere. So, there are two ways. I am stuck with this direction which is going down. I know that if I project it with South pole, I will get a point outside. So, let us call this OP, so its projection let us say P prime is outside, y double prime. P prime is outside; two ways of solving this.

See, if we are only interested in the direction and not in the sense which often happens. In the Miller indices also, we said that 100 direction or minus 100 direction if we are only interested in the X axis line and not on the X axis sense then 100 and minus 100 is the same. So, if you are not interested in the sense but only one the entire line, then the line is not only extending in the southern hemisphere, line is also extending in the northern hemisphere.

So, look at the opposite end of that line. If you look at the opposite end of the line that will obviously hit in the northern hemisphere. You can happily join by S and you get the projection point P double prime. So, that we say that now let us not go down. Let us look at the opposite direction of this line.

But if you insist, sense is also important to me. I should know that whether I am going down on this line or I am going up on this line. If that is the case, then, then either you live with that you have to... So, if you want to show directions going down all directions going down or outside the primitive, plot outside the primitive. But if you again insist that no I do not want to go out of the primitive also but I want to keep track of the sense, then one way suggested is that you project from the North pole.

Instead of South pole, you change your projection point itself and use North pole as the projection. If you use North pole as the projection, then the point will come here. But then, if you are using for some point, your formula tan theta by 2 will stop working because that was for the South pole. You have to derive a different formula for the North pole. It will work but you have to figure it out not so difficult.

So, all the same point the same point here if it was a projection from North pole was this direction, but if it was a projection from South pole, then I will join this. See where it hits the sphere join O, so that was this direction. So, this point is it the blue direction in the Northern Hemisphere or the red direction in the Southern Hemisphere. So, you have to tell me that whether you are projecting this point from the South pole or from the North pole.

So, if you are using both you will have to use some notation or some colour, open circle and closed circle or dots and crosses or red and blue to identify whether the points are plotted from South pole or plotted from North pole. So, we will end here.