Crystals, Symmetry and Tensors
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Lecture No. 7b
Stereographic Projection – I

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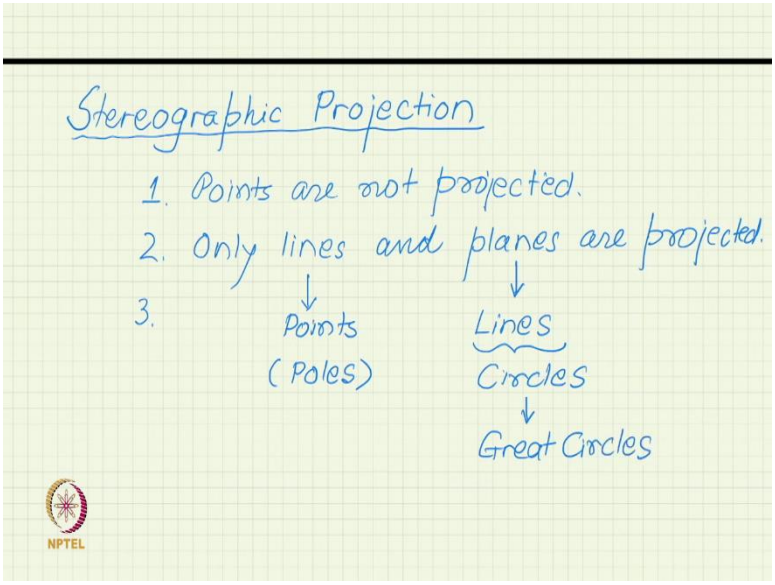

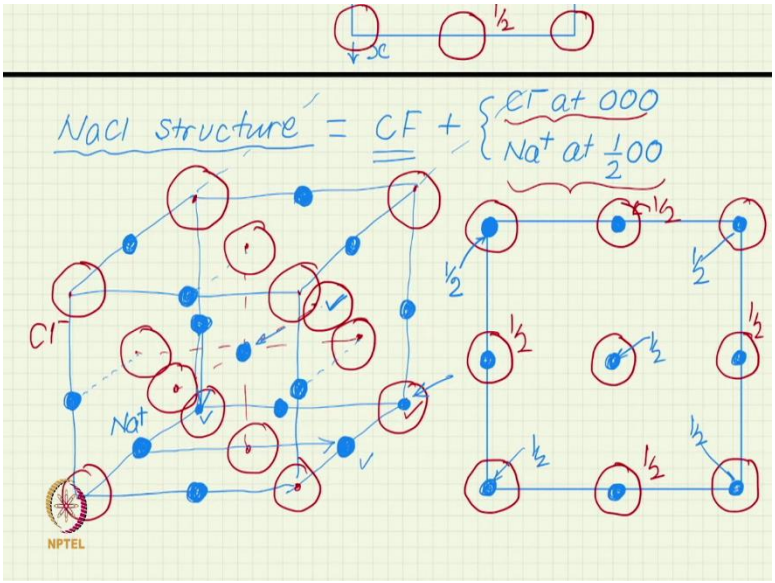

Now, let us go into another interesting projection which is quite common in crystallography but not so common outside crystallography. I am not really right means it is actually used in other places also but is still not very common. It is used in astronomy for example, in star charts. So, that is the stereographic projection, in fact it came from astronomy. Crystallographers adopted it later because this is one way of showing where the stars are located in the sky. So, that is called an stereographic projection.

Also, nowadays in one of the techniques, experimental techniques which you use the EBSD, the Electron Back Scattered Diffraction, there also this is very very useful stereographic projection. And mathematicians also like to use it in complex analysis. So, if you look or go deeper into complex analysis you will find that they have a stereographic projection.

So, what exactly is a stereographic projection? So, let us construct an stereographic project, let us give some brief information. First of all, one distinction between orthographic projection and stereographic projection, that in stereographic projection you do not project points. There is no way that you say what is…. I was asking here the question what is the projection of this point? What is the projection of a given point? But now I do not have an answer to that when I am talking of stereographic projection.

So, points are not projected. We will make some exception to this, we will add some points on a stereographic projection but they are not proper stereographic projection particularly when we want to represent inversion centres. So, inverse and centres are points, we will show them on stereographic projection but they are not actually stereographic projection. So, points are not projected. Only lines and planes are projected.

And in orthographic projection, if there was a point it projected as a point, if there was a line in 3D and you projected that entire line that will be a projected line. So, the dimensionality is preserved, point projects as point and line projectors line in orthographic. But in stereographic it is again not true because line will project as point so one dimensionality is being reduced that is the reason why we cannot predict a point because point is already zero dimensional, so where will it go?

So, lines become points, line project size points and planes project as lines and these points in the language of prediction sometimes called poles and the lines planes which become lines are very special lines, they are not arbitrary lines, they are not a straight line, sometimes they are straight lines but actually they are always circles. So, line projects such circles and these circles are known as great circles. So, those are the characteristics of stereographic projection.

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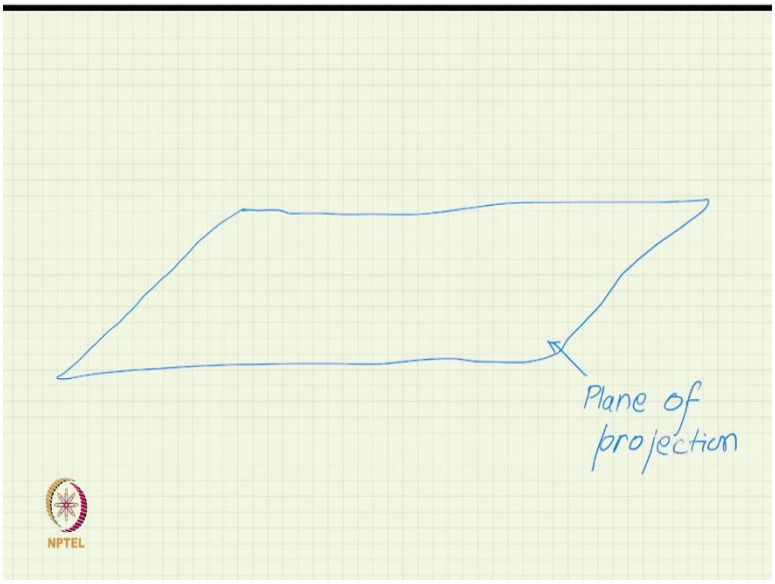

So, how do we do this? How do we create a stereographic projection? So, let us look at that. First of all, here also you are projecting on a plane. So, you need a projection plane so first of all if you want to create a projection, you need a projection plane but I do not need a rectangle. So, you need a projection plane. So, let us call this plane of projection, then I am interested in projecting certain lines.

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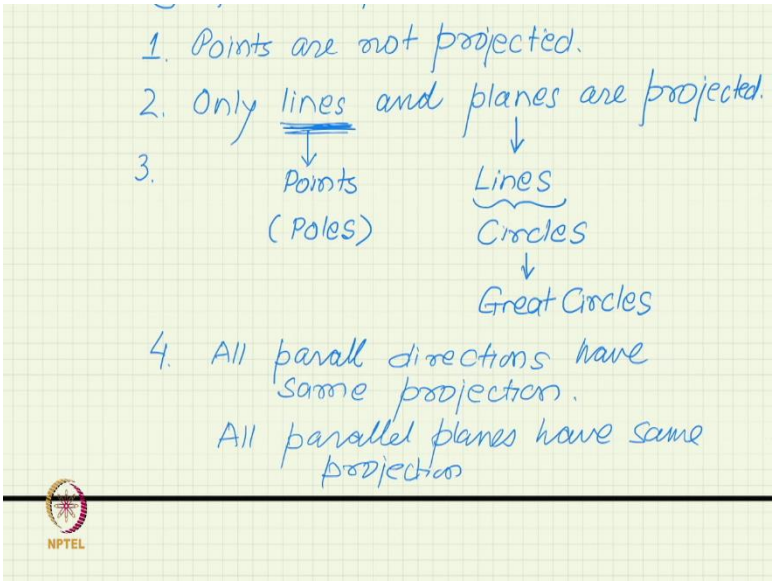

So, one more point here. When lines are projected, all parallel lines will be projected in the same point and all parallel planes will project in the same circle. So, really individual line means in the orthographic projection if you had parallel lines, they will have different parallel

projections but in a stereographic projection we will not distinguish between this line and the parallel line and a parallel line. We will only be interested in a direction.

A direction going normal to the back wall of this room to the front wall. Whether it is located there or whether it is located there is immaterial. So, something like it is reminding you of our definition of Miller Indices. Miller Indices also if this is a 100 direction, this is also 100 directions. It become 001 when it goes up. But then any direction going up is 001. So, all parallel directions have same projection and all parallel planes also have same projection.

So, that means what does it mean in other words, in other words if the orientation which is being represented not the exact location because whether the direction the vertical direction is this way or here or whether the vertical direction is there, both are vertical and by vertical we are specifying an orientation. We are not specifying the exact location so locations cannot be captured in a stereographic projection only the orientation.

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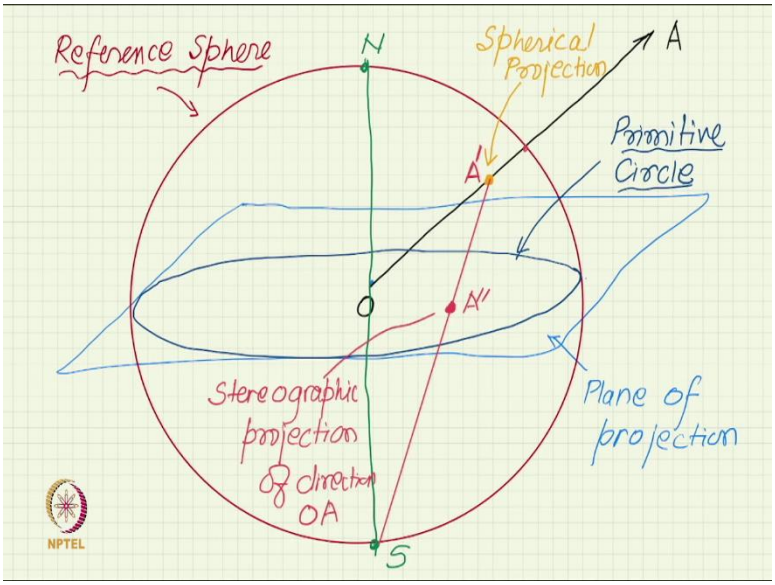

So, since only orientation is important and I want to project a given direction. So, I should have a direction drawn from some origin. I select the origin on the plane of the projection and then I draw the direction of interest from that origin. So, let us say that this is the direction which I want to project somewhere on this projection plane. I want to find and the direction is going to be project as a point so somewhere on this plain of projection, I want to find a point which will represent this entire direction or the orientation of the direction.

I should be able to find out by looking at that point what is the inclination of the direction with respect to let us say vertical or with respect to horizontal or whether it is going forward

or to the right. So, there should be a point on this projection plane which will represent the direction. So, let us call this direction some direction, let us say OA. So, I want to find a projection of OA.

The way to do that is we introduce a so-called reference sphere centered at the same point. So, this is my reference sphere. Obviously, if there is a reference sphere and there is a projection plane there will be an intersection of projection plane with the reference sphere and I will get a circle. A sphere always intersects a plane in a circle and it will intersect a plane passing through the center into a largest possible circle.

Any plane will cut a sphere in a circle but the circle will be smaller if the plane is off center. The largest circle, largest cross section of a sphere you will get when the plane is passing through the center of the sphere. So, you will get from the center of the sphere, till it is not good like get rid of it and live with my little less than perfect circle.

So, it intersects, the references sphere intersects the plane of projection in a circle, we give that a name we call that a primitive circle. These are just jargon which one has to know when one is approaching a new subject, every subject develops its own jargon, means it is just a circle but it is named primitive circle. So, primitive circle is nothing but intersection of your references sphere with the projection plane.

Now since the direction is starting from origin and going out into infinity somewhere it will hit the sphere. So, we locate that point, we locate the point where the direction hits the sphere. Suppose on the sphere you give me this point and I know that all directions emerge from the center of the sphere so a point on the sphere is a good enough representation of the direction because any point on the sphere, so I have already made a projection but that projection is on a spherical surface that every point on the sphere represents a direction which you get by joining that point to the origin or to the center of the sphere.

So, this itself is a projection of the direction and that is called a spherical projection. But now if you want to communicate or if you want to show this projection, you will have to have a three-dimensional spherical model and will show that this is the point which I am considering. So, although you have reduced from line to a point, those points are lying on the surface of a sphere and still not very convenient.

So, to reduce further to this point to bring this point to the projection plane what we do, we draw an axis which is normal to the projection plane and passing through the center. That

axis will cut the sphere in two points arbitrarily we can call with reference to imagining it as a globe. We can call it one point as North pole and another as South Pole. This just sort of a mnemonic or so you could have called it X and Y or whatever points. It will hit a normal to the projection plane will cut the sphere in two points and we are labelling those two points.

Then we say now the final step of projection stereographic projection is going to happen where we bring this spherical projection down to the projection plane. So, what we do we say that join the South pole to this spherical projection. Join the South Pole to the spherical projection. So, let us say that this OA direction got projected into on the spherical projection it was A prime.

So, on the sphere A prime represents the entire direction OA because if you give me A prime, I can connect A prime to O and extend that direction, I get OA. So, the point A prime on sphere is a complete representation of the direction OA, the entire line OA. But now, I have connected A prime to S, the South pole, that line will cut the plane of projection somewhere. That line will also cut the plane of the projection somewhere, that point we will now call the stereographic projection of the point. So, point A double prime is the stereographic projection of the direction OA. So, this A double Prime is the stereographic projection of the direction OA.

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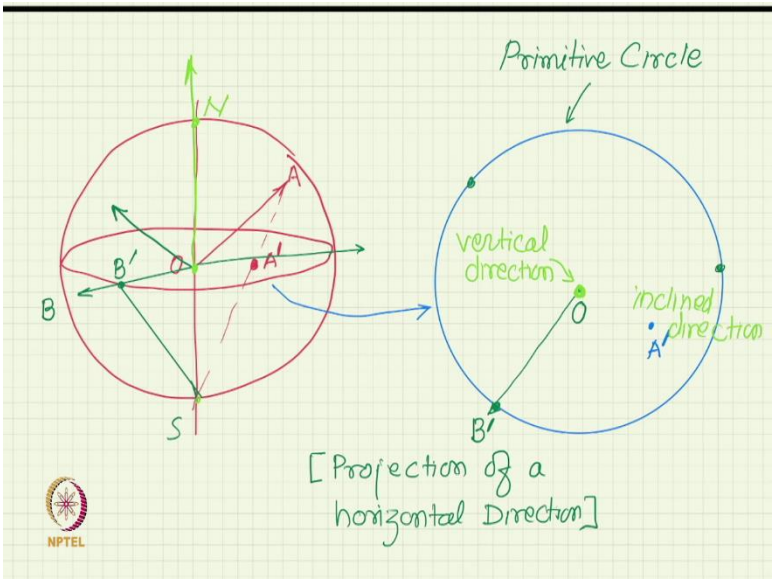

Now you can see, so that is the difficulty of drawing 3D and 2D. That is you will always come, let me change the language, I use double prime. So, this is OA, this is OA, this is A prime. So, A prime represents OA. Let me draw it in the circular form, so if we are looking down the projection plane we are looking down the projection plane then this is my primitive

circle. This is my primitive circle, so here the circle was also looking like an ellipse but now I am looking from the top. So, it will look like and my A prime was somewhere there. Where will horizontal direction project? A horizontal direction means a direction lying in the plane of projection.

So, let us construct a direction lying in the plane of projection. So, suppose this direction, so this is a horizontal direction, where will this project? At the edge of the circle, on the primitive circle because this circle was named. So, you start using these jargon so that you get familiar. So, this circle was named primitive circle, if there is a horizontal direction and we join it with S. So, if we join it with S, I do not get any new point I get that same point with the plane. So, if there was a horizontal direction OB, then its projection B prime is exactly on the circle.

So, what is B prime? This is a projection of a horizontal direction. How many horizontal directions you have in a space? Infinity. How many points you have on a circle? Infinity. So, different points on the circle are just representing different horizontal direction. So, if there was other inclined horizontal direction that way, you will get some point there. If there was a horizontal direction this way, we will get some point there and so on.

So, all points on the primitive circle are actually projection of horizontal directions that is their projection of directions lying in the horizontal layer cooked up word because there is no horizontal vertical in the crystal space or in the projection space. So, horizontal here is being used as a simple mnemonic for projection plane instead of using two words or little bit more complicated word projection plane, we say horizontal plane.

So, all horizontal directions, so there is a one to one correspondence you can see. Every horizontal direction is a point on the primitive circle. Every point on the primitive circle is representing the horizontal direction. How will you find that horizontal direction which is being represented by this projection? You simply connected to the center. Because since this is a horizontal direction, it is actually lies in the projection plane.

So, in this case you can actually draw the direction and say that this is the direction which is being projected in B prime. I do not have that freedom with A prime because A prime was a space direction. So, it is a direction somewhere inclined that way which is being projected by the South pole there. So, I cannot join O and A prime and say that is my direction which I am projecting. So, the OA direction is actually at an inclination to my projection plane.

Now let us look at another special direction. This time let us took this vertical direction. So, vertical direction, where will it hit the surface of the sphere? Exactly at the North pole and if I join with South pole, where will it cut the primitive circle? At the center of the primitive circle. So, the center of the primitive circle is also a direction… all points are directions now. Directions have been projected into points and there is a one-to-one mapping of all the points on this stereographic projection to directions in space and we are just picking out some special ones to start getting a feel of it.

So, the direction at the center is the vertical direction. Again, vertical is a jargon. What do we mean by vertical direction? A direction normal to the plane of projection. So, direction lying in the plane of projection or on the primitive. Direction normal to the plane of projection is the center of the primitive. All other directions like A prime are directions which are neither lying in the plane of projection nor is vertical or perpendicular to the plane of projection. So, they are inclined direction. So, vertical direction, horizontal direction and an inclined direction.