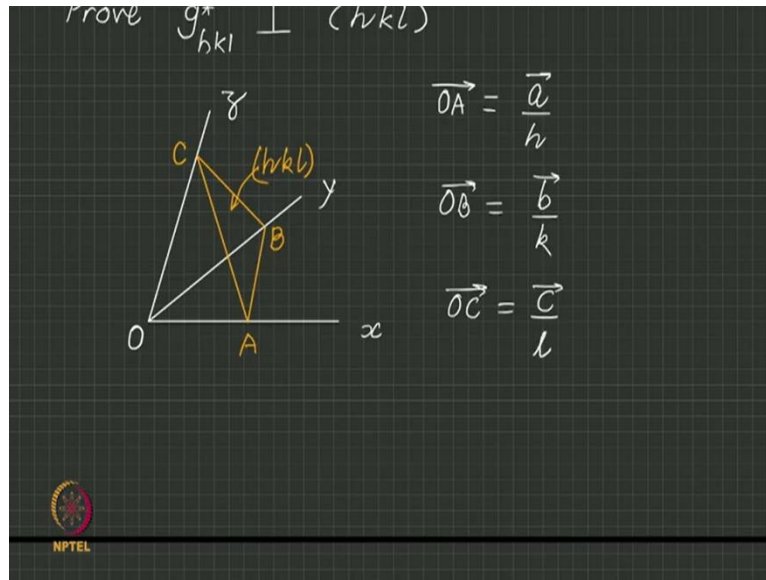


Crystal, Symmetry and Tensors
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Lecture 5a

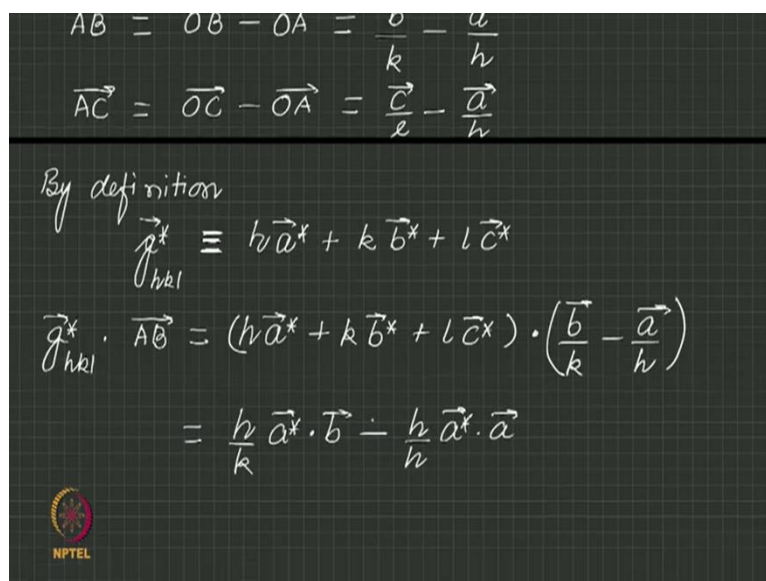
Reciprocal Lattice Vector (G^*_{hkl}) is perpendicular to plane (hkl)

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Prove that g^*_{hkl} is perpendicular to the plane hkl. So, these are our x y and z axes. Again, we have the plane a, b, c. We have already seen in the previous example also that the vector OA if this plane is hkl then its intercepts are the vector intercepts are a by h, b by k, and c by l.

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So, let us find out vector lying in the plane vector lying in the plane. So, two vectors lying in the plane hkl is AB for example AB and AC. So, vector AB is nothing but OB minus OA. So, this is b by k minus a by h and vector AC is OC minus OA is equal to c by l minus a by h. So, and now we already and by definition g star hkl is a vector. We can use three horizontal lines also to show that this is actually a definitional equality is ha star plus kb star plus lc star.

Now, we just take the dot product of ab and g star hkl. So, let us take g star hkl dot ab. Now, we have an expression for both of them, so, we just write them out and take the dot product. So, do the algebra so that is g star and ab is b by k minus a by h. We can expand this. So, h by k a star dot b minus h by h a star dot a plus k by k b star dot b minus k by h b star dot a plus l by l c star dot b minus l by h c star dot a.

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$$\begin{aligned}
 \vec{g}_{hkl}^* \cdot \vec{AB} &= (h\vec{a}^* + k\vec{b}^* + l\vec{c}^*) \cdot \left(\frac{\vec{b}}{k} - \frac{\vec{a}}{h} \right) \\
 &= \frac{h}{k} \vec{a}^* \cdot \vec{b} - \frac{h}{h} \vec{a}^* \cdot \vec{a} \\
 &\quad + \frac{k}{k} \vec{b}^* \cdot \vec{b} - \frac{k}{h} \vec{b}^* \cdot \vec{a} \\
 &\quad + \frac{l}{l} \vec{c}^* \cdot \vec{b} - \frac{l}{h} \vec{c}^* \cdot \vec{a}
 \end{aligned}$$

Again, you can use the relationship between a star and b. So, a star dot b is 0, b star dot a is 0 and b star dot a is 0. Sorry this last one did I make a mistake here I was making a mistake so let me correct it. So, these are c stars, lc star we have. So, c star dot a and c star dot b so both of which is 0.

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The image shows a chalkboard with handwritten mathematical derivations. At the top, there are some partially visible equations: $\vec{g}_{hkl} \cdot \vec{a}$ and $+\frac{l}{l}\vec{g}_{hkl} \cdot \vec{b} - \frac{l}{h}\vec{g}_{hkl} \cdot \vec{a}$. Below this, the derivation continues with the following steps:

$$\begin{aligned} &= -1 + 1 = 0 \Rightarrow \vec{g}_{hkl} \perp \vec{AB} \\ \text{Similarly} \quad &\vec{g}_{hkl} \cdot \vec{AC} = 0 \Rightarrow \vec{g}_{hkl} \perp \vec{AC} \\ &\vec{g}_{hkl} \perp (ABC) \Rightarrow \boxed{\vec{g}_{hkl} \perp (hkl)} \end{aligned}$$

An NPTEL logo is visible in the bottom left corner of the chalkboard image.

So, you can see that only two terms are left two non-zero terms are left first one is h by h a star dot a with a minus sign. So, that is minus one. The second one is k by k b star dot b that is plus 1, minus 1 plus 1 that is 0.

Similarly, so I have done it in detail but you can convince yourself now for the other relation that g star hkl dot ac also you will find to be 0 when you expand and work it out like this. So, what is it indicating? It is indicating that g star hkl is perpendicular to the first zero indicated that g star hkl was perpendicular to ab and the second zero indicates that g star hkl is perpendicular to ac. So, that means g star hkl is perpendicular to two vectors in the plane.

So, g star hkl is perpendicular to the entire plane abc which is the hkl plane. So, that is the, it was supposed to be first relation according to my plan because we need this in the proving the other relation so, that is why I was calling this the first relation.