Crystals, Symmetry and Tensors

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Lecture 3b

Weiss Zone Law

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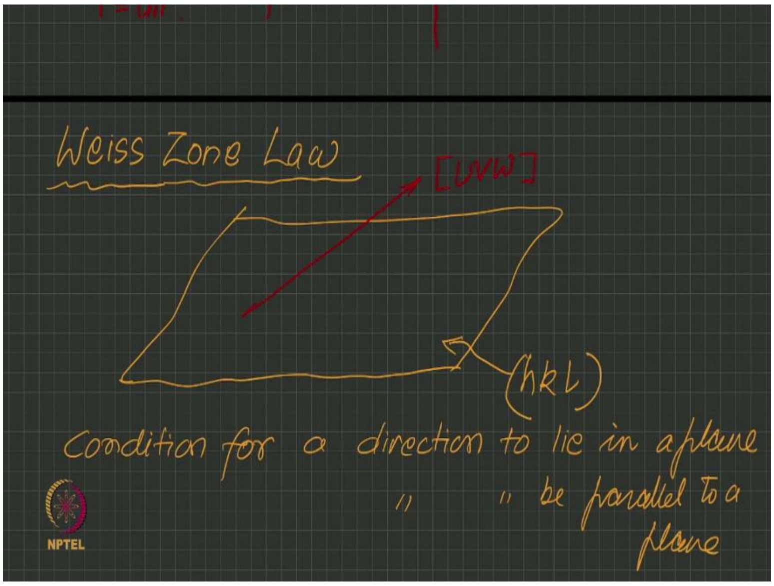

What does Weiss zone law says? this says that, if you have a plane, suppose you have a plane hk,l and you have many directions in this plane, so suppose one direction uvw, Weiss zone law tries to explore the relationship, that is there a condition, is there a relationship between uvw and hkl, if uvw lies in the plane hkl, so condition for a direction to lie in a plane which is the same thing as a direction to be parallel to a plane. Lying in plane and parallel to plane crystallographically are equivalent terms, why?

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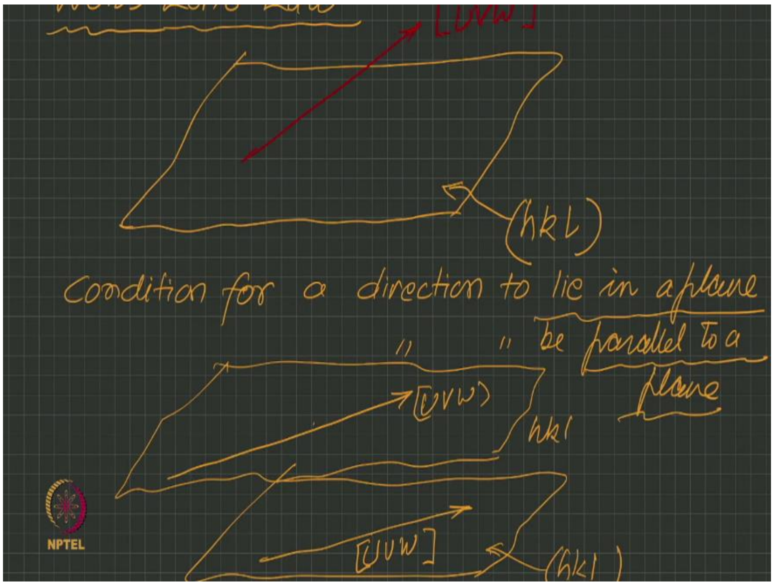

Because we have said that we are not distinguishing between parallel direction and we are not distinguishing between parallel planes. So, if you insist that this is my plane hkl but my direction is here and is not lying in the plane, I will say lift the plane up to pass through the direction and this plane being parallel to this plane is still hkl, or pull the direction down to lie in the original plane, and since this direction is parallel to that direction, this direction is still uvw. All parallel direction and all parallel planes have the same Miller indices.

So, that is why there is no distinction between the phrase parallel to a plane or lying in a plane, they are considered to be equivalent in crystallography. Unless and until there is some application where you want to make a distinction then you have to take care of that, but otherwise in general like in the Weiss zone law, we are seeing.

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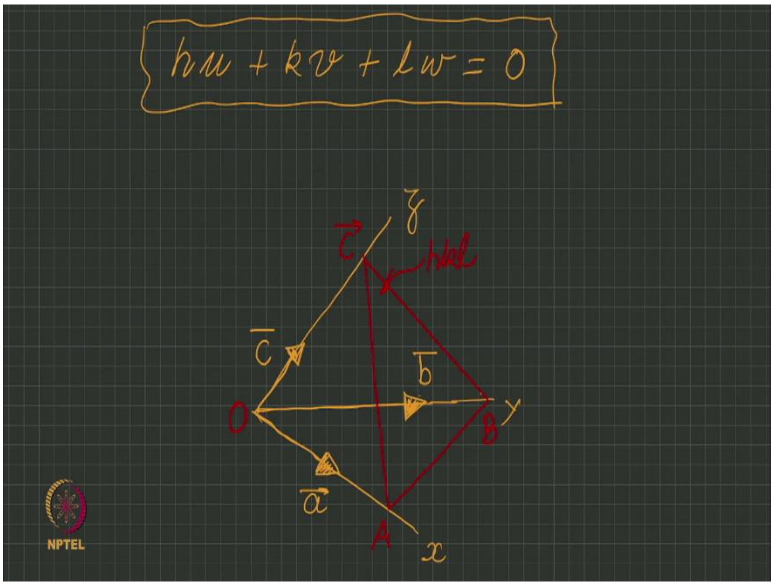

So, what is that condition? That condition is also very simple so that says that hu plus kv plus lw is equal to 0 is a very very important law. Very very important for crystallographic calculation, crystallographic interpretation, many cases you would like to check whether the direction is lying in the plane or not, or you may like to find the intersection of two planes which we did last time, we will redo that exercise using Weiss Zone law. So, that was a geometrical method of finding intersection of two planes, you can do it purely algebraically by using Weiss zone law.

So, let us see how do we prove this now, more or less we will use the same technique. Let me not begin with the orthogonal axis anymore, so let me try to draw some non-orthogonal axis, not very easy to draw but let me try. So, this should not give you any impression that the x, y, and z are orthogonal and then I take unit vectors, unit, not unit vectors, basis vectors a, b, and c, and that also I am, I should not give you any impression that they are equal or something in any sense.

So, all three are of different length and different direction. And then I have my plane, I have my plane hkl. You have just seen that how to write OA, sorry OB, and OC, so same results we will use, so we do not have to, sorry, we can only, we cannot use now I J K vectors because that is not available to us.

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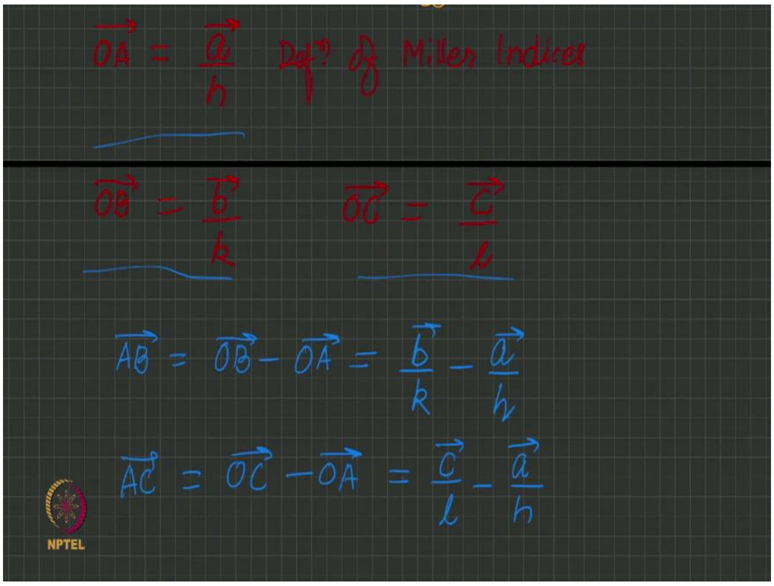

So, we, because A vector is not necessarily I, so we write OA is equal to just a by h, that is Miller indices, definition of Miller indices. Because the Miller indices of the plane is hkl, h is the reciprocal of the intercept in terms of a. So, reciprocal of h multiplied by a gives you the intercept, the vector intercept. So, you have OA, you have OB, and you have OC, and then we go exactly the same way of finding AB and AC. So, b by, this would be b by h, and b by k minus a by h, and c by l minus a by h, just from here.

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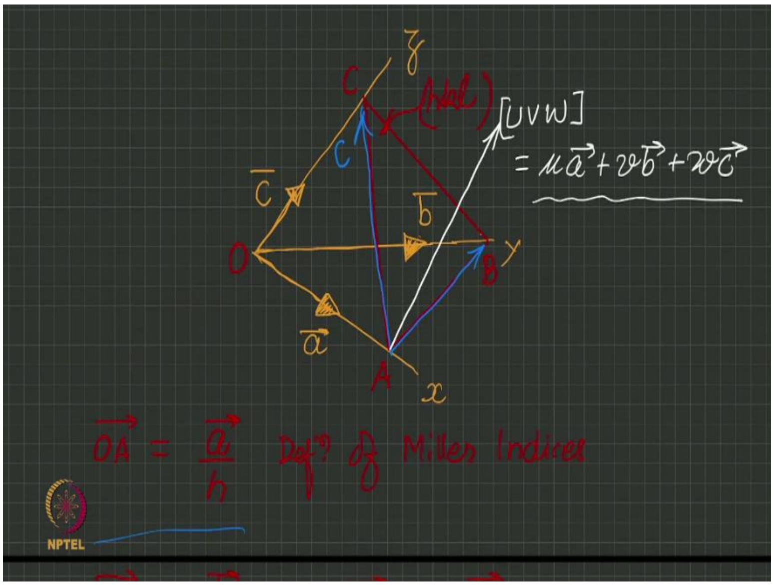

So, now I have, till now we did the same thing, now only thing is that, now our Vector with which we are interested in, that Vector is not perpendicular to the plane but is lying in the plane, so uvw is lying in the plane. So, if I have two non-parallel vectors AB and AC and a third Vector also is lying in the plane defined by AB and AC, what can we say from vector algebra, as a linear combination of these two vectors?

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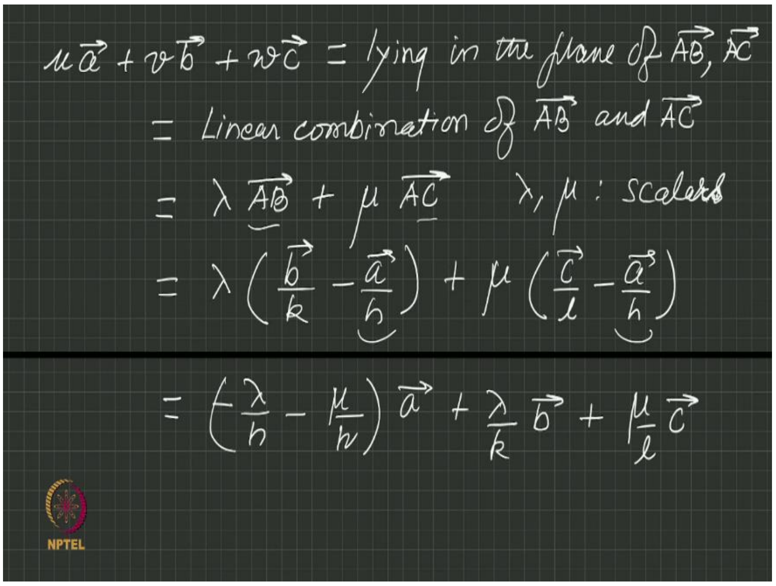

So, ua plus vb plus wc, since it is lying in the plane, so this Vector should be a linear combination AC. Which means some Lambda times AB plus mu times AC should give me the same vector.

So, now we use our AB and AC, so Lambda times b by k minus a by h, mu times c by l minus a by h is what we had found for AB and AC, just because just based on the definition of the Miller indices of the plane in which AB and AC are lying in terms of h k and l. So, we can now collect the terms in terms of a b and c.

So, what do we have? We have Lambda by h minus Lambda by h for a and minus mu by h a, and then for b we have Lambda by k b and for c we have mu by l c. But AB and C are also linearly independent vectors, because that was the basis vectors for our original coordinate system. So, whenever if they are linearly independent vectors and you have a quality like this, the component should be equal that you know from.

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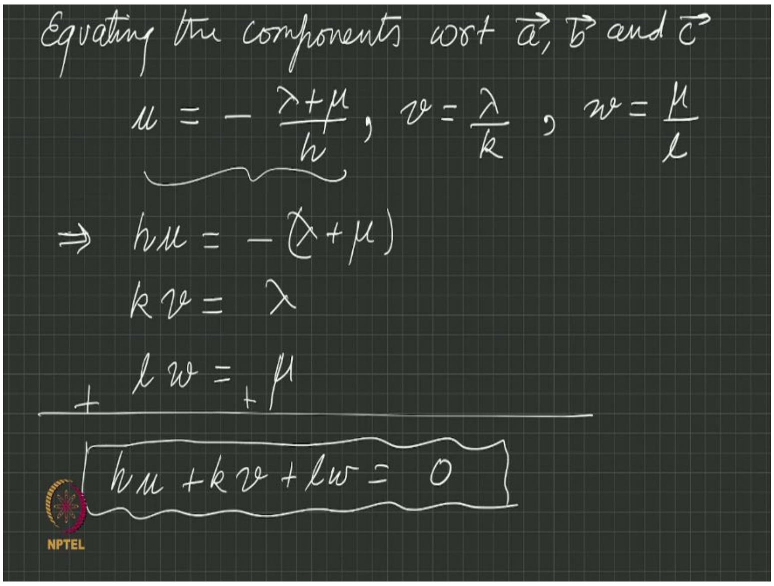

So, equating the components with respect to a b and c, just like you equate the component with respect to i j k, if two vectors are equal their components are equal, that was in the Cartesian coordinate system, Crystal coordinate system is no different except for the fact that a b c are not equal and their angles are not 90 degree, but if two vectors are equal with respect to these vectors, the corresponding components have to be equal. So, we get u is equal to minus Lambda plus mu h, v is equal to Lambda by k, and w is equal to mu by l.

For hu is equal to minus Lambda plus mu from here, kv from the second equation is Lambda, and lw is mu, all you have to do now is to add these three equations hu plus kv plus lw is 0, the Weiss zone law.