Introduction to Materials Science and Engineering
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Lecture - 84
Error function solution of Fick's second law

In the previous video, we saw an established Fick's second law, which was a differential equation. We will look at in this video; we will look at one form of the solution, which is the error function solution of this differential equation.

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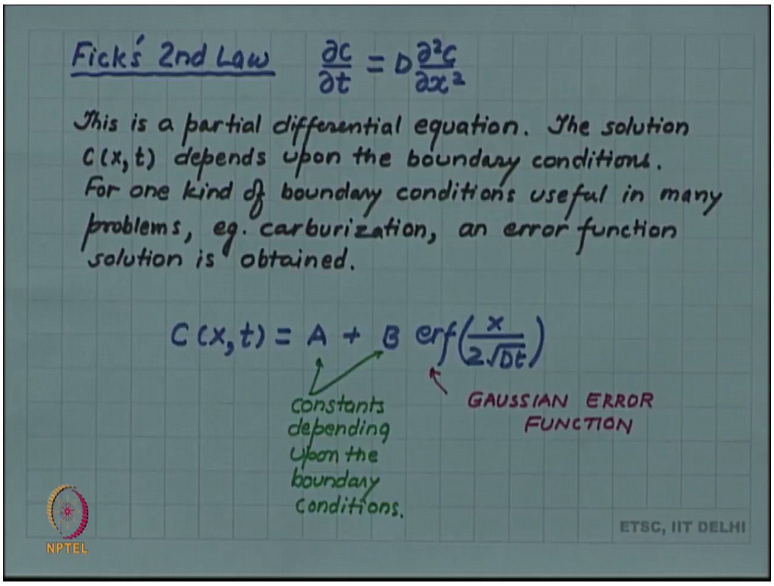

So, recall that Fick's second law was just this differential equation del C by del t is equal to D del 2 C by del x square. Now this is a partial differential equation, equation the solution of this equation will mean finding the function C x, t. So, the solution the solution will be how does the concentration change with position and time.

So, this is the solution, and this will depend not only on the differential equation, but also on the boundary condition this. So, the solution C x, t depends upon the boundary conditions. For one kind of boundary conditions, for one set of boundary conditions, which are useful in many problems for example, carburization, which we discussed while introducing diffusion, error function solution the error function solution is obtained. What do we mean by the error function solution?

The error function solution is so the solution can be written as C x; t A plus B error function, A and B are constants, depending on the boundary condition. So, both A and B constants conditions, and this erf erf is the Gaussian Gaussian error function.

So, this whole thing inside the bracket is the argument of the Gaussian error function, it is just like it is just like A function like sin theta, cos theta, which for any given are argument theta can be evaluated. Similarly, the error function can be evaluated for this value of the argument argument is x by 2 square root of D t, D was the diffusion coefficient. So, let us look at, what is what is this Gaussian error function.

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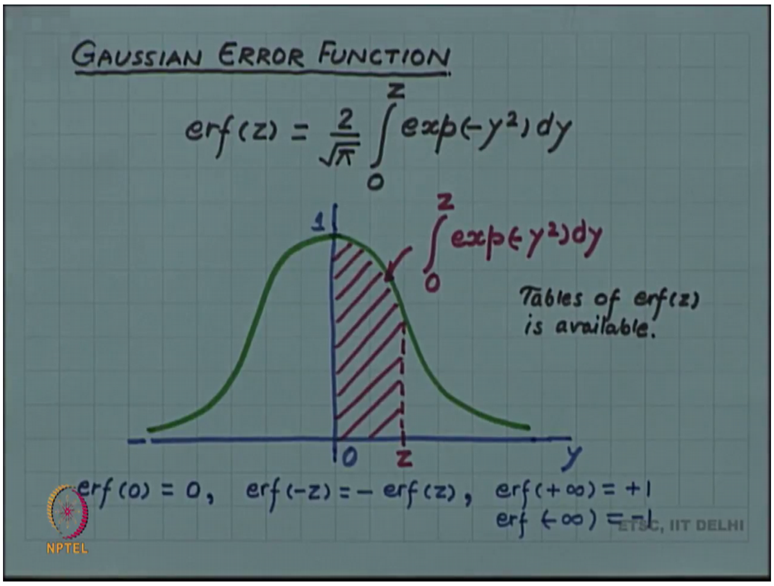

So, let us introduce this Gaussian error function. The definition of Gaussian error function is error function for any given argument z is 2 by square root of pi is numerical constant integral of the limit 0 to z integral of function exponential function minus y square dy.

So, error function is defined as integral of this exponential function exponential minus y square. And, if you integrate it from 0 to z, you get the error function value at z of course, multiplied by this numerical factor 2 by root pi, which is also called the normalizing factor. Let us look at this argument exponential minus y square and let us plot it. I could have written it as x, but I have written y; so as not to confuse with the x which was there in our argument x by 2 square root D t.

So, I have taken this x axis variable as my y now; and if you look at this function exponential minus y square. So, this function is a nice symmetric function at plus y and minus y to have the same value and at y is equal to 0, it will have exponential 0 and that will be a value 1. So, at y is equal to 0, it is a value equal to 1. And then this function will decrease, as you increase y because of this negative sign function will go like this, so it is the Gaussian bell shaped curve and by the very definition of this function; so you have to find the area from 0 to z.

So, if I want to find the error function at z. So, all I have to do you have to find area of the curve up to z. So, this area is nothing, but is what is representing represented by that integral 0 to z exponential minus y square D y. So, this area can be found and if this area is multiplied by 2 by root pi you get the Gaussian error function. So, some of the properties of this function you can see very quickly, that the error function of 0, because then the limit will be 0 to 0. So, you have both the upper and lower limit at 0. So, the area will be 0.

So, error function 0 will be 0 and because of the symmetric curve error function of minus z will be minus of error function of z, because the areas will be the same, but you will take D y in an opposite negative sign , so that will give this negative number. And error function of infinity will be plus 1 and error function of minus infinity will be minus 1.

And for all other values of error function values can be numerically found and a tabulated. So, tables of error function is available; you can we will find it in books and also on the web, it is just like tables of sin theta or cos theta. So, tables of or in many mathematical software you can have this function defined also.

So, this is your error function and this error function appears in this solution C x, t A plus B error function of x by 2 square root D t and the constants A and B have to be determined by a specific boundary condition. So, we will take up one example, of carburization and see how A and B can be determined for a particular specific situation.