Introduction to Materials Science and Engineering
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Lecture - 83
Fick's second law

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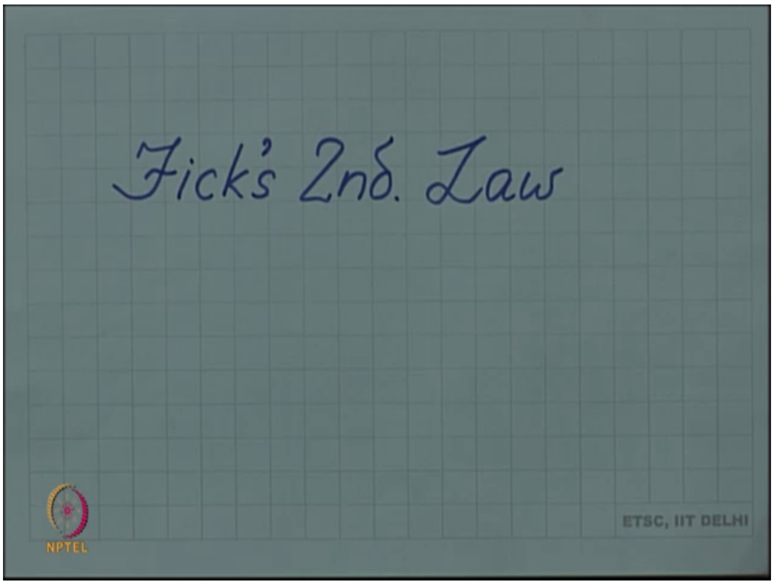

So, we are looking at diffusion and we discussed the Fick's first law of diffusion, which was relating flux to the concentration flux to the concentration gradient. We now we will look at Fick's second law which second law can be easily derived on the basis of mass conservation. So, let us look at what is Fick's second law. So, let us consider a block.

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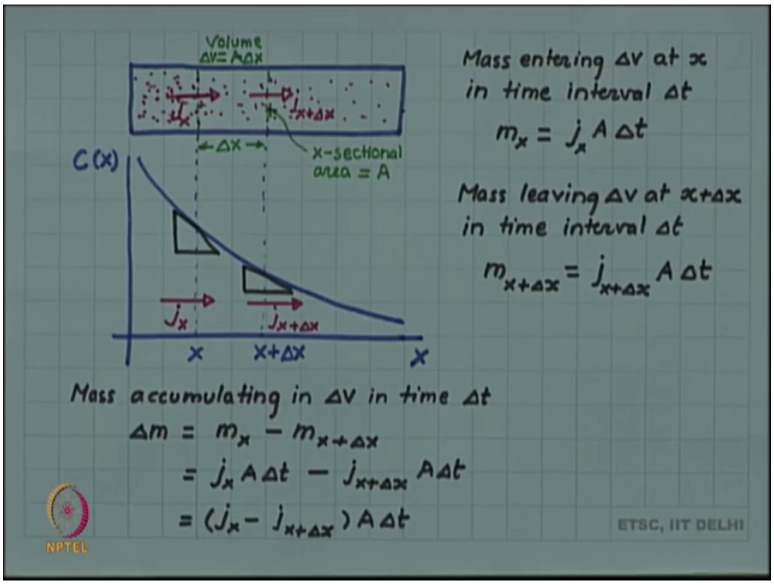

And let us assume that there is a concentration gradient which is existing in this block. So, concentration of some diffusing species is high at one end and in low at the other end and it is gradually decreasing. And let us plot that concentration gradient a function of distance along the bar. And let us say that concentration profile is something like this, this is the concentration as a function of x and this is the distance x.

I am deliberately drawing not a straight line, but a curve because I want to have different slopes at different locations of x. So, let us assume that at some location x , let us say that this is a location x and let us look at another nearby location x plus delta x. So, I am considering what is going to happen in this volume of width delta x let us say that the cross-sectional area cross sectional area is A.

So, we have a volume A delta x, we have a volume delta v which is A delta x. Now if we apply we have we already know our Fick's first law. So, if we apply the Fick's first law we see that at the x we have a certain concentration gradient. And at x plus delta x we have another concentration gradient. So, based on this concentrate these concentration gradient there will be a flux, there is a flux of these species. So, there will be a flux at x. Let me call that jx which will be transporting material into this volume.

And then at x plus delta x we have another slope so we will have another flux, which will now be transporting the material outside this volume and I am calling that j x plus delta x because that is at a location x plus delta x.

So, in a small we want to find out what changes happen in the concentration in this volume delta v as a function of time. So, in time delta t how much does the concentration of this volume changes. So, look let us look at that concentration change in this volume. So, mass entering delta v at x in time interval delta t. So, let us call that mass m x and what will that mass be. So, let us recall the definition of this flux jx plus flux is mass per unit area per unit time.

So, if we take the flux jx and if we multiply it with the area so, it is over the cross-sectional area A and if we multiply it with the time interval delta t we will know the total amount of mass which is entering in this time interval into this volume delta v at the cross-section x.

Similarly, if we now write mass leaving delta v at x plus delta x in the same time interval then we have m x plus delta x j x plus delta x only the value of j is changing the cross-sectional area is the same and the time interval is the same. So, if we take the difference of these 2 we will find what is the mass accumulating in this volume. So, let us do that so mass accumulating in delta v in time delta t. So, this mass let us call that delta m is nothing but mass in and mass out minus mass out. So, this is jx A delta t minus j x plus delta x A delta t.

So, we can write this as j x minus j x plus delta x A delta t.

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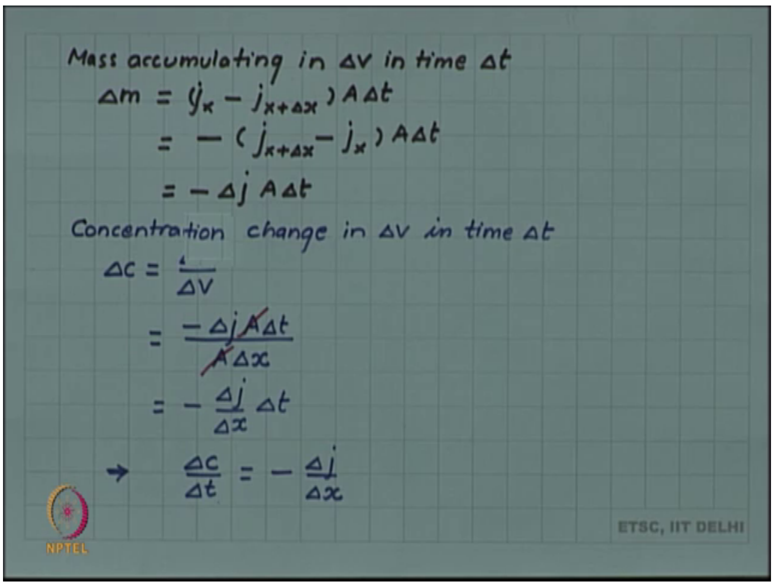

So, we have mass accumulating in volume delta v in time interval delta t delta m equal to j x minus j x plus delta x A delta t, we just rewrite it as minus of j x plus delta x minus x A delta t, we are rewriting like this because this expression in the bracket now is the difference that what we will call delta j. So, this is the difference delta j A delta t. So, this is the mass which has accumulated in the volume in time interval delta t. So, we want to find out the concentration change in time t.

So, the concentration change let us look at the concentration change now. So, concentration change or change in concentration in delta v in time t, now remember that in Fick's law we are using the concentration as volumetric concentration. So, by very definition the concentration change delta c a mass divided by volume so the delta m by delta v, delta m is the change in mass and delta v is the volume of the element which we are considering and if we write this as minus delta j A delta t and delta v is nothing but A delta x.

So, A cancels from numerator and denominator and we can write this now as minus delta j by delta x delta t and if we transpose delta t on this side we can find the rate of change of concentration. So, we can write this as delta c by delta t is equal to minus del j by del x.

So, this is a nice equation we have arrived at which is giving me that average rate of change of concentration in the time interval delta t is the rate of change of flux with the x coordinate. So, the spatial rate of change of flux is being linked to the temporal rate of change of concentration. So, we already have arrived at our equation only these are the average values and we want to convert it into a differential equation. So, all we have to do now is to take the limit.

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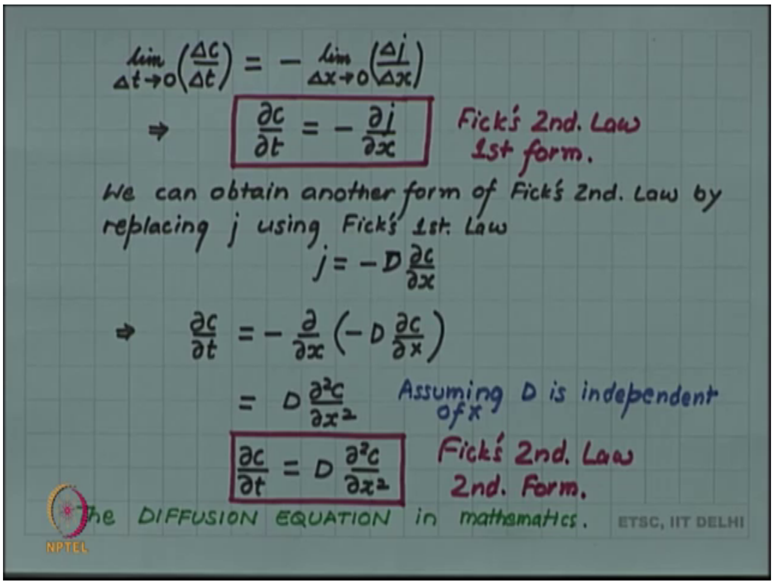

So, if we take the limit of delta t tending to 0 of del c by del t. So, instead of average over the time interval we will now have the instantaneous rate of change of concentration and similarly on the other side we take the limit delta x tending to 0.

So, instead of average change over a finite distance delta x we will have the rate of change at a particular position in a space. So, this quickly then changes this process of taking the limit will then change it into the differential equation we now have del c by del t is equal to minus del j by del x. So, this is one of the forms of Fick's second law. So, it is it is saying that rate of change of concentration with time is the negative rate of change of flux with space. So, this is one form of Fick's second law let us write it Fick's second law first form.

Another form of the equation can be found simply by replacing j using Fick's first law. So, we can obtain another form another form of second law by replacing j using Fick's first law. And remember that in the Fick's first law we had established that j is equal to minus D del c by del x. So, if we do that we find del c by del t equal to minus del by del x minus D del c by del x.

So, and if we now assume that D is constant independent of x we can take it out of the differential. So, we will get this as D del 2 c by del x square. So, we have another form del c by del t is equal to D del 2 c by del x square Fick's second law, Fick's second law second form and this of course, is assuming D to be constant. Assuming D in independent of x, so, essentially Fick's second law is a differential equation and it is a very famous and important differential equation in mathematics and it is because it comes in diffusion problem this equation itself is known as the diffusion equation.

So, this is the diffusion equation mathematics that is if any variable any variable c it need not be concentration if any variables c if it is differential coefficient with respect to time is related to it is second differential coefficient with respect to space through a constant like this mathematically is called a diffusion equation.

And we have seen that we arrive at this simply by the mass balance or all that was done is to see how much is coming into the volume and how much is leaving and the difference is what is getting accumulated and that is what get converted into a concentration change. We will look at a kind of solution of this equation in the next video.