Introduction to Materials Science and Engineering
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Lecture - 06
Symmetry 1

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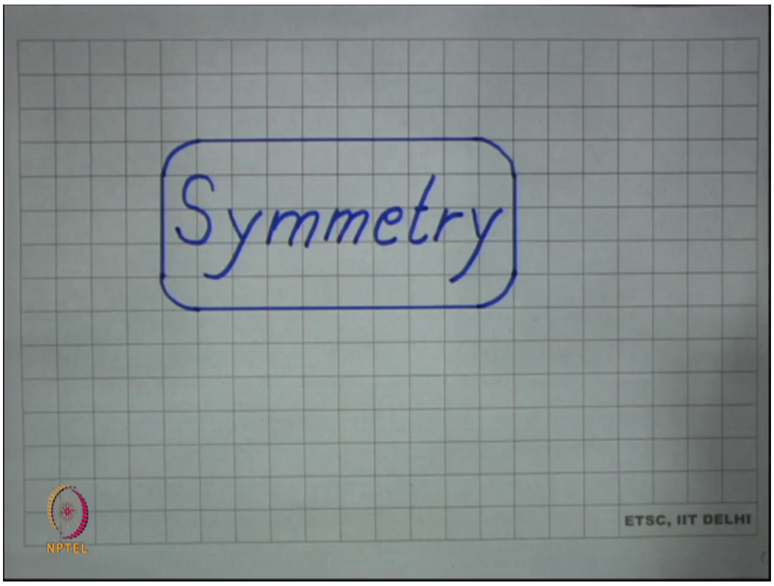

In the last lecture we said that the shapes of the unit cell cannot be used as a basis for classification of lattices. The real basis for classification of lattices we claimed is the symmetry, to justify that claim we in this lecture we develop the basic ideas of symmetry. Let us begin with the definition. So, we define symmetry.

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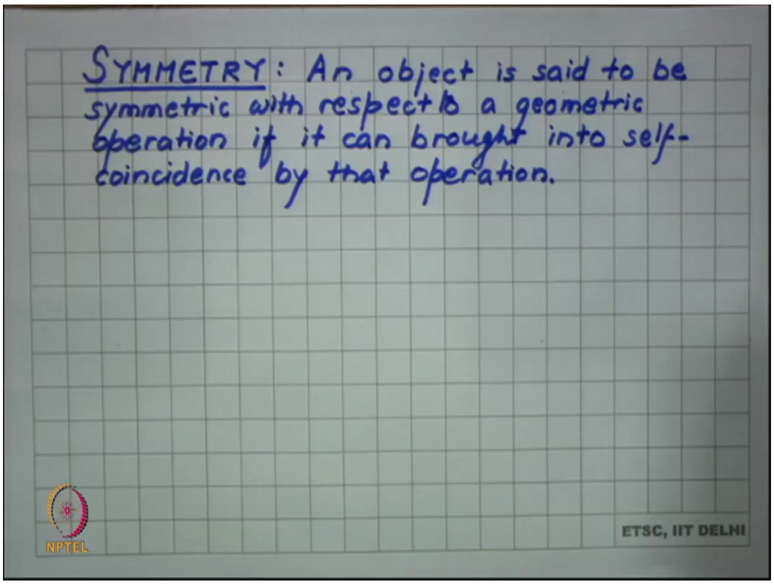

Let me try to put a definition an object is said to be symmetric with respect to a geometric operation, if it can be brought into self coincidence by that operation. The object is said to possess symmetry with respect to some geometric operation, then the definition appears to be little bit abstract.

So, let us try to see, but it will become concrete, if we look at some example. So, we begin with the example.

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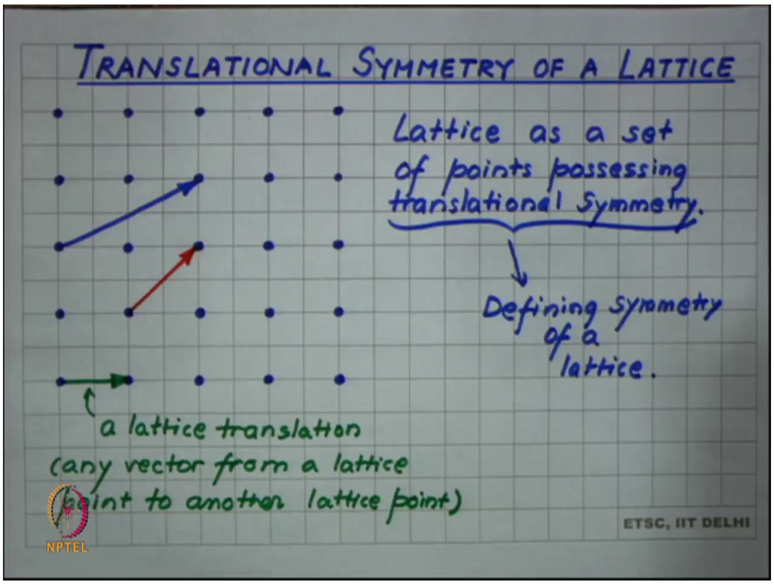

Of this 2 d lattice so, familiar to 2 dimensional is square lattice any vector from a given lattice point to another lattice point is a lattice translation. So, let me mop a lattice translation. So, from one lattice point to another lattice point I draw this vector, this is an example of a lattice translation. A lattice translation any vector from a lattice point to another lattice point.

Now, if we shift all the lattice points by this vector, the lattice will come into self-coincidence. Let me illustrate this with the help of this transparent sheet. So, I put this transparent sheet and I make a copy of this lattice on the transparent sheet, just copying it in red. So, the original and copy can be seen distinctly. So, I now have a copy of the original lattice on this transparent sheet.

Now, if I start shifting the sheet in the direction of this green vector, then I find the lattice is shifted and the 2 lattices on the sheet, and the transparent paper are not coinciding, but if I continue the shift and if shift is equal to the green lattice translation, then I find that the copied lattice is exactly occupying the same points as the original lattice. So, this shift by a lattice translation vector is the symme is a symmetry operation of this lattice; this is the translational symmetry of the lattice.

Of course, a lattice will have many other translations. So, let me mark another translation let us say this diagonal vector starting from this lattice point to a diagonal point, this is also a lattice translation and if I had shifted my copied lattice by this vector, again I would have got a self-coincidence. So, if I put this here again and instead of going by this green vector now if I go by this red vector again I come into self-coincidence. So, the lattice is self-coincident with respect to many different lattice translations all of them are symmetry operations of this lattice. In fact, the translational symmetry is the definitional symmetry of the lattice, in other words lattice can be defined we can define lattice as a set of points possessing translational symmetry.

So, translational symmetry is a defining symmetry of a lattice, a lattice apart from these translational symmetry may possess other non-translational symmetry also.

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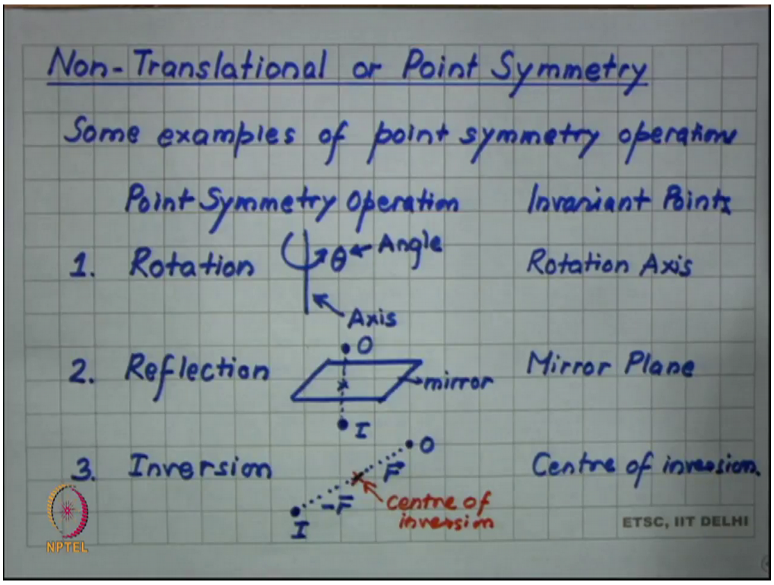

These non-translational symmetry are called point symmetry, because in contrast to translational symmetry which move all the point, point symmetry leave some points unchanged. We will look at some examples of the point symmetry let us list that, some examples of point symmetry operations. So, for example, let us take this list point symmetry operation invariant points, one of the important point symmetry operation is the rotation, rotation about an axis, by some angle theta, if this brings the object into self-coincidence we will say that the object has rotational symmetry; obviously, rotation moves all points except the points on the axis. So, the rotation axis is left invariant by the rotation these points do not move upon the action of rotation.

Similarly, we have another point symmetry reflection, the reflection is in a mirror plane we have a plane we have an object, that object will be imaged at equal distance on the other side of the mirror plane, this is the mirror again every point moves to some other point, but the points on the mirror plane remain invariant, the points on the mirror plane do not move anyway finally, we have inversion, by inversion we mean that we have a centre of inversion let me show it with a little red cross, that is my centre of inversion and if there is any point go that point will be imaged through the point at a distance equal to on the other side of the point.

So, a vector at r position vector at r, will be mapped to the position vector minus r. So, that is the inversion again all points are imaged to some other point, but the inversion centre the centre of inversion will not move. So, the point which is left invariant is the centre of inversion. Let us look at little bit more details of the rotational symmetry.

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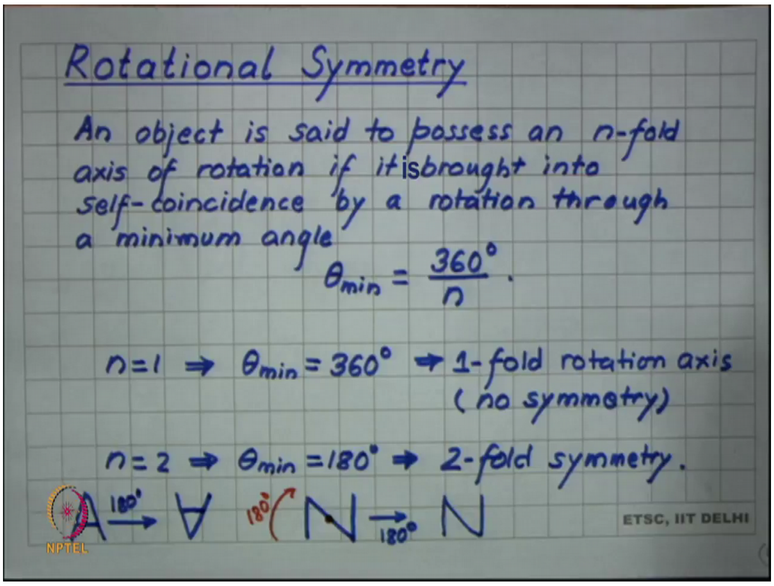

We will now consider rotational symmetry and object and we will define we will make a definition now, an object is said to possess an n-fold a new jargon which we are introducing an object is said to possess an n-fold axis of rotation, if it is brought into self-coincidence by rotation, through the minimum angle theta min 360 degree by n. For example, let us take n is equal to 1, if you take n is equal to one this means theta min is 360 degree, but any object; however, irregular it is shape if it is rotated by 360 degree; obviously, it will come into self-coincidence. So, this is this can be called 1-fold rotation 1-fold rotation axis, but this is equivalent to saying no symmetry, instead of saying instead of making a negative statement that the object has no symmetry, you can make a positive statement it has and one-fold rotation axis, because by 360-degree rotation everything will come into self-coincident.

Let us take n is equal to 2 now the theta min will be 180 degree. So, the object will have a 2-fold symmetry, 2-fold symmetry or 2-fold rotation axis we can say, I will just write 2-fold symmetry consider for example, English letter n.

Now, if I rotate about this central point by 180 degree then n will come into self-coincidence it will still appear as n. So, it has a 2fold symmetry after 180-degree rotation it will still appear as n. So, it possesses 2fold symmetry, if this will not be true for say some other letter for example, a if I rotate a by 180 degree I will get upside down a. So, a does not possess 2-fold symmetry, n possesses 2-fold symmetry.

Let us illustrate this rotational symmetry by examples of some 2 dimensional figure. So, here I have a rectangle.

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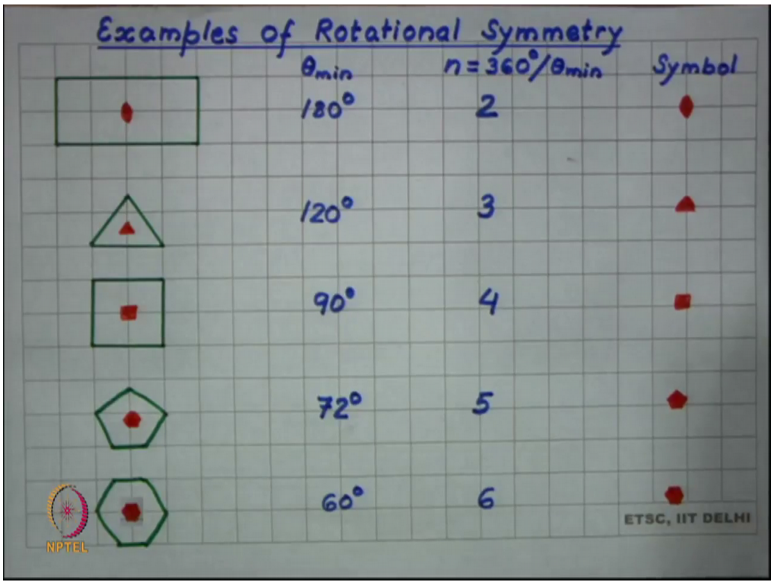

Rectangle if I rotate by 180 degree it will come into self-coincidence. So, n 360 by 180 will be 2. So, it is a 2-fold axis of symmetry, and crystallographers use a little lens a small lens symbol to represent this 2-fold symmetry, and to indicate that this rectangle had a 2-fold symmetry passing through it is centre I put this little lens here at the centre, an equilateral triangle will come into self-coincidence by 120-degree rotation.

So, that represents a 3 fold axis of rotation, and it this will be represented by a small little triangle. So, I put the triangle in the centre to indicate the bigger triangle the green triangle has a 3fold symmetry in the centre this square; obviously, will have will be brought into self-coincidence by 90-degree rotation. So, it processes a 4fold symmetry axis, and the symbol either go to the square and I put that square there, a pentagon repeat angle 72 degree the fold of the axis 5 symbol a little pentagon.

So, it is intuitively obvious now what will be the symbol for all other rotation axis, we can have a regular n sided figure and that will have an n fold axis of rotation. A regular hexagon coming into self-coincidence by 60 degree has a 6 fold axis and it is symbol is, a little hexagon.

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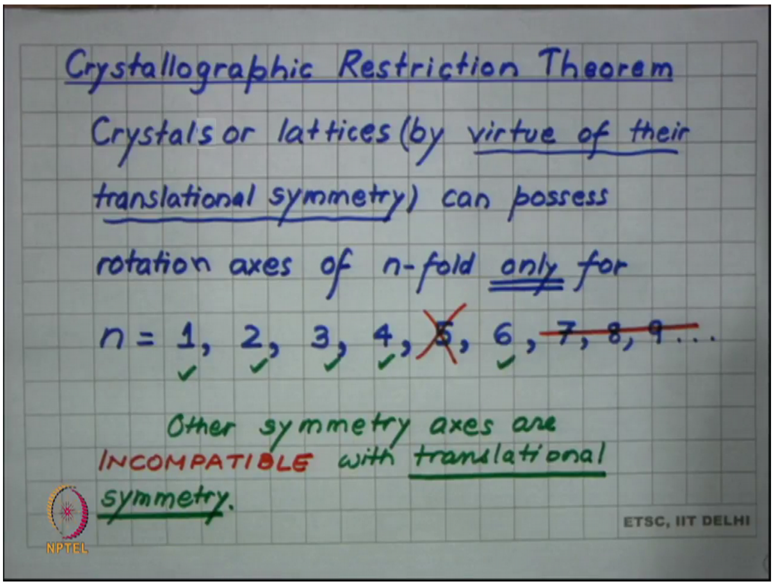

Now we have already seen that in terms of regular polygons, if we have an n sided regular polygon it will have n fold symmetry, we saw the examples up to 6. So, are equilateral triangle had a 3fold symmetry, is square had 4fold symmetry, regular pentagon 5 regular hexagon, 6 and so on. So, you can find objects of any symmetry, but if you want the symmetry to be present in a crystal, for a symmetry for a rotation axis; crystals or lattices by virtue of their translational symmetry can possess, rotation axis of n fold only for now this is a very, very restricted a restriction on the symmetry axis which we are putting, and fold only for n is equal to 1, 1 is allowed, 2 is allowed, 3 equilateral triangle allowed, 4 is square allowed, 5 pentagon not allowed, we will not look at the proof we are just listing the theorem 6 again allowed, 7, 8, 9 any other higher symmetry not allowed.

So, crystal can have only crystal or lattice can have only 1, 2, 3, 4 and 6-fold axis, this is highly restricted and that is why the name of the theorem itself is crystallographic restriction theorem. We will not prove this, but it is interesting to know that only these symmetry axis can be present in a crystal, no other and as I have written here this is by the virtue of their translational symmetry any other symmetry axis.

Let us note that down other symmetry axes are incompatible, with translational symmetry. We have seen the translational symmetry is the definitional symmetry of the lattice. So, unless and until the rotation symmetry is compatible with translational symmetry, it cannot be present in the lattice and the theorem states that only these 5 axes, 1, 2, 3, 4 and 6 are compatible with translational symmetry. So, we end this lecture here we will continue our discussion of symmetry in the future class.