Introduction to Materials Science and Engineering
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Lecture – 14
Inter-planar spacing

Hello, in this video, we will define an important concept called the Inter Planar spacing of a given set of lattice planes and we will derive this for the cubic system.

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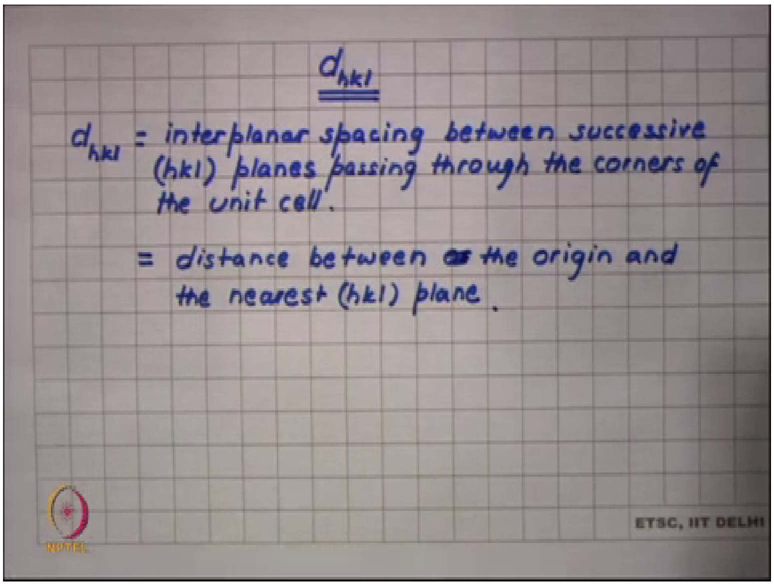

So, this inter planar spacing has a universal symbol dhkl. We will use that it is inter planar spacing between successive hkl planes. If you have parallel planes passing through the corners of the unit cell facing excessive planes passing through the corners, passing through the corners of the unit cell. Another way of looking at it is the same distance will also be equal to distance between origin distance between the origin and the nearest hkl plane. So, once we see some example it will become clear.

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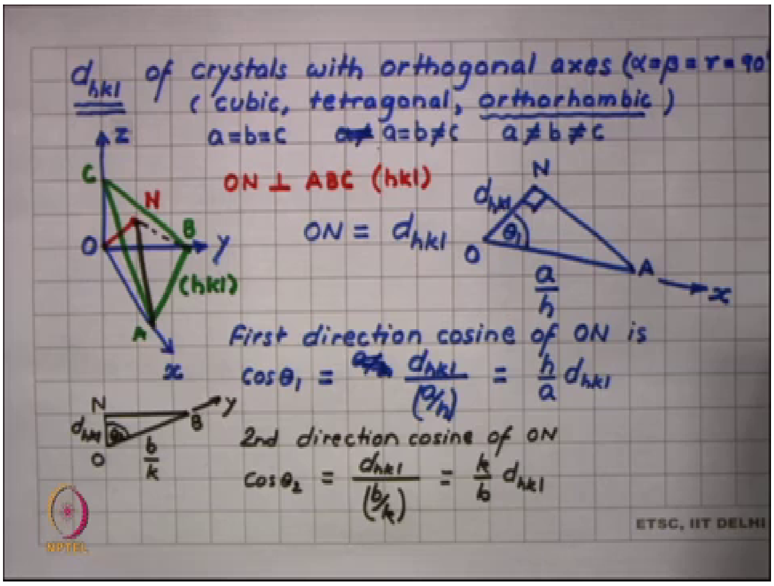

So, let us first look at dhkl you can you can derive formula for dhkl and in the books you will find for various crystal systems, but the geometry becomes very complicated for crystals which are not having axes which are orthogonal. So, we will derive this for the simple case where the axes are orthogonal. So, we are assuming the crystal to have orthogonal axes and that is a crystal system in which all the 3 interaxial angles are 90 degree, you know that such systems there are 3 out of 7 the cubic tetragonal and orthorhombic for these systems it becomes simple the derivation becomes simple.

So, we will do that let us say that this is the origin and these are the 3 axes of my orthogonal system could be cubic, could be tetragonal, and could be orthorhombic. So, to keep it general let us start with orthorhombic later on we will specialize what a tetragonal and cubic. So, all of them have alpha beta gamma 90 degree. Orthorhombic the 3 lattice parameters are not equal a not equal b not equal c tetragonal sorry a equals b and not equals c and in cubic all are equal this is the only difference.

So, let us start with the general case of orthorhombic system. So, we have 3 orthogonal axes, and let us say that the first hkl plane away from the origin is this it intersects the x axes at A,y axis at B and the z axis at C. So, ABC is my hkl plane and again like in the last exercise for finding the plane normal we had done we draw a perpendicular O N onto the plane. So, ON is perpendicular to ABC which has miller indices hkl.

Now, let me connect N and A. And let me look at this triangle O NA. So, remember first that ON is the distance from the origin to the hkl plane and that is why by the definition which we gave on is equal to dhkl now let us look at this triangle ONA. So, I am redrawing this triangle O N and then A note that O N, N is the foot of the perpendicular from o to the plane. So, O N is perpendicular to the plane ABC. So, O N and N A is a line lying in the plane. So, O N and N A are perpendicular. So, I have a right triangle O N A, O A you now know can be written the magnitude of O A can be written as a by h. That is the intercept the reciprocal intercept hkl is the plane. So, the first number is the reciprocal of the intercept. So, that is 1 by h, but this intercept is in terms of the corresponding lattice parameter. So, we have a by h. So, O A is a by h, O N by our definition is dhkl.

Let me call this O A was also the direction of my x axis let me call this angle theta 1. So, the first direction cosine of O N is the cause of the angle O N makes with the x axis. So, this is cos theta 1; which is nothing but the base is dhkl by the hypotenuse which is a by h. So, dhkl by a by h which we can write as h by a dhkl.

Similarly, the second you can form a similar triangle let us say with b and think of the triangle on b let me do that I have O N let us say here is the b this direction is the y axis this is theta 2 O N A is still dhkl and NOB, OB which is the hypotenuse is the second interceptor of the plane. So, this is b by k the reciprocal is of k is 1 by k and multiplied by the corresponding lattice parameter which now since we are talking of orthorhombic is different from a it is equal to b. So, OB becomes b by k; so our second direction cosine of O N cos theta 2 becomes dhkl this time b by k.

So, I have k by b dhkl. Now we have seen the pattern

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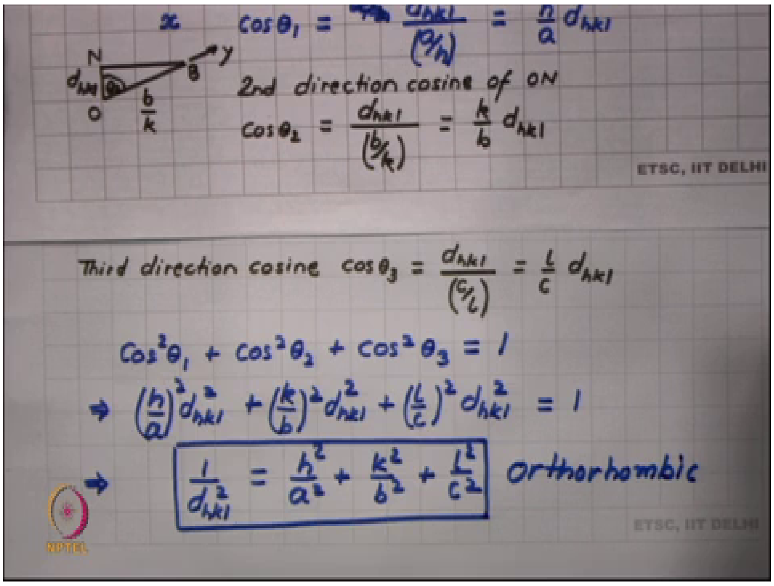

So, I can write the third direction cosine simply by looking at the pattern cos theta 3. Corres lattice parameter now each C the miller indices is L. So, I divided by C by L. So, I get L by C dhkl. So, if we know the direction cosines have the property that sum of the squares of the direction cosines cos square theta 1 plus cos square theta 2 plus cos squared theta 3 is equal to 1, there is a property of direction cosines. So, we use this property we have already seen that cos theta one was h by a dhkl y square e cos theta 2 was k by b dhkl. So, I get k by b square dhkl square and I get l by c square dhkl square.

So, you can quickly see from here, but now we have derived the formula, I can take dhkl common and take it on the other side. So, 1 by dhkl square is nothing but h square by a square k square by b square L square by C square this is for orthorhombic. We can quickly derive the formula the special cases for super orthorhombic.

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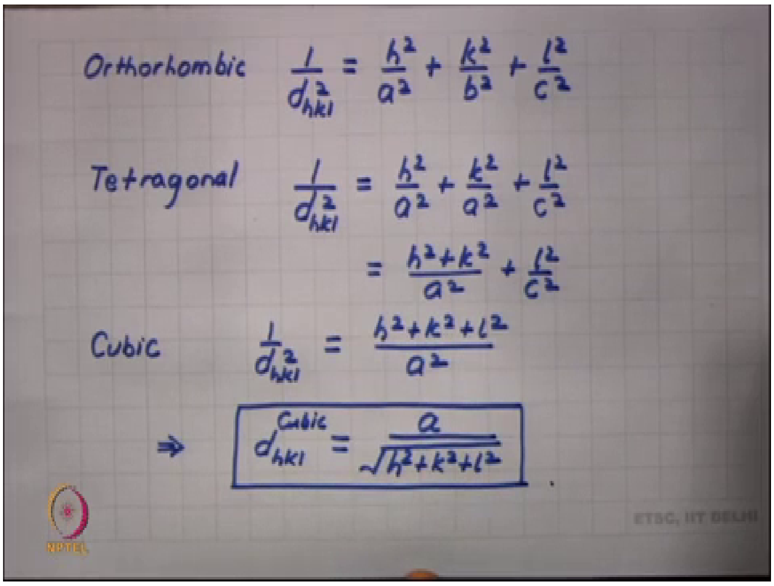

For orthorhombic we just derive that 1 by d hkl square is h square by a square by b square l square by c square for tetragonal all you have to now result for tetragonal you can get simply by substituting for b equal to e h square by a square c square or you can write it h square plus k square by a square C square, and for cubic all the 3 parameters will be equal over cubic you get even simpler formula if we equate all of them, or this sometimes is simplified writing directly in terms of dhkl dhkl rearrange and take the square root h square k square and this is for cubic.

So, we will use these dhkl is spacing particularly in the topic which we are now going to move that is the topic of x ray diffraction. So, there this; the concept of dhkl the inter planar spacing between planes will form an important role.