Introduction to Materials Science and Engineering
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Lecture - 136
Isostress modulus

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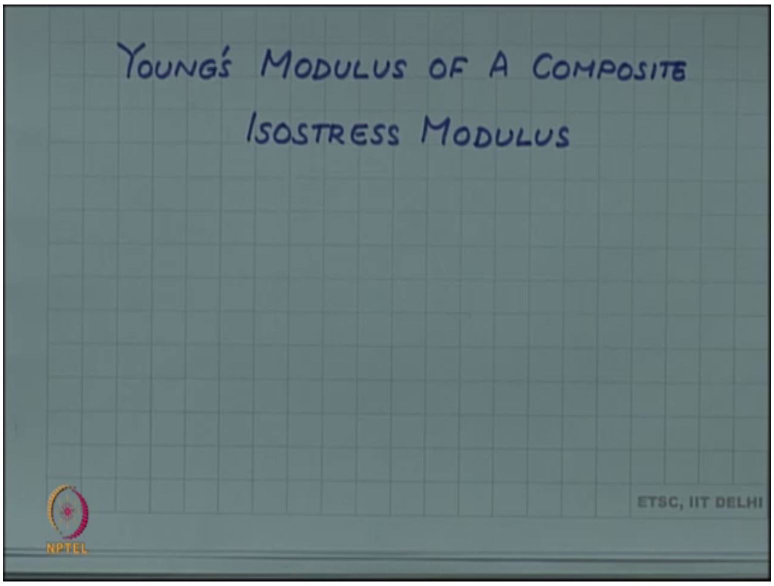

So, let us continue the discussion on Young's modulus of Composite. Now, let us look at Isostress Modulus, by isostress modulus, we mean a loading system which gives the same stress in the matrix as well as in the reinforcement. This is not possible with fibre reinforced composites.

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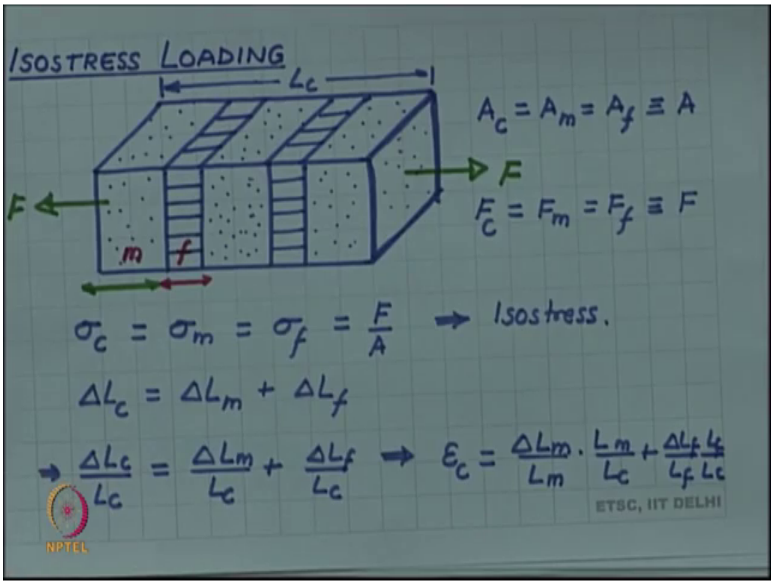

So, we look at a different kind of composite, which is the sandwich composite, here we have slices of the two phases. So, let me call this the matrix phase this be the matrix phase and let this be the fibre phase.

So, this is we are not really now calling this slice as a fibre is not good terminology. So, f is being used as a symbol for this, but you should understand that this is now, in the slice of the sandwich is it is a lamella, or a plate and it is not fibre, but we using f as for reinforcement, for representing reinforcement. Similarly m the matrix is no more continuous here, matrix also is coming as slice and is discontinuous. So, now, in this situation if you see the area of the composite, an area of the fibre, area of the matrix, area of the reinforcement all are the same.

So, let us call that A. So, that is this area, this area is the area for matrix fibre as well as composite. And this is the area on which we will put the load, the area on which we will put the load. So, you can see this load will be transmitted to both matrix and fibre. So, the loads are again the same. So, F in the composite the force in the composite is force in the matrix is equal to force in the fibre, or reinforcement and let us call all of them as F so; obviously, because of this situation, they stresses will all be the same.

So, stress in the composite will be the same as stress in the matrix will be the stress in the reinforcement will be F by A. So, this is what then leads to an isostress condition. Now, since we want to get to the modulus, let us look at the modulus let us look at the strain.

So, the strains now you can see both the reinforcement and the matrix will elongate. So, this will elongate as well as the matrix will elongate.

So, the total elongation, total elongation of the composite will now be the sum of elongation of the matrix and that of the reinforcement. Now, to convert these into strains, we divided by the length of the composite, which is this and so, the left hand side is of course, the strain in the composite, but for right hand side we again use multiplication and division by L m to convert them into strain.

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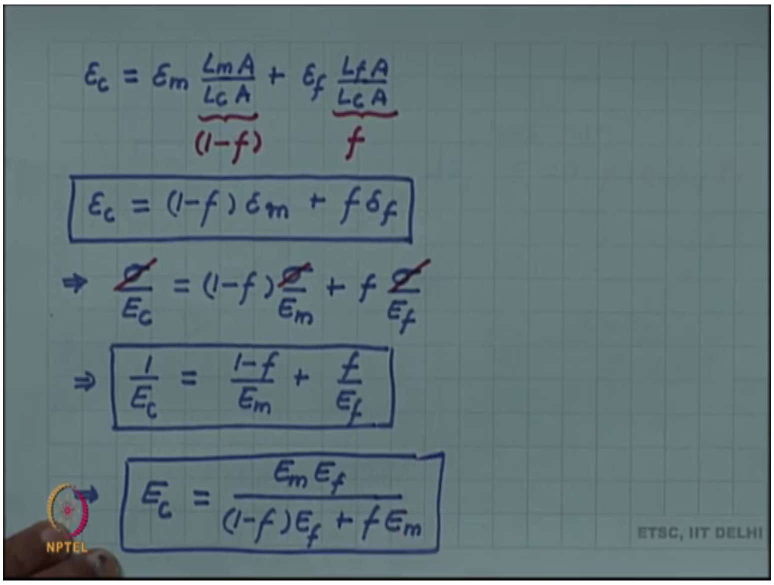

So, we continue with this so, we get strain in the composite. Now, you can see delta L m by L m is the strain in the this term is strain in the matrix and this term is the strain in the reinforcement. So, we use that.

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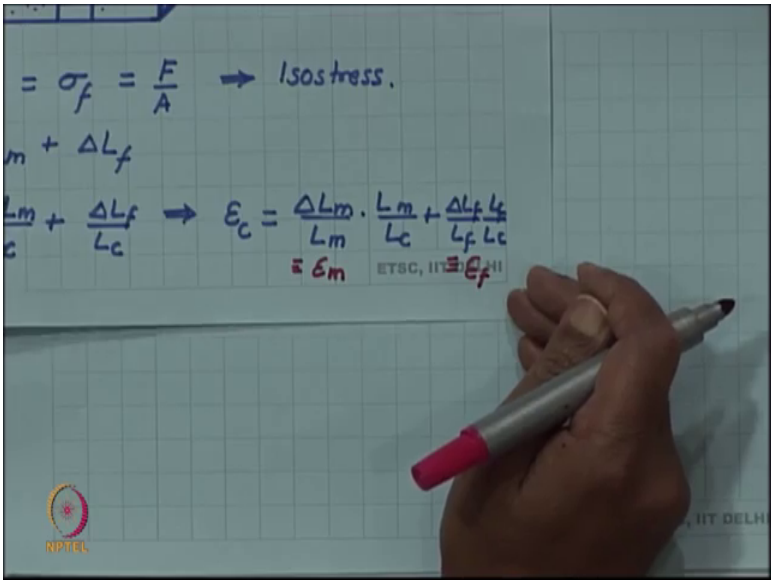

So, epsilon c is epsilon m L m by L c plus epsilon f L f by L c. Now, we multiply and divide these terms by A to convert them into recognizable volume. And, now you can see that L m A the length of the matrix times the cross sectional area of matrix, this is the volume of the matrix and length of composite multiplied by the cross sectional area is the volume of the composite.

So, this is volume of the matrix divided by volume of the composite. So, this is the volume fraction of the matrix, which is 1 minus the volume fraction of the reinforcement and, this is nothing, but volume fraction of the reinforcement f. So, we can write this now the epsilon c is equal to 1 minus f, epsilon m plus f epsilon f.

So, we get similar rule of mixture for a strain, in this case like we had got rule of mixture for stress in the isostrain case. So, for the isostress case the strain goes by rule of mixture, but since we are now interested in modulus we will convert this. So, for epsilon c we can write stress divided by stresses are the same.

So, we use the same stress we do not use the subscript. So, stress in the composite divided by modulus of the composite is a strain in the composite, 1 minus f stress in the composite divided by modulus stress in the matrix divided by the modulus of the matrix plus f times, stress in the reinforcement divided by modulus of the reinforcement.

The stresses are all same; the stresses are all same they can be cancelled. So, we get a relationship 1 by E c is equal to 1 minus f by E m plus f by E f, or if you show is if you want to write it as instead of reciprocal of E c. So, you can solve this, you will find E m into E f 1 minus f into E f f into E m.

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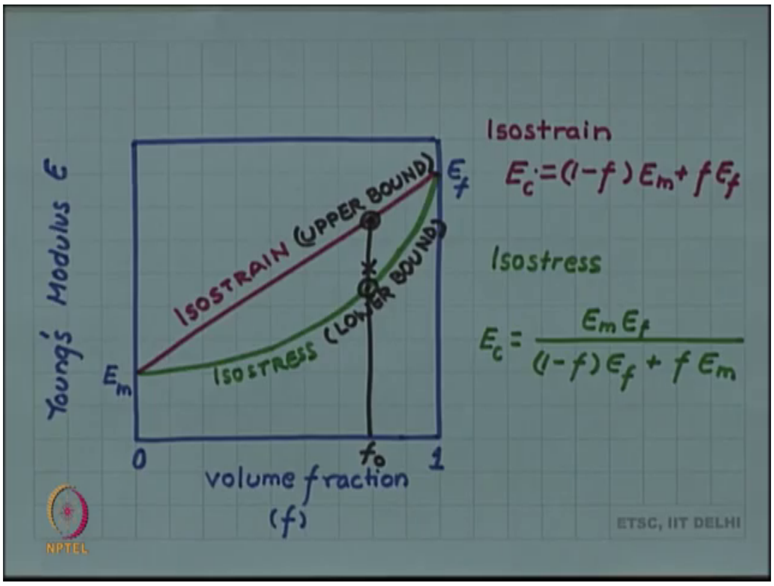

This slide is going to summarize whatever we have derived about the modulus of composite. We have already derived isostrain which was given by this formula, which when plotted is simply a straight line between E m and E f.

Now, let us plot also the isostress which is given by the second formula, which also we have derived, but when plotted this curve comes always below the isostrain case, with the shape something like this that is the iso stress case the isostress case, is always below the isostrain case. And an important result in composite mechanics, we will not prove this is that the isostrain case is the upper bound for composite materials. This is the upper bound whereas, the isostress is the lower bound, which simply means that if we take a composite of any given volume fraction.

Let us say we take a composite of this volume fraction, f naught then if we make a uniaxially aligned continuous fibre composite and load it axially along the fibre that will be the isostrain case, then we will get a modulus on this red line.

However, if we make a sandwich composite with parallel layers and, load it normal to the layers we will get a lower modulus on this green curve. However, if we make any other composite out of the same two material with the same E m same to E m and E f values, then that composite will have some value in between these two.

It cannot be more than the isostrain, or cannot be less than the isostress. So, a particulate composite, let us say a particulate composite will neither be a pure isostrain, or a pure isostress case. So, it is value will come somewhere are between these two extremes.

So, with this we end our study of composites.