Introduction to Materials Science and Engineering
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Lecture -135
Isostrain modulus

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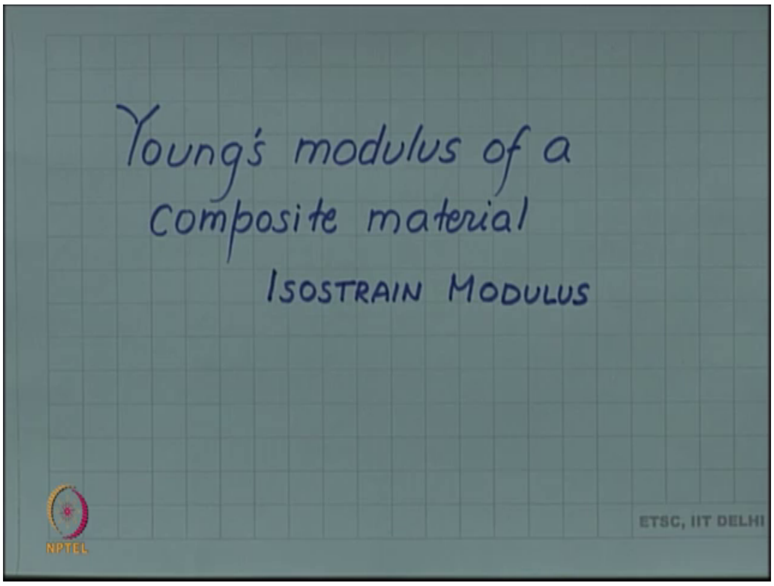

As an example of modification of a property of a composite material due to addition of reinforcement, let us consider Young's modulus of a composite material; Young's modulus; particularly for example, we look looked that polymer matrix composite, the glass fibre reinforced plastic GFRP. So, in this the polymer itself has very low stiffness very low Young's modulus, but glass has higher stiffness.

So, when glass is added the stiffness of the composite the Young's modulus of the composite increases. Two standard models are used for calculation of the Young's modulus; one is the Isostrain modulus and another is Isostress modulus, we will look at both of them.

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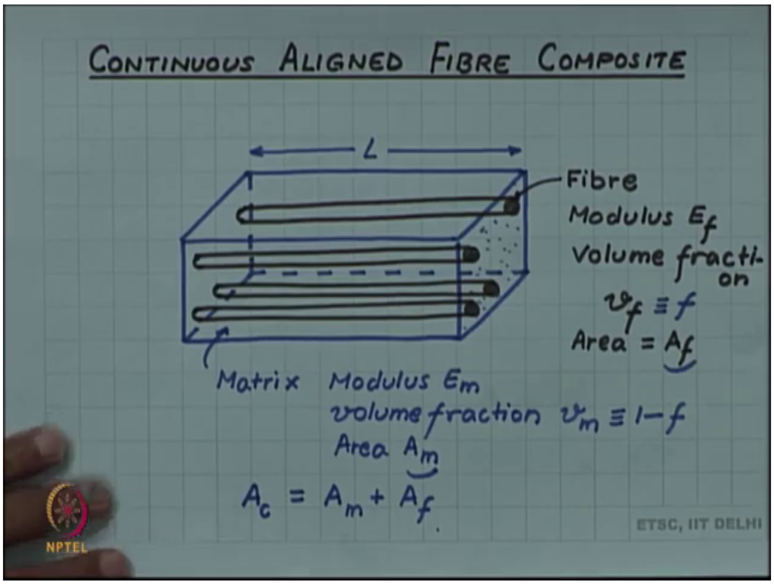

Let us look at for the isostrain modulus a good model is of continuous aligned fibre composite. So, in this sketch, I have shown a sketch of continuously aligned glass fibre composite. So, the polymer is of shown as a block of length L and the fibres are also of length L running from the left end of the composite to the right end. So, these fibres are of the reinforcing phase. So, they have a different modulus the modulus is E f, whereas, the modulus of the matrix is E m. Also the volume fraction of the two phases is important in determining the modulus of the final composite and the volume fraction of fibre; let us call that v f or much more simply as let me denote it as f.

The volume fraction of the matrix is v m which will; obviously, be 1 minus f because there are only two phases, the volume fraction of matrix is 1 minus the volume fraction of fibre. On the cross sectional area a part of the area is taken by part of the area is taken by the fibre these areas are taken by the fibre. Whereas, rest of the area is taken by the polymer; the rest of the area is the polymer area. So, we can every service separately denote the area of the fibre as A f and the area of the matrix as A m. So, the overall area of the composite A c will be sum of these two areas.

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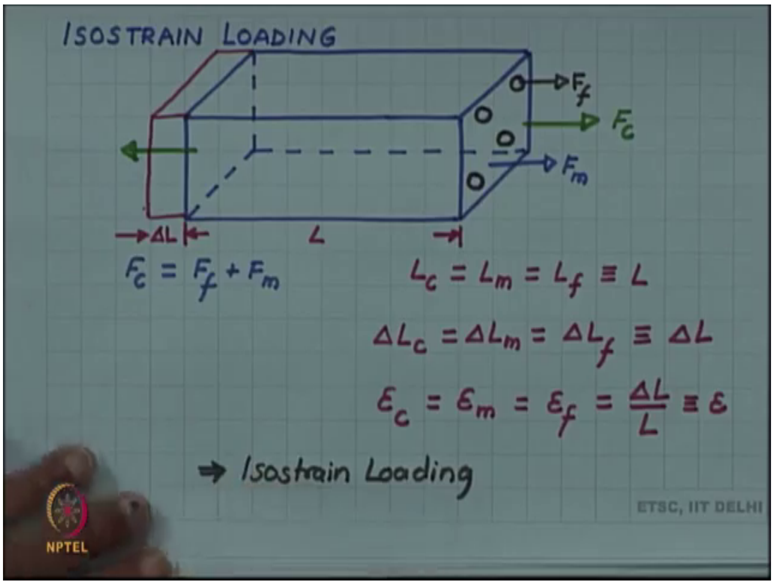

Now, let us look at what we mean by this isostrain loading. So, for such a composite if I if I put a load if I put a load F c on this composite material, then this load will be distributed on the fibre and in the matrix. Let us say that in the fibre the forces F f that is the sum of not on just this fibre by F f I mean, the sum of the forces acting on the entire cross section given by the fibre and similarly on the on the matrix there is a force F m.

So; obviously, this F c will be the sum of these two forces. So, the total force on the composite is sum of the force on fibre plus the force on matrix. Now let us look at due to this force what extension the composite undergoes. So, let us say that there is an extension delta L due to this force, the original length of the composite was L. So, you can also see here that the original length of the composite L c is same as the original length of the matrix and the length of the fibre; all of these are same and so, we are designating all these equal lengths by L. Similarly, if there is no debonding between the fibre and the matrix, the fibre will also extend by the amount delta L and the matrix will also extend by amount delta L.

So, the change in length change in length delta L in the composite change in length of the matrix and change in length of the fibre; all these quantities also are same and let us call them that as delta L. So, this of course, leads to equality of a strain in all these phases. So, a strain in the composite is equal to the strain in the matrix is equal to strain in the fibre, all of which is equal to change in length delta L divided by original length. So, let us call all these strains as by a single symbol epsilon.

So, this is the reason why we are calling this loading as an isostrain loading. So, this results in an isostrain loading.

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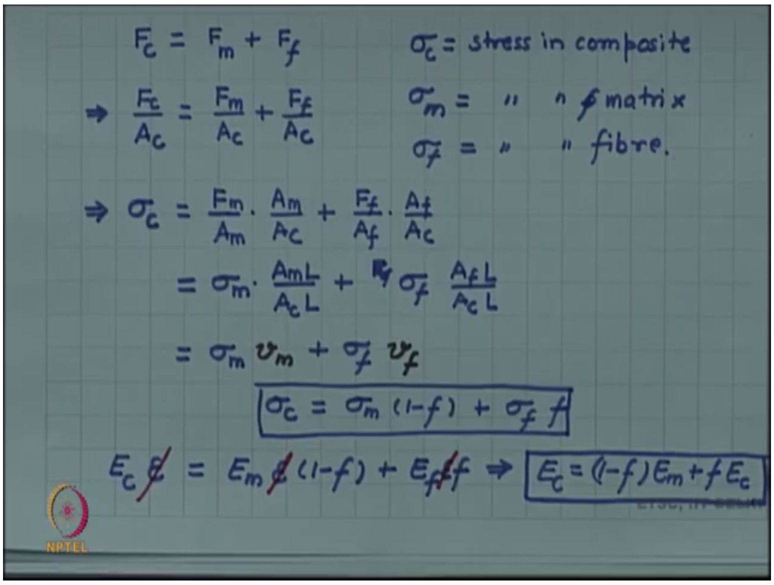

So, for this isostrain loading we have seen let us begin with the force; we saw that the force on the composite was sum of the force acting on the matrix cross section and on the fibre cross section. Now since we are interested in stress we will divide it by the composite cross section F c by A c; A c is the entire cross section this entire cross section on which the force F c is acting. So, we write it as F m by A c F f by A c; this can of course, to be changed this we can call this force on the composite cross section by the area of the composite cross section is they stress in the composite cross section sigma C.

So, sigma C is stress in the composite. So, this will equal to now F m by A c does not simplify to anything non. So, I write it as F m; F m by A m which will become the stress in the matrix. So, I have to then multiply it by A m by A c. Similarly, I write this as F f by A f into A f by A c. So, sigma; now this can be simplified into F m by A m can be written as sigma m and I multiply for this fraction, I multiply the numerator and denominator by L to convert them into recognizable volumes; similarly F f by A f is sigma f and then I have A f into L by A c into L. The sigma m is stress in stress in matrix and sigma f is stress in fibre.

Now, you can see this can further be converted into what is this quantity; say A m L; A m is the cross sectional area of the matrix and it is being multiplied by the length of the matrix. So, A m L is the volume of the matrix, whereas, A c L is the volume of the composite. So, we are dividing the volume of the matrix by volume of the composite. So, this is nothing but volume fraction of the matrix is nothing but volume fraction of the matrix plus sigma f and similarly this is nothing but volume fraction of the fibre.

So, we finally, get a very simple and interesting relation that the stress in the composite is just weighted average of the stress in the matrix and stress in the fibre with weighting factors being the volume fraction and if I use my symbol f for V f; I can write this as sigma m 1 minus F plus sigma f into f, but we are interested in the modulus. So, I write stress as modulus of composite times strain in the composite, but a strain in the composite, we have seen is strain in the composite a strain in the matrix and a strain in the fibre are all same and is equal to epsilon.

So, I write it as epsilon; similarly sigma m I write as E m into epsilon 1 minus f plus sigma f as E f into f sorry E f epsilon into f. So, this epsilon of course cancels out from this equation finally, leading us to a relation identical to the stress that E c is nothing, but 1 minus f times E m plus f times E c; again the rule of mixture or weighted average kind of formula for this modulus also.

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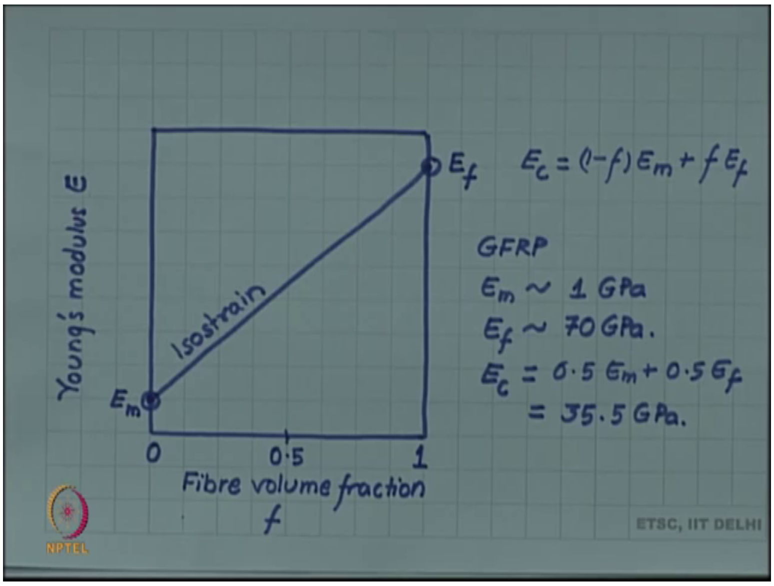

So, let us look at how does that modulus look like. So, the composite modulus is 1 minus f times modulus of matrix plus f times the modulus of the fibre. Now, here I am trying to plot with x axis as the fibre volume fraction running from 0 to 1.

So, at 0 volume fraction of fibre, you have pure matrix and the pure matrix has a modulus E m; E m is the modulus of the matrix and the pure fibre at volume fraction 1; you have only fibre, so, the pure fibre has modulus E f and if you plot this. this is nothing, but a equation of a straight line running from E m to E f. So, all you get is a straight line joining these two points. This is a nice simple relationship and this is based on the isostress loading when they stress sorry isostrain loading when the strain in the fibre and composite and the matrix are all the same.

So, this is the isostrain modulus for the composite. In a typical glass fibre reinforced composite for example, let us say GFRP E m will be very low, let us say 1 to 2 Giga Pascal. So, this may be 1 Giga Pascal, whereas, the glass E f has a very high modulus. So, 70 Giga Pascal, so, you can see that in this formulation E c; let us say for 50 percent for the 50 percent composite E c will become 0.5 E m plus 0.5 E f which will be 35.5 Giga Pascal's.

So, by putting 50 percent by 50 percent of the reinforcing glassfibre, we can raise the stiffness of matrix which was very low from 1 Giga Pascal to 35.5 Giga Pascal which is a very high volume. However, even if we required a high stiffness, we do not want the brittleness of the glass. So, composite overall is giving us a better property in the sense that we have an stiffness a very high stiffness which could not have been obtained in plastic, but at the same time we are avoiding the brittleness of glassfibre, there are several glass fibres. So, if any glass fibre in the matrix even if it breaks the other glass fibre will still remain intact and the whole thing will not break as a brittle material.