Introduction to Materials Science and Engineering
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Lecture – 130
True stress and true strain

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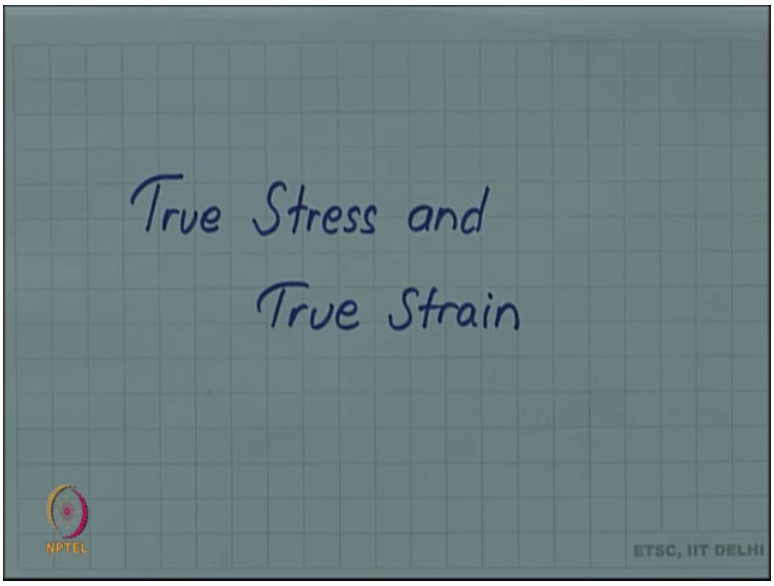

We will discuss true stress and true strain; we have already discussed stress and a strain.

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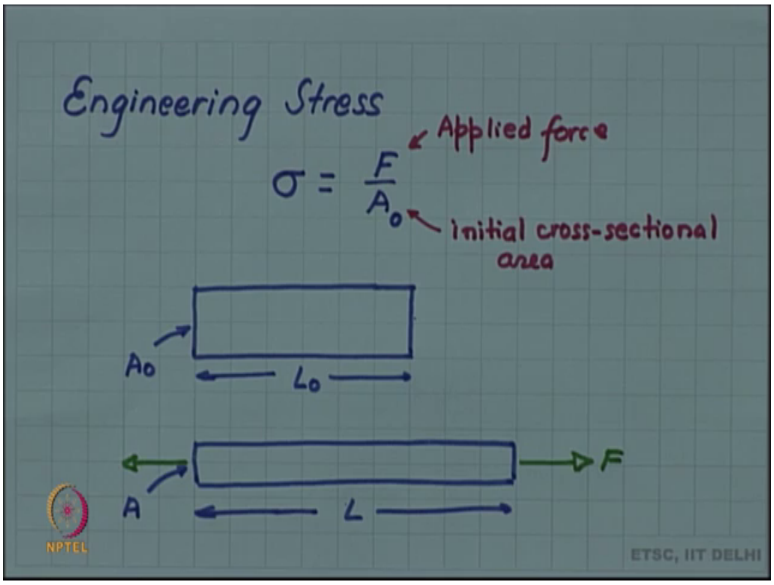

But those stresses and strain were engineering stresses and engineering strain. And when we define engineering stress then that was defined as F by A naught where F was the applied force and A naught was the initial cross sectional area. But there is a problem in this definition, you can see that suppose this was your initial sample of length L naught and cross sectional area A naught and then you applied a force on it such that it is elongated to a new length L.

But you can see that as it elongates to L, its cross sectional area also decreases to some new value A. So, the force the current force F is not acting on the original area A naught, but is on acting on the current area A.

So, in this sense, this engineering stress is not really true representation of the stress being seen at this point, we can define the true stress simply by sigma is equal to F by A, where A now is the current area, on which the forces acting not the original area, but the current area.

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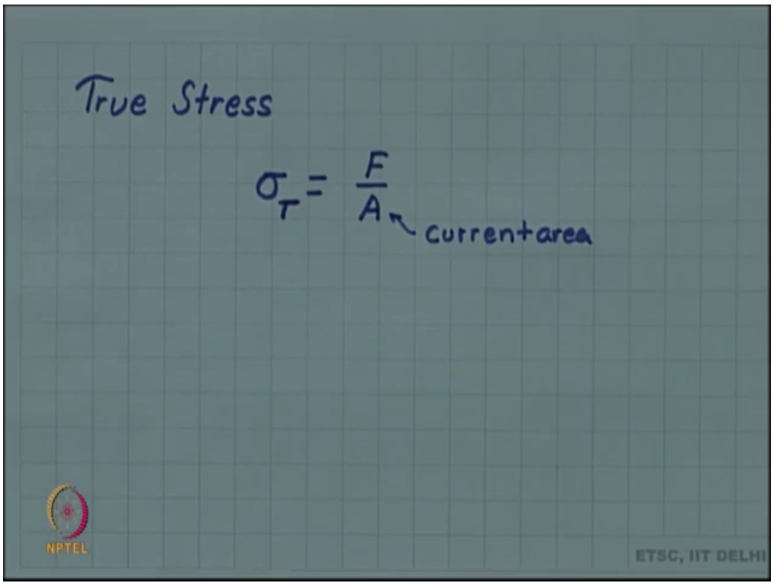

So, I have now put the subscript T to distinguish it from sigma defined earlier as engineering stress.

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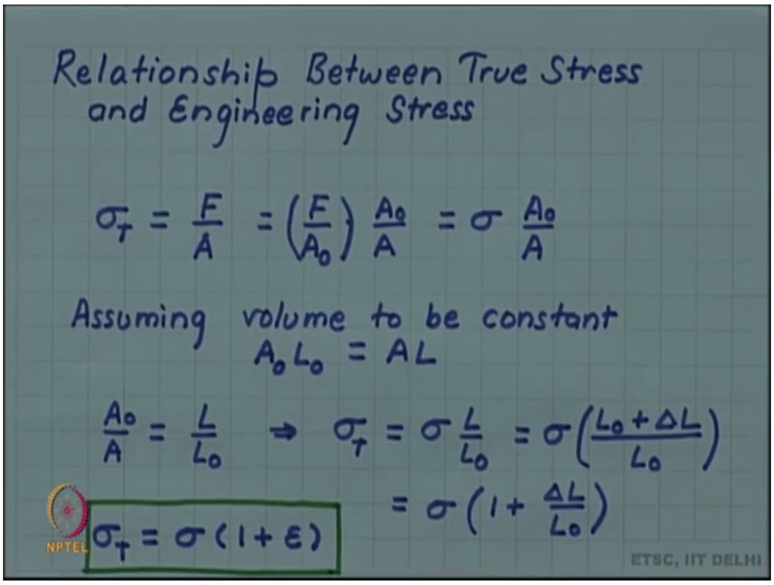

You can establish a relationship between the two by looking at it that sigma T was F by A, if I want to relate it to the engineering stress, I bring in A naught. So, I simply multiply and divide by A naught to get this expression. So, for this; for the first factor in this expression, we can write it as the engineering stress. So, it is engineering stress times A naught by A, we can make further progress by the assumption, which is true in plastic deformation of volume constancy.

So, assuming volume to be constant, we can write this as here we can use the relationship. So, A the cross sectional area A times the initial cross sectional area A naught times the initial length L naught was the initial volume, and the current cross sectional area A multiplied by the current length L is the current volume and if the volume is constant during deformation, these two will be equal this assumption of volume constant is quite true in plastic deformation not so true in elastic deformation.

But then significant part of deformation; if we are in the plastic deformation regime, then this assumption will be quite true. So, using this constancy of volume, we can now express A naught by A naught by A as L by L naught. So, with this substitution we can write the true stress as engineering stress times L by L naught, but then I can write L as the current length L as L naught plus increment in the length delta L divided by L.

So, you can see now that we will have sigma 1 plus delta L by L naught, but then delta L by L naught changing length divided by the initial length is nothing, but engineering strain. So, we can write our final relationship as true stress is engineering stress times 1 plus engineering strain, this is a nice simple relationship, but based on the assumption of volume constancy.

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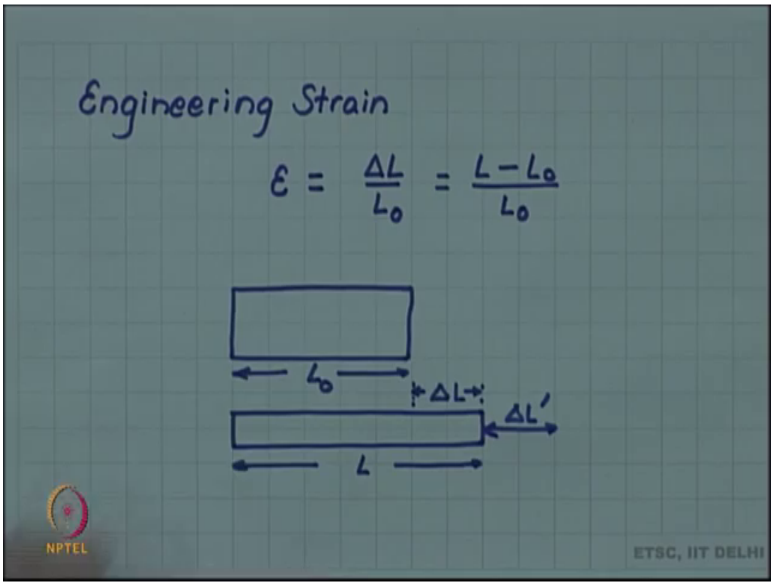

So, now we will look at the engineering strain, which we defined as delta L by L naught. So, this will be L minus L naught by L naught. So, delta L is the increment in length. You can see that the engineering strain depends on what is the original length. So, if delta L was the increment over original length L naught; I get the engineering strain delta L by L naught, but suppose I implemented by further delta L, then now this increment also will be divided by the original length L naught. Although; now this increment the subsequent increment delta L prime; let us say is being applied on the current length L and if I divide delta L by the delta L prime by current length, then I will get a different value.

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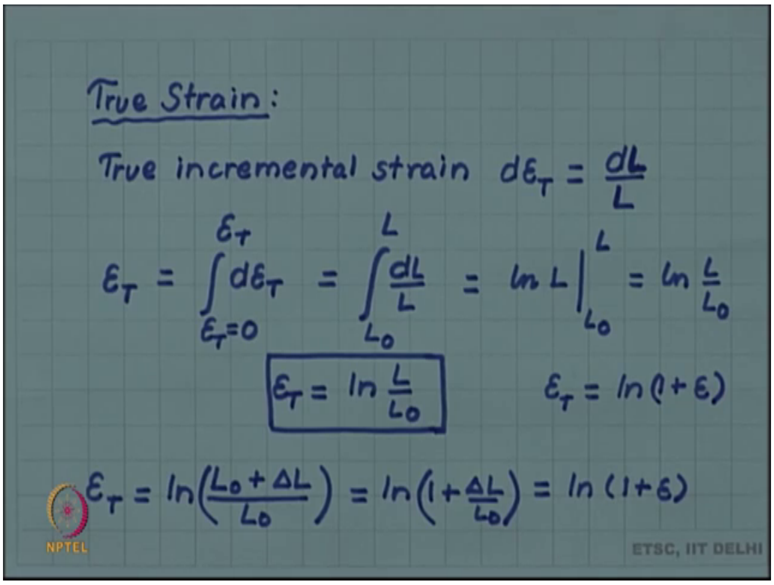

So, we define true strain by taking the current length into account. So, we will say true incremental strain d E true is a small incrementing length L divided by let me write capital; I was using capital small increment in length L divided by the current length L. So, this will be the true increment the true incremental strain for the same delta L the incremental engineering strain would happen d L by L naught which will be a different quantity.

So, now to find the strain at any given length E T all we have to do is to integrate this incremental strain from a strain of 0 to the current strain. So, if we write this as now d L by L, then the limits change from L naught to L, but this is a simple integrand, you know that this integrates to log of L. So, we have log of L; this is a definite integral with limits L naught an L.

So, finally, I get a value log of L by L naught. So, the true strain is simply logarithm of current length divided by the initial length; let us now try to establish a relationship between true strain and the engineering strain. So, this is simple to do because you can see that E T is simply log of L by L naught, but on the numerator L can be written as the initial length plus the change in length divided by L naught.

So, this then gives you log of 1 plus delta L by L naught, but delta L by L naught was nothing, but engineering strain. So, you can write it as log one plus epsilon. So, the relationship between true strain and engineering strain can again be written as epsilon T is log of 1 plus the true strain is log of 1 plus engineering strain.

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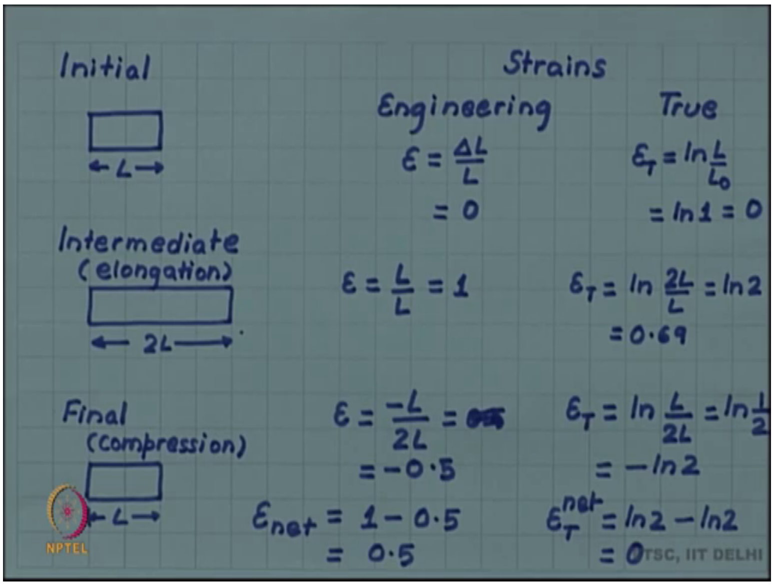

So, let us compare the engineering strain and true strain for an example situation here shown. It is extreme situation where I start with an initial length and then extend it or elongate it to twice its original length. So, we go from L to 2 L and then finally, again compress it back to L and let us see; how the situation is described in terms of engineering strain and true strain. So, initially of course, there is no change in length. So, delta L is 0. So, the engineering strain is 0 and again since there is no change in length. So, L and L naught are the same. So, you have log of one which is also indicating 0. So, we are starting with 0 engineering strain and 0 true strain, but the intermediate stage of elongation above up to a length of 12 is described differently by engineering and true strain.

So, the engineering strain will give me now the change in length is equal to the original length. So, it will be L by L. So, which is 1. So, engineering strain is 1 which is 100 percent strain; whereas, true strain gives me a more conservative estimate of this here, we have log of the final length is 12 and the initial length is L. So, you have log 2, which is only 0.69. So, it is saying about 69 percent strain, whereas, the engineering strain is giving me 100 percent strain, but the interesting situation comes when we describe the final compression.

So, now in this stage from intermediate to final the engineering strain will give me a change in length, which is equal to minus L, but the original length now is 12 because we are talking of a strain from intermediate to final this will give me minus 0.5 negative sign is indicating compression whereas, in the case of true strain, I get log of final length which is L and the initial length which in this case is 12. So, we get log of half and log of half is nothing, but minus of log 2. So, if I add the total strain in the process of going from initial to intermediate to final the total strain.

So, let us say E net. So, E net will come out to be if I add the two strains, then I will get 1 minus 0.5, which will still give me 50 percent strain, whereas, the net strain is 0 because there is no change in the initial and final length. So, engineering strain is not able to capture this kind of change in length, but if you look at the true strain epsilon 2 net, then you can see the intermediate stage was log 2 and then subsequently we had minus log 2. So, it truly describes that the true strain finally, is 0.

So, you can see the true strain actually is the real strain and engineering strain can give you erroneous value, if the deformations are very large and that is why for careful scientific theories of plastic deformation, if you are trying to develop, you will always use true stress and true strain and not engineering a stress and engineering strain, but for practical purposes, since using initial length and initial cross sectional diameter is always easier.

So, for practical purposes engineering stress and engineering strain are quite good, we do not get a situation of 100 percent extension followed by compression in most practical situations. So, this problem will not come.