Dealing with Materials Data
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Lecture 57
Lifetime and Exponential distributions

Okay, welcome to Dealing with Materials Data, in this course, we are looking at collection, analysis and interpretation of data from material science and engineering. We are looking at probability distributions.

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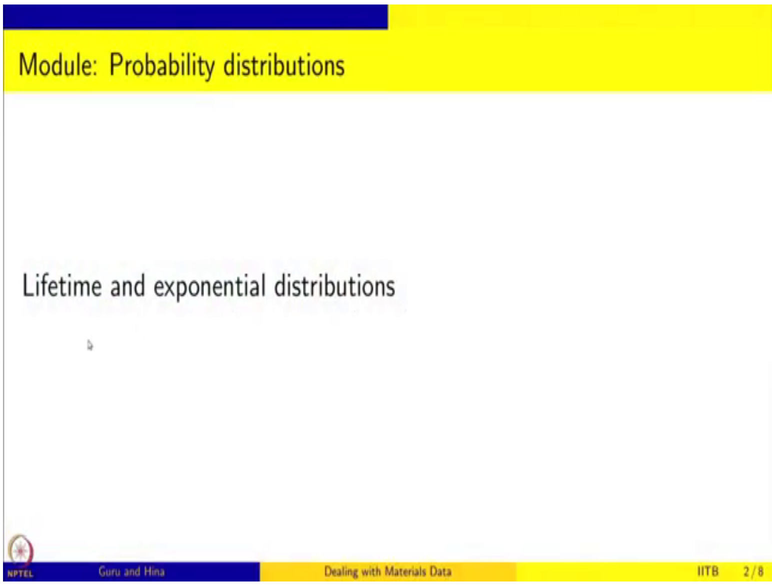

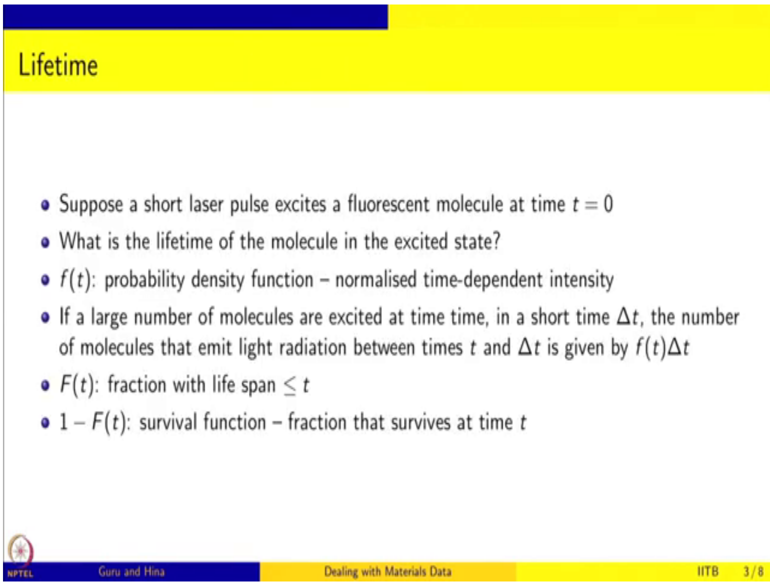

And in this session, we are going to look at lifetime and exponential distributions because they are also important in many material science and engineering problems. Suppose, what is lifetime? Suppose, let us consider a short laser pulse that excites a fluorescent molecule at time t equal to 0.

What is the lifetime of the molecule in the excited state is a question that you can ask, so, it is going to stay in the excited state for a while and then it is going to come back by emitting the fluorescent light. Now f(t) is the probability density function which gives the normalized time dependent intensity of radiation or light.

Suppose, if you have large number of molecules that are excited at time t equal to 0 in a short time delta t, how many molecules will emit light radiation, let us say between t and t + Δt. So, that is given by f (t) * Δt. So, f(t) is the probability density function or normalized time dependent intensity function. Capital F(t) which is a cumulative distribution function of the normalized time dependent intensity.

Basically gives the fraction with lifespan less than or equal to t and 1-F(t) which is called the survival function gives the fraction that survives at time t. So, we are saying that at time t equal to 0, we excite a large number of molecules and then by the time you reach a time t, what is the fraction which would have come back by emitting light to there from the excited state and how many survive in the excited state. So, this is given by F(t) and 1 minus F(t).

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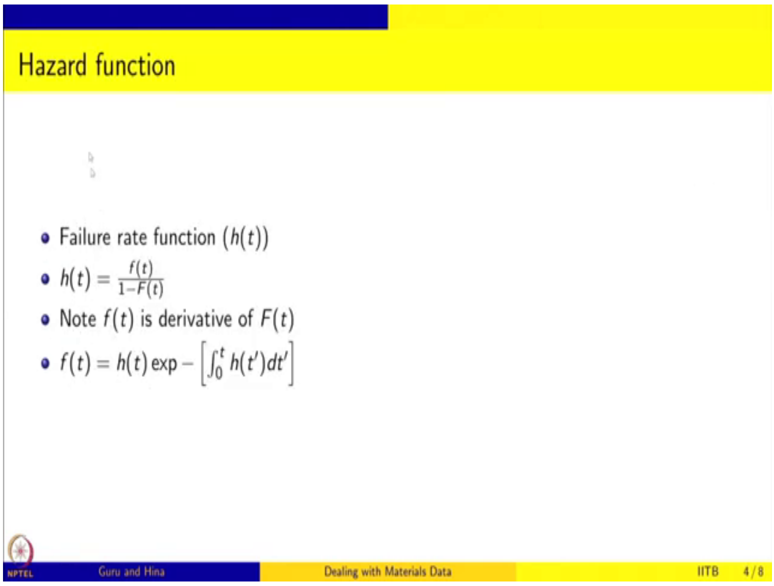

We can also define what is known as a hazard function, it is a failure rate function and it is defined as

$$h(t) = \frac{f(t)}{1-F(t)}$$

Okay f(t) is a derivative of this function because this is obtained by integrating F(t). So, this is so, the f (t) is basically a derivative of this function.

$$f(t) = h(t) \exp - \int_0^t h(t')dt'$$

So, in terms of the hazard function or failure rate function, you can write the probability distribution function and this is the relationship and that happens to be exponential.

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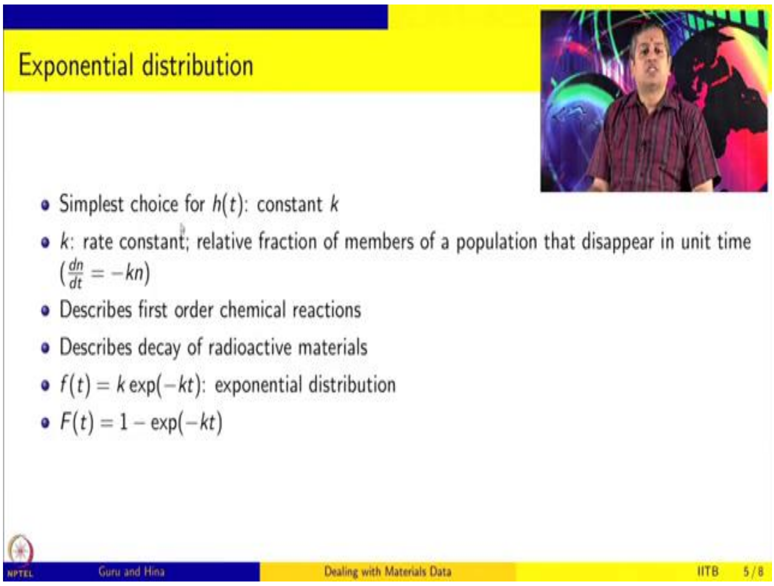

Now, the simplest choice that you can make for the hazard rate function is that it is a constant k and what does the k, the rate constant tell you? It tells you the relative fraction of members of a population that disappear in unit type, right

$$\frac{dn}{dt} = -kn$$

So, dn by dt by n is basically k which is the rate constant, it is a relative fraction of members of a population that disappear in unit time. So, this kind of description is for first order chemical reactions and radioactive decay of materials.

$$f(t) = kexp(-kt)$$

$$F(t) = 1 - \exp(-kt)$$

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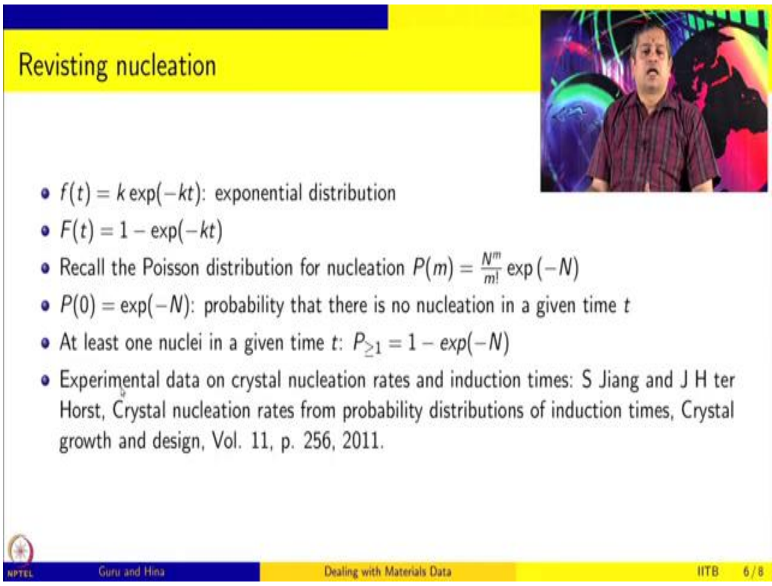

$$f(t) = kexp(-kt)$$

$$F(t) = 1 - \exp(-kt)$$

$$P(m) = \frac{N^m}{m!}\exp(-N)$$

$$P(0) = \exp(-N)$$

$$P_{\geq 1} = 1 - \exp(-N)$$

So, you can see that experimental data and crystal nucleation rates and induction times for example, follow this distribution which is the exponential distribution. And one example is this crystal nucleation rates from probability distributions of induction times. It is crystal growth and design and there is data that is given. And, you can look at the numbers and analyze and look at how these distribution functions look like.

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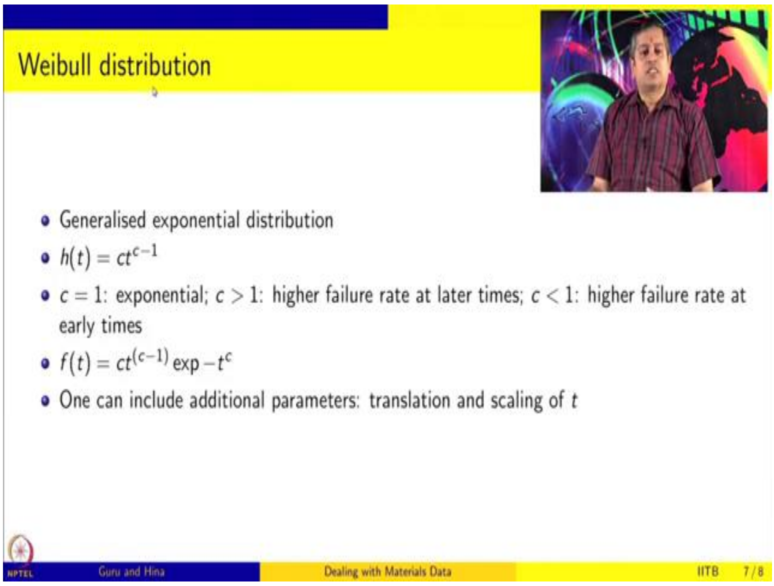

Weibull distribution is a generalized exponential distribution, we have already worked with

$$h(t) = ct^{c-1}$$

$$c = 1: exponential; c > 1: higher\ failure\ rate\ at\ later\ times; c < 1: higher\ failure\ rate\ at\ early\ times$$

you expect higher failure rates at early times.

$$f(t) = ct^{(c-1)} \exp(-t^c)$$

Weibull distribution is very important in materials because failure for example, many a times is described using Weibull distribution, we will work with some data and show that you can describe the failure using the Weibull distribution function.

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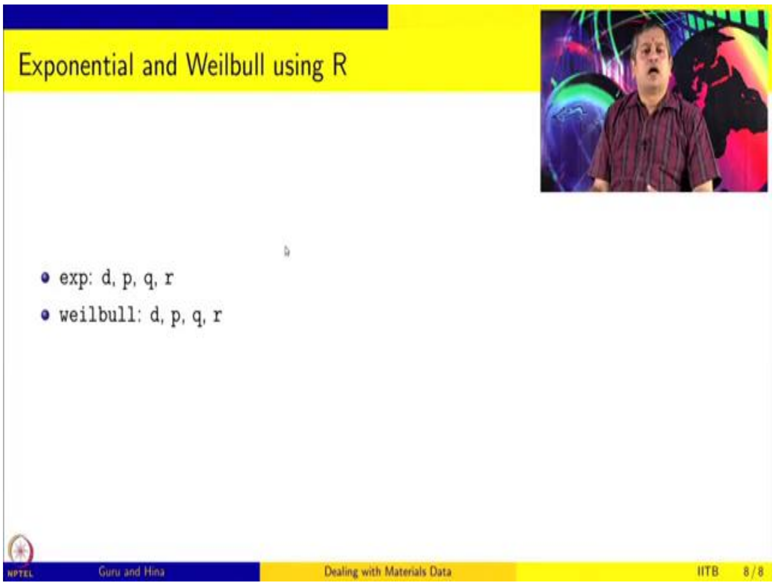

Of course, you can use exponential and Weibull using R, e x p is the 1 for exponential, so dexp, pexp, qexp, rexp will work. And Weibull, we have already used and d, p, q, r is for the Weibull distribution function.