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Phase field modeling; the materials science, mathematics and computational aspects

Prof. M P Gururajan
Department of Metallurgical Engineering
And materials Science, IIT Bombay

Module No.13
Lecture No.56
Variational derivative

Welcome, we are trying to understand a variation calculus so we will start with differentiation what is differential suppose.

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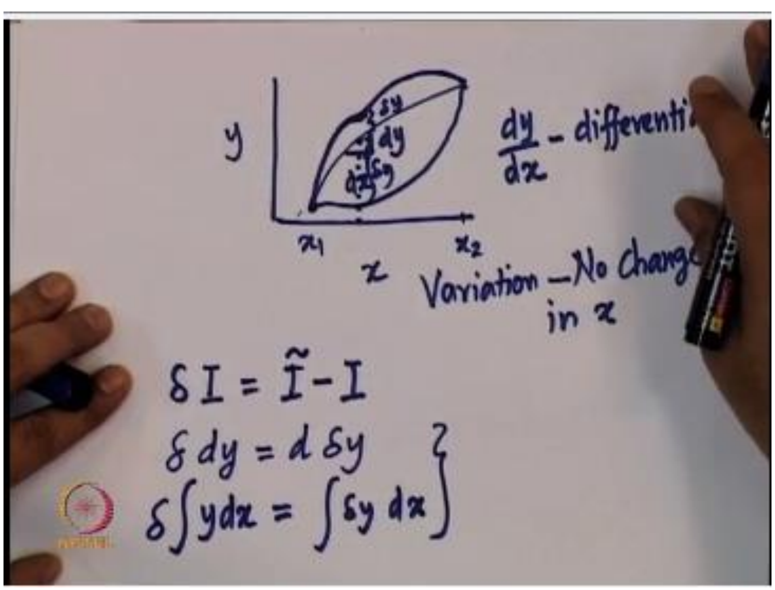

If you have a function right y as a function of X so that is some curve now when we take derivative what we do is that we take a small change in X and find out what is the corresponding change in Y and dy/dx is basically said to be the differential right this is the process of differentiating on the other hand when we are looking at variation. What we are looking at is that we are looking at the same x value but we are looking at the curves which have different slope at the same x value and different Y at the same x value so basically we are looking at for example there could be another thing which goes like that so at the same x value it has a different Y value and it has a different slope so you can also consider another function which is like.

That which means that the same x value it has a different Y value and it has a different dy/dx one so these are known as variations so in other words when we are looking at a variation okay no change in X so are going to look at the same x value at the same x value we are looking at functions which have different y + dy/dx value at that point so that is why y+ dy/dx become the independent variables and we are trying to look.

At the what happens to these things so at this point where for the same X the Y and the dy/dx are not changing are said to have no variation right there is 0 variation on at these two points okay those are our X1 and X2 points in our definition yesterday. So there is away to make it a more formal so like we say dy/dx the d/dx which is the differential so similarly we define what is known as a delta operator and delta operator.

Operating on I for example is nothing but the varied I minus the original I now we define this Δ operator in such a way that it obeys certain properties and one of the properties is that if you take Δdy then it is the same as d of ΔY that is the variation on the differential is the same as the differential on the variation similarly we also define Δ acting on Ydx integral is nothing but integral of ∂ydx that is the variation on the integral is nothing.

But integral on the variation so if these sort of properties are fulfilled then we can define what is known as a delta operator and the sink the derivation that we are going to dousing the Δ operator is the same as what we did in the last lecture for obtaining the variation except that we are making the process more formal and hence it becomes you know probably you have done it when you did the derivative initially.

The derivative is defined as a limit some hedge tending to zero F(x)+H-F(x)/H and every time you want to take the derivative a given function then you have to evaluate this quantity and then you have to find out what the derivative is but after a while then we come up with formula like d/dx of sinx is this or d/dx of x squared is this and soon and so forth. So it becomes more mechanical in a similar fashion it is possible to make the process of taking variational derivative a little bit more formal which is what I want to do so.

The derivation proceeds identical to what we did in the last class except that the notation and the terminology we use is a little bit different from what we did so in that process so let us take the original functional that we considered yesterday and try to do the variational derivative.

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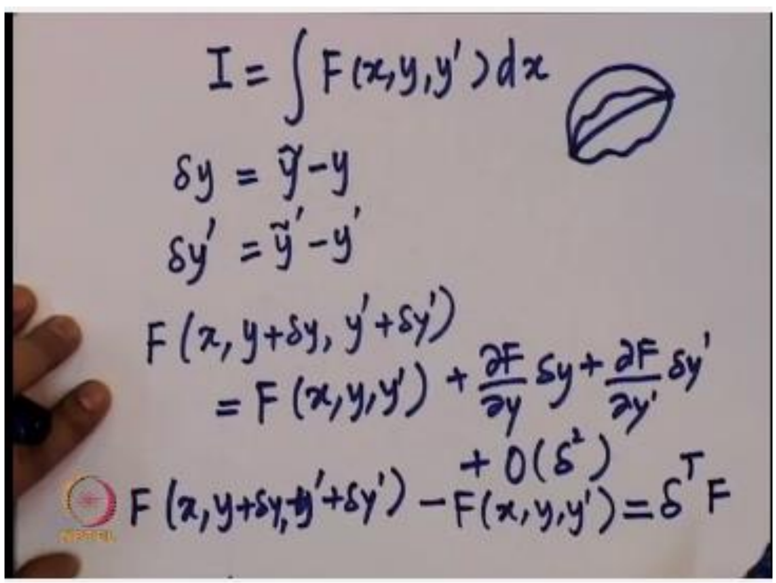

So what is the function will be considered we considered I is equal to integral f of x y'dx okay so we want to consider the so we want to define as the variation on yΔy is nothing but Y tilda minus y okay, y tilda are the varied pass remember we had the one optimizing path which was identified as Y and all the varied paths about that we're basically defined by Y tilde so y tilde minus y is Δy so Δy'.

And now because we also have all this commutation between Δ and D so also has this property so it is y tilda prime minus y' okay. So we now look at F so let us take fxy + Δy'+ Δy' what is this quantity. So we are going to tell it expand like earlier so it is xy'+ ∂/∂ y Δy + ∂/∂y'Δy' plus higher order terms in Δ. Now if we take this so f of xy + Δyy' + Δyy' minus this f of x, y, y'b are going to call this as the total variation on earth. So now what is this total variation on F by the expansion that we have written.

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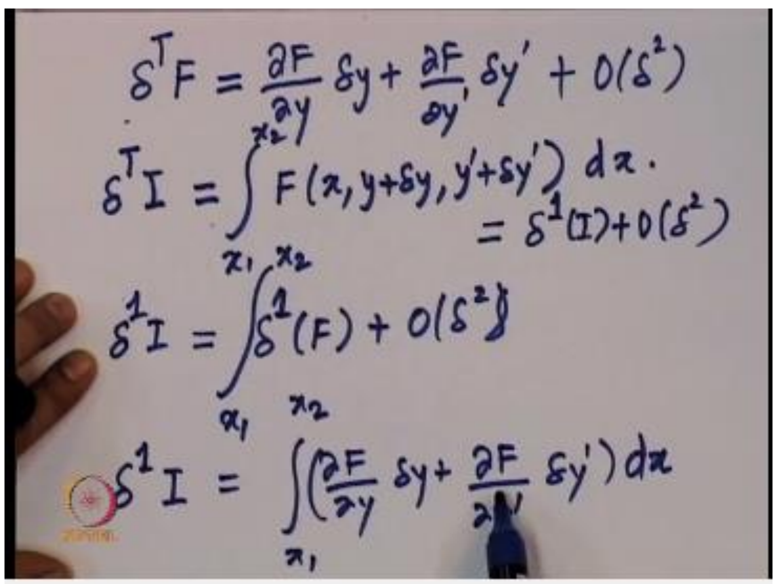

We can see that Delta total on F is nothing but 0 f/yyΔy+∂/∂y'Δy' plus order of Δ^2 now I am also going to define what is known as the Δ total acting on I which is nothing but the, so we have to integrate between x1 to x2. And so we had this yes so X0y+Δy+Δy' right, so this is what we are going to call the total Δ so that is nothing but Δ so this quantity if I call as Δ1 of f plus order of Δ^2.

So this quantity delta total of I is nothing but an integral x1 to x2 which is Δ1 of I okay, so what is the Δ1 of I so the first variation on I is nothing but X 1 to X 2 ∂/∂y Δy+ ∂/∂y'Δy' integrated over dx okay. So that Δ total of I is nothing but Δ1 of I plus order Δ^2 +Δ1 of I we identified with this Δ1 of because this is tilted total of F. So this is Δ1 of f order Δ^2 so Δ1 of I will be this integral now we can do now the integration by parts on this quantity.

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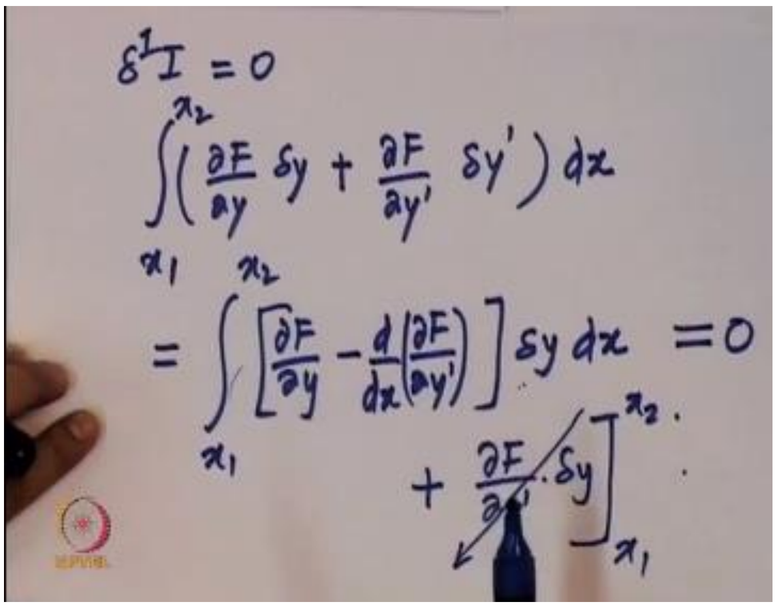

So we have Delta one of I should be equal to zero which is a necessary condition so that means X 1 to X 2 ∂/∂yΔy- you know Δ and D are interchangeable. So when we so let me write the first full quantity and then we will do that Δy'dx. So which I am going to write as integral x1 to x 2 ∂/∂Y Δy I am going to write outside and ∂f/∂y'Δy' and Δ are interchangeable.

So I am going to make it as d by dx(Δy) and that d/dx I am going to bring on to this side by integration by parts. So that gives me minus d/dx(∂f/∂y'dx) and this integration by parts we do we also have an extra surface term which is nothing but ∂ if by ∂y'Δy evaluated at points x1 and x2 and the variation ΔY is 0 at the x1 and x2.

So we are going to take this quantity off remember it need not be 0 you still have ∂f/∂y' to be 0 on x1 or x2 is sufficient we will look at that case with respect to the action principle that we derived just now at the end of this derivation. So if this one t becomes zero then we know that this should be equal to zero because the first variation should be equal to 0 and the same argument as earlier works because this Δy is an arbitrary variation in Y and this total integral from this x1 to x2 should be zero.

Then within that domain this should be identically equal to zero which is basically the Euler Lagrange equation that we have derived

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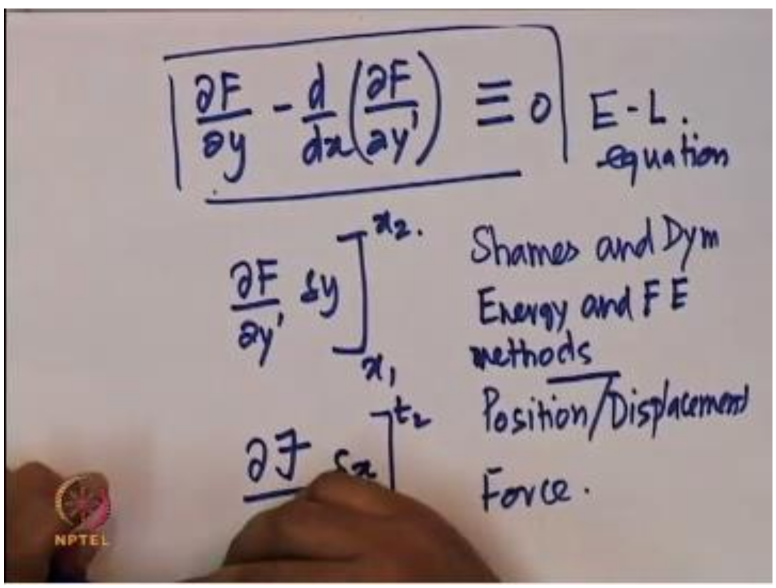

So by so we have this ∂f/∂y – d/dx of low f by ∂y' should be equal to zero and that is the so-called Euler Lagrange equation this is what we derived and we derived this in the last lecture so we have done the same thing except that we have used this so called Δ operator formalism. Now let us go back to the term that we made 0 on the surface, so that term was ∂f by ∂y'Δy at X1 and X2 let us consider this quantity with respect to the action principle that we were looking at in the case of action what is this quantity so this quantity was something like this and this was Δx right.

So at these p1 t 2 so this Δx then is the variation in X variation in position so this is if you make this equal to 0 that is equivalent to prescribing the position at time t1 and t2 so this is or prescribing the position or displacement boundary condition so you can prescribe them then in which case there is no variation allowed at time t1 and t2 at these points so they will become 0 on the other and you do not have to prescribe the position you can expect this ∂ f by∂.

X dot to be prescribed and if you look at that quantity that is nothing but the force at that point okay so you can also describe the force at any given point you can give both at the point so either you can prescribe a force or you can prescribe a displacement at a given point in either case of prescribing them makes this term on the surface to go to zero so you get the Euler Lagrange equation.

So this becomes very important there is a nice text book by shames and dim called the energy and the finite element methods thirds in structural mechanics I think so which basically deals with the boundary conditions that one can naturally derive from the variational calculus okay so in any case so if you assume that your variations are prescribed at the endpoints you can assume this quantity to be zero so you get the Euler Lagrange equation which is what we derived in the last lecture also so this is the variation derivative.

So basically variation derivative or the Euler Lagrange equation is the first variation on the functional that we are considering so this brings us to the question as to why in the case of modeling we need to deal with variations why is it that we are dealing with functional and not with functions so that is what we will discuss in the next part of this lecture. Thank you.

NPTEL
Principal Investigator
IIT Bombay

Prof. R.K Shevgaonkar

Head CDEEP

Prof. V.M Gadre

Producer

Arun Kalwankar

Digital Video Cameraman
&Graphics Designer

Amin B Shaikh

Online Editor
&Digital Video Editor

Tushar Deshpande

Jr. Technical Assistant

Vijay Kedare

Teaching Assistants

Arijit Roy

G Kamalakshi

Sr. Web Designer

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Research Assistant

Riya Surange

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Bharati M. Sarang

Web Designer

Nisha Thakur

Project Attendant

Ravi Paswan

Vinayak Raut

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