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Phase field modeling; The materials science, Mathematics and Computational aspects

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Module No.13
Lecture No.55
Optimization of functional 11

Welcome, we are looking at variational calculus specifically we looked at functionals are functions of functions an example is the action functional which is used in classical mechanics. In which we found that the kinetic energy - potential energy which is defined as the action for the particle at every point at every instant of time for example the particle has a position at that position you can have any velocity so positions as well as velocities become independent variables.

And with respect to time and so we could write a functional and in such quantities we were looking at the equivalent of the condition in the case of single valued function like f(X) = 0 some polynomial in X then we know how to look at the extrema points for this polynomial we take the first derivative we equate it to 0 we take those solutions we put it in the second derivative we look at this sign and then we decide whether it is a minim our maximum. In a similar fashion you can define the first variation for functionals which is what we derived and which goes something like this.

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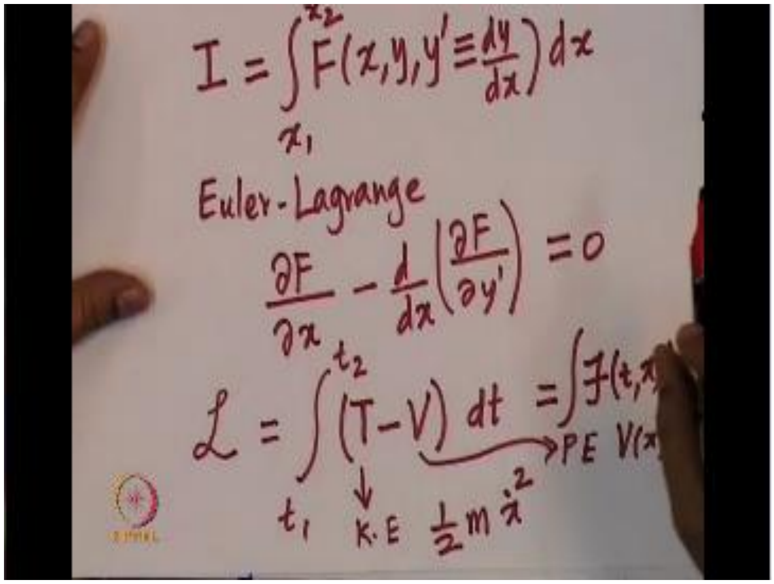

So we define the functional I $_{x1}\int^{x2F}(x,y,y^1$ which is nothing but the dy/dx)dx . So this is what we defined and in this case we said that the Euler Lagrange equation which is the first variation so that we said is nothing but 2f/2 x- d/dx of (2f $2y^1$)= 0 .This is how we define the Euler Lagrange equation so this is for the first variation. In this case now we want to consider the action principle. So let us define the action as this quantity goes from some t1 to t2 and we said that it is (t – V) dt.

The principle of minimum action says that where t is the kinetic energy so it is given as 1/2 M $X^{.2}$ and V is the potential energy and it is just a function of X. Now given these two now if we are trying to optimize this function so this function can now be written if we want to write it in this form as this so some F(t,x,$x^{.}$) which is nothing but which is nothing dx/dt. So this is what we have integrated over DT so we can use for this functional and now the Euler Lagrange equation and we can try to write it.

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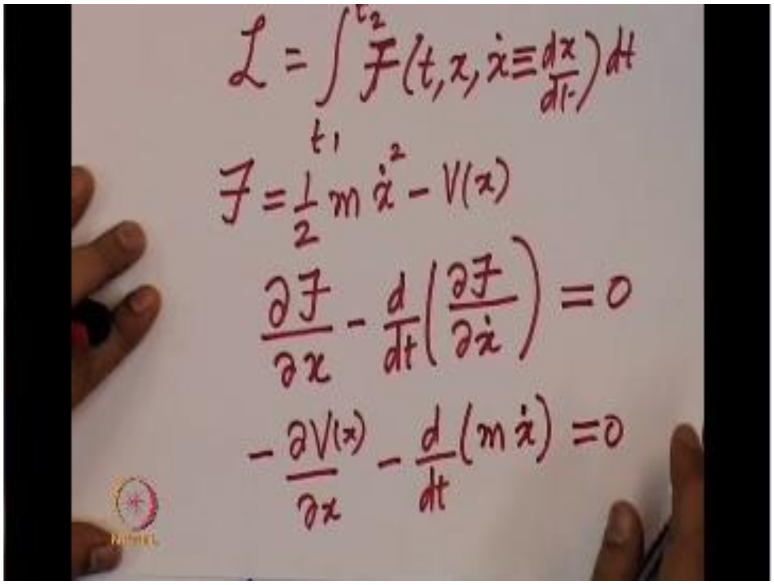

So if we write it then we will see what happens okay so the principle states that this action should be minimized and where this f is nothing but $1/2mx^{\cdot 2}$- V(x). If we want to minimize this functional then we know that minimization means the Euler Lagrange equation in this case because T happens to be the independent variable the Euler Lagrange equation would be this $2f/2x-(2f/2x^{\cdot})$. If so if you take this functional and if we do this so this should be equal to 0.

This is the Euler Lagrange equation so do this functional by ∂ X then you see that this is the only explicit function of X. So you have – ∂/∂X V which is a function of X- d/ dt (∂) /∂ $X^{\cdot}$ so if we see this is the only term which has $X^{\cdot}$ if you assume that m is a constant now 2 & 2 goes off so you get X= 0.So if we now take this that leads to the following expression.

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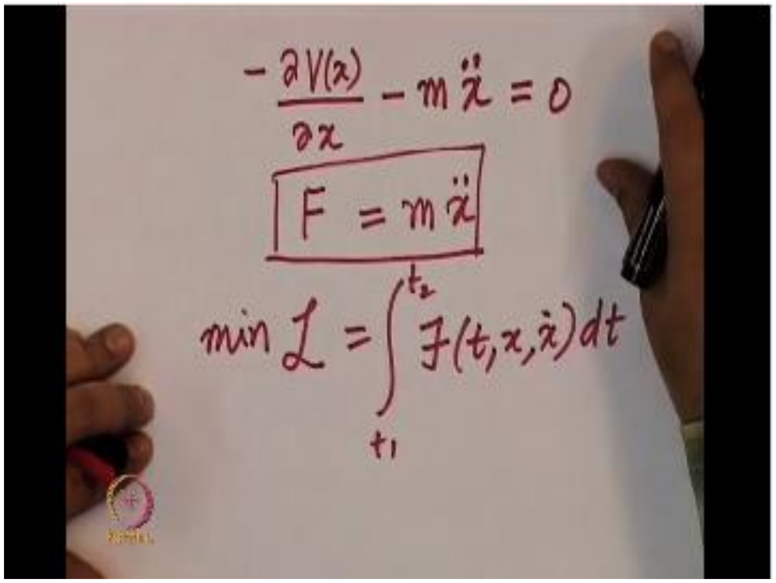

So we get - ∂ V(X)/ ∂X - again M is not changing with time if you take that then you have $m\ddot{\ddot{x}}$= 0 in other words. If we know that the negative gradient of the potential is nothing but the force =$m\ddot{x}$. This is the Newton's equation, So we find that the classical action principle so you can define a functional called action and that is nothing but integral between time t1 to t2 the kinetic energy minus potential energy integrated over time and we say that now we want to extreme eyes this functional.

And we use the Euler Lagrange equation because that is the necessary condition so Euler Lagrange equation is like making the first derivative of a function equal to zero and getting the solutions to find out where the extreme a are so if we do the same thing with this functional now we obtain Newton's law f equal to mass times acceleration and this came from minimizing the action which was defined as dt.

So there is an ice description of this functional and the so-called action principle in finance lectures on physics so you can take a look at it so this is an example of how the functional is written how I learn a garage equations are used and what they give us so you can see that this is a nice elegant principle solving which basically we attain Newton's laws so classical particles will obey Newton's equation but because Newton's equation is obtained as an minimization over this functional called action.

So minimization of action is basically the principle from which you can get the Newton's laws so this is a way of looking at the classical dynamics so it demands that minimization of the action is basically equivalent to solving the Newton's equation so this is an example so in the next part of this lecture we will go or take a closer look at what is this very as I and there is a formalism known as delta operator formulism which makes this process of taking variational derivative a little bit more methodical and hence mechanical so we will look at those two aspects in the next part of this lecture thank you.

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