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Phase field modeling;
the materials science,
mathematics and
computational aspects

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Module No.12
Lecture No.52
Representation quadric

Welcome let us consider the quantity which is known as a quadrant okay.

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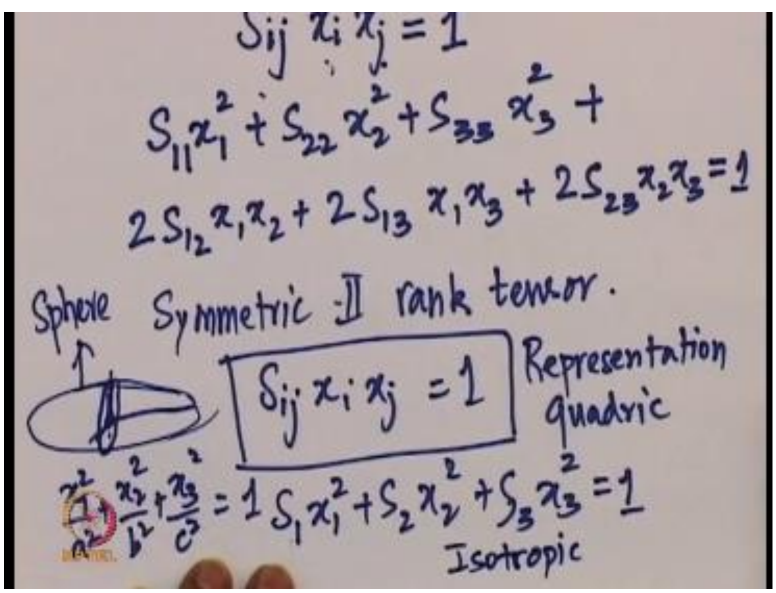

So it is defined as follows SI j x I XJ is equal to 1 okay we are using Einstein summation convention suppose if I and J run from one to three so what is this quantity expanded it is nothing but s 11 x1 squared plus s 2 x2 squared plus s 3 X 3 squared plus s 1 2 s 1 2 X 1X 2 plus 2 s 13 x 1 x 3 plus 2 s 2 3 x 2x 3 is equal to 1 so this is the generic representation of a geometric object which if all this one 1 two 2 three 3etcetera or real basically is an ellipsoid right so we have already seen this we have said that first rank tensors or those quantities which transform like the position vector of any point second-rank tensors are those quantities which transform like the multiplication of the pairs of points right now if you look at the representation quadric.

X I x j suppose if I did a coordinate transformation and I went to x i prime x j prime so they will have some a 2 as multiplying sij that will be equal to SI j prime so this basically transforms like a second rank tensor but notice because when I multiply x I x j x1 x2 and x2 x1 is the same so that is why I have a to hear the quadric written in this fashion transforms exactly like a symmetric second rank tensor because this quadric transforms like asymmetric second rank tensor this is known as a representation quadric for.

So this is nothing but representation quadric for a symmetric second rank tensor now a general property of such a quadric is that you can always find three directions in which if you write all this so then these all these cross terms will go away and you will get s 1x1 squared plus s 2 x2 squared plus s 3x 3 squared equal to 1 okay this can always be transformed by choosing appropriate x1 x2 x3 in this form wherex1 x2 x3 will be perpendicular to each other so what this means is that this can always be represented as an ellipsoid right which is like that.

So what is this s1 so it is nothing but the1 by a square root of this hop distance right so you have an ellipse for example the semi-major semi minor axis so this is a relief side so it has three s1 s2s3 three perpendicular axis so basically 1 is 1 by so 1 by square root of s 1is nothing but the a right so this if you compare with x1 square by a square plus X 2 square by B squared plus x 3square by C squared equal to 1 which is the equation of such an ellipsoid so you can identify s1 with one by a is thing but 1 by square root of s 1 B is nothing.

But 1 by square root of s 2 C is nothing but 1 by square root of SD in other words the symmetric second rank tensor transforms like this quadric and the property of the quadric is that it can always be represented in the form of an ellipsoid like this now all symmetric second-rank tensors have this representation if you consider now in a cubic crystal you have the symmetric second rank tensor namely diffusivity so the diffusivity.

If you want to represent as a quadric you realize that because there should be four threefold axis if you try to manage for threefold axis on a generic ellipsoid like that it will become a sphere this is the only way the unless the representation quadric for asymmetric second rank tensor is a sphere it cannot have four threefold axis and for 34 taxes is essential if that symmetric second rank tensor is a material property of the cubic crystal which automatically implies because it is a sphere now s1 s2 s3 are all the same which automatically implies that you are symmetric second rank tensor in cubic system has become isotropic or it has only one.

So instead of all these three half axis you have the radius of the sphere as the only in dependent quantity so this is basically the same information as we argued out using. The earlier argument where we try to take the define the symmetry of a given crystal property that we did by taking these stimuli and response and looking at them in the original unit cell and In a symmetrically transformed unit cell and we try to get the property to be the same and that gave us the property that cement symmetric second rank tensor properties are basically isotropic because they also remember we use the symmetry property we said X Y should be equal to Y X.

But it happens to be minus y X and that is the reason why x YY x terms became zero the off diagonal terms begin 0 so symmetry is already assumed why this property should be symmetric a Chanel together different argument we will not get too into it now, what this point but it is an interesting argument by itself but if you assume that we are looking at asymmetric second rank property tensor property material property tensor.

Then in cubic system it has isotropic symmetry that you can prove two ways one is by looking at the stimuli response doing a coordinate transformation which is a symmetry transformation on the crystal and matching the properties the second one is by looking at the representation of the property tensor which is a quadric and this representation quadric if you represent it in terms of its what is known as principal axis that is there are always three mutually perpendicular axis in which if you take the x1 x2 x3 the all the cross terms will go to 0 in such are presentation if you take because that represents the property.

Now for a cubic system that representation quadric should have the symmetry with symmetry for threefold axis if you now demand that there should be four threefold axis for an a generic ellipsoid of that type you notice that it already becomes a sphere and the sphere will have the symmetry of isotropy so this is another way of proving that a symmetric second rank tensor is isotropic in cubic systems it is isotropic in isotropic systems also but what is more it is isotropic even in cubic systems which obeys Neumann principal.

Neumann principle says that the property should have asymmetry which includes the point group symmetry isotropy certainly includes the cubic symmetry so it is allowed where as for example a tetragonal symmetry is noting including the cubic symmetry less than cubic symmetry so no proper you can have tetragonal symmetry in a cubic crystal for example so that is the meaning of the anointment principle. So we will we will later see how these kind of properties the information about the property tensors are helpful for us in defining certain material properties when we are dealing with face full models thank you.

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