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Phase field modeling; the materials science, mathematics and computational aspects

Prof. M P Gururajan
Department of Metallurgical Engineering
And materials Science, IIT Bombay

Module No.12
Lecture No.48
Group theory

Welcome we have been looking at the normal principle which says that the point group symmetry of a given material and the symmetry of any given property in that material that the symmetry of the property should include the point group symmetry so it can be bigger than that or it can be larger than that but it should at least include the point group symmetry of the crystal so in this context we wanted to understand what is a symmetry element and what is the point group etcetera so let us start with the notion of a group so let us consider a set which consists of four elements namely plus one minus one plus I the imaginary number or complex number and minus.

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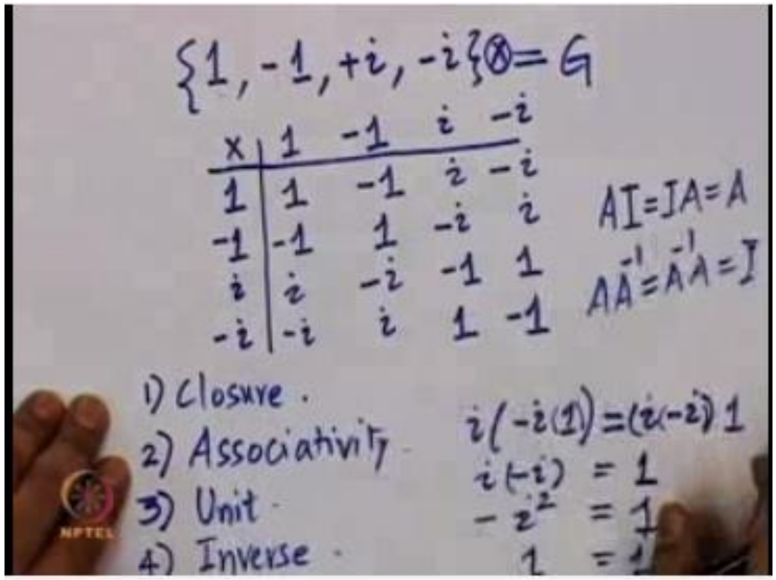

I so this is a set so let me write it down 1 minus 1 plus I and minus I now we can form a multiplication table consisting of these four numbers and this is a multiplication operation and then we write the same four numbers and we fill this table so 1 times 1 is 1this is minus 1 this I this is minus I and this is minus 1 this is plus 1 this is minus I plus I and then I minus I is qua red is minus 1 minus I squared is plus 1 and this is minus I then minus I squared is plus 1 and minus I into minus I is plus I squared which is minus1.

Now there are certain properties which are satisfied in this kind of multiplication table first one is known as closure entire table the multiplication table consists of only one minus one I and minus I which means that our set consists of these four numbers under the multiplication operation you get the same four numbers in different combinations so this property is known as closure second property is known this basically tells at the order in which you multiply is not important for example let us take I into minus I into one.

For example which is equal to I into minus I multiplying 1why is this so this is I this will be minus I and I into minus I will give you minus I squared that is minus 1 I squared is minus 1 minus I squared is actually plus 1 so this will give you one and here you again have minus of Is qua red equal to 1 and that is one so you get so the order in which we multiply whether you multiply minus I with one first and then multiply it Y or you multiply minus I with I first and then multiply that with one the order does not matter so this you can check for many different combinations and you will find that they always give you the same result this is known associatively then there is the notion of a unit element.

So what we mean by that is that for every element there is one element which if you multiply you get the same element that is for every element let us say ye h in a group the unit element is an element which has the property that you multiply it left multiply or left right multiply you get the same element right so in the case of14 example you multiply it with one you get one and if you want to get minus one and you can multiply it with one and you will get minus 1.

For example with one will give you I minus I with one will give you minus I and it is the same in the other way also so minus I with one will also give you minus I with one will give you I and minus one with one will give you minus 11 with one will give you one so there exists an element which is known as the unit element which if you multiply left multiply or right multiplied gives you the same element so this is known as unit and then there is an element which is known as the inverse.

So the inverse for any element a has the property that a inverse or a inverse a will give me I the unit element because in this case the unit element is one for one for example multiplication with one gives 1 4-1 multiplication with minus one gives one for I multiplication with minus I gives 1 and 4 minus I the multiplication with I gives one so the unit element is recovered by these elements so that those are known as the inverse syllables so we define a group as follows if you have a set of elements.

And if you can define an operation on these set of elements in this case it is called multiplication sometimes it is called group multiplication or group addition and under the action of this operation if the given elements these four properties namely that the multiplication table such that only those elements appear so this is known as closure so the operation between different elements results in elements of the same set being reproduced and the operation is associative it does not matter in which order you do this and there is a unit element such that for every element left multiply or right apply this unit element you get the same element back.

And for every element that is an inverse element left multiply or right multiply this inverse with the element you will get the unit element so if you have a set with given operation which obeys all these properties then that is known as a group this set now but to define the group we also have to identify what is the operation in this case this is the multiplication operation. Now this is one of the groups of this type you can also think of another group for example I can consider the angular addition modulo 360 so let us consider another set.

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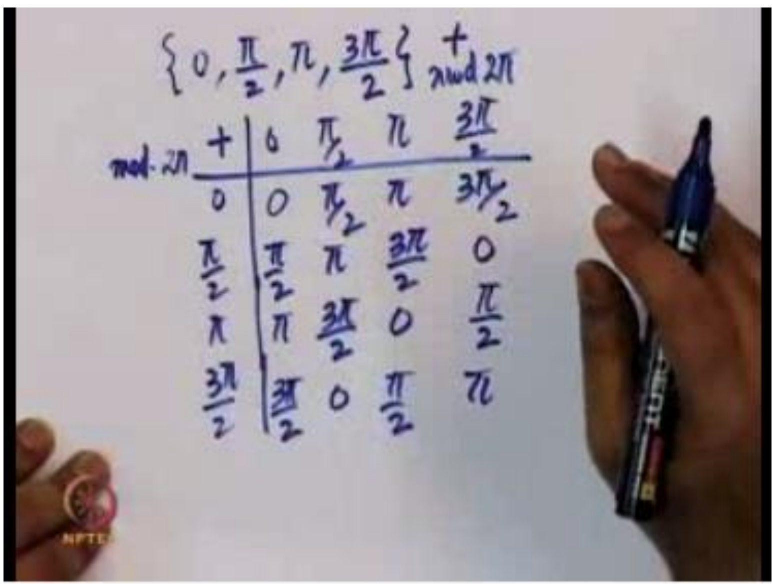

Which consists of four elements 0 PI by 2 y +3 PI by 2 right so if I have addition but addition this is modulo 2 PI right so 0 PI by 2 pi and 3 PI by 2 and here it is 0 5 by 2 pi and 3 PI by 2 so 0plus 0 will give 0 plus PI by 2 will give me pi by 2 0 plus x give me pie and this is 3 5 by 2 pi by 2 will give me 5by 2 then it will give me pie give me 3PI by 2 and this will give me 4 PI by 2which is 2 pi which is nothing but 0.

Now this will give me pie and then this will give me 3 PI by 2 and 2 pi is 0 and then PI plus 3 PI by 2 which is 5 PI by 2 so4 PI by 2 is 0 so it will give me a pie by 2 and now this will give me 3 PI by 2and then this will give me 0 and so 2 pi so it will give me pi by 2 and 3 pi by 23 pi by 2 6 pi by 2 so that is 3 PI so 2pi goes so it will me a pie now you can say that the set consisting of 0 PI by 2pi and 3 PI by 2 where the group edition is addition modulo 2 pi forms a group because it has closure and for every element.

For example if you add 0 you get the same element back and for every element you can find an inverse element which if you add you will get the same element back for example if you add that inverse element to this element you will get the unit element which is 0 in this case okay so this is another multiplication table which has all these properties so closure unit and inverse is available here also you can also think of another such multiplication table let me consider.

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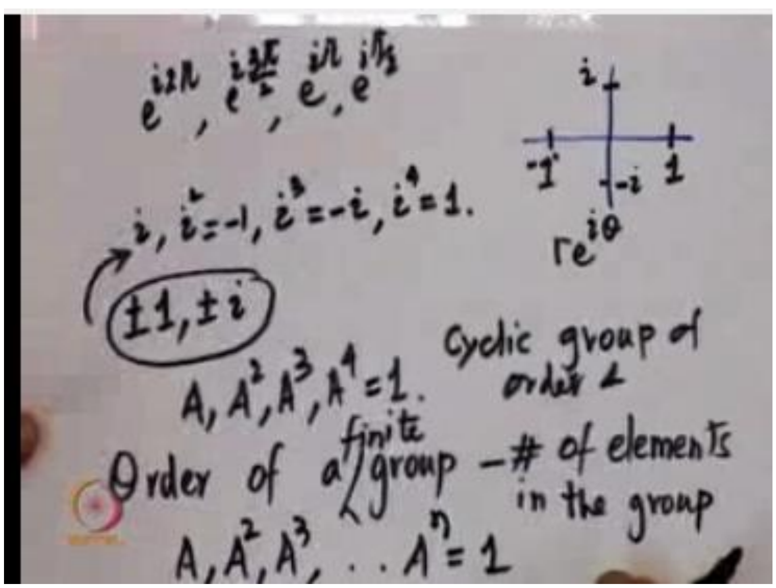

The complex plane and let me consider a unit vector of length 1 and if you do a90 degree rotation that is going to become I to do another 90 degree rotation that is going to be minus 1 and if you do another 90 degree rotation that is going to be minus I now they can be represented as for example e power I 2pi that will be one e power i 3 pi by 2and e power i pi and e power I pi by 2 8pi by 2 pi 3 pi by 2 to PI.

So you can also think of so this is basically are a power I theta form so you have a unit vector a 90 degree rotation of that is going to so under this operation again you will find that you can put a 4i two pipes and do the multiplication able there is it another way of thinking about this you can think of this as I i squared which is equal to minus 1 I cube which is equal to minus II power 4 which is equal to 1 okay.

So now I mean this so you can represent all the elements like in the in the first group that we had which had plus or minus one and plus or minus I so the elements are all represented in this form in other words you have a group elements of which can be represented as a squared a cube and a power for which comes back to unit itself right so now the order of a group right if it is a finite group is the number of elements in the group that is the set over which the given operation is defined in that set how many elements are there that is considered as the order of group.

Now this kind of group is order for so it is also called a cyclic group of order for cyclic group because it has this kind of property a s squared a cube a power for equal to 1 so cyclic group of order n will be for example that goes something like a cube etcetera up to a power n which becomes one now. In a group now there are certain things that one can define the first thing that you can define is what is known as a subgroup so let us take the multiplication table that we defined.

For the 1-1 I minus i1 minus 1 I minus I so I had 1 minus 1 I minus i minus 1 1minus I minus I min us 1 minus I I1-1 now in this you can define what is known as a subgroup for example the entire group itself is a subgroup but within that I mean non-trivial II there is a proper subgroup which is made up of by these two elements right because between 1 and minus 1 again there is closure there is associatively there is unit and there is inverse okay.

And then of course then tire 1-1 I minus I becomes a trivial subgroup and this is known as a pro per sub do so within a group a set of elements which form a group are known as subgroup and there is also this notion of isomorphic that is if you have multiplication tables so we have seen till now three different multiplication able though all three tables are identical except for the way in which b the elements so they are all said to be isomorphic to each other.

So these are identical multiplication tables except for symbols now the notion of a symmetry group is that for certain geometric objects there are symmetry operations and those operations form a group so what do we mean by that we well do that in the next part of this lecture thank you.

NPTEL
Principal Investigator
IIT Bombay

Prof. R.K Shevgaonkar

Head CDEEP
Prof. V.M Gadre

Producer
Arun Kalwankar

Digital Video Cameraman
&Graphics Designer
Amin B Shaikh

Online Editor
&Digital Video Editor
Tushar Deshpande

Jr. Technical Assistant
Vijay Kedare

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Research Assistant

Riya Surange

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Vinayak Raut

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