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Phase field modeling; The materials science, Mathematics and Computational aspects

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Module No.11
Lecture No.45
Coordinate transformation

Welcome we are looking at the description of vectors and there are several descriptions one is to say that vectors are quantities with magnitude and direction other one is to say that vectors are quantities with one index when you refer them with respect to some coordinate frame of reference and our problem now is to get rid of this dependence on frame of reference because frame of reference is arbitrary anybody can choose some frame of reference but the vectors have a life of their own these are physical quantities like flux for example that does not depend on which frame of reference you choose to describe it okay.

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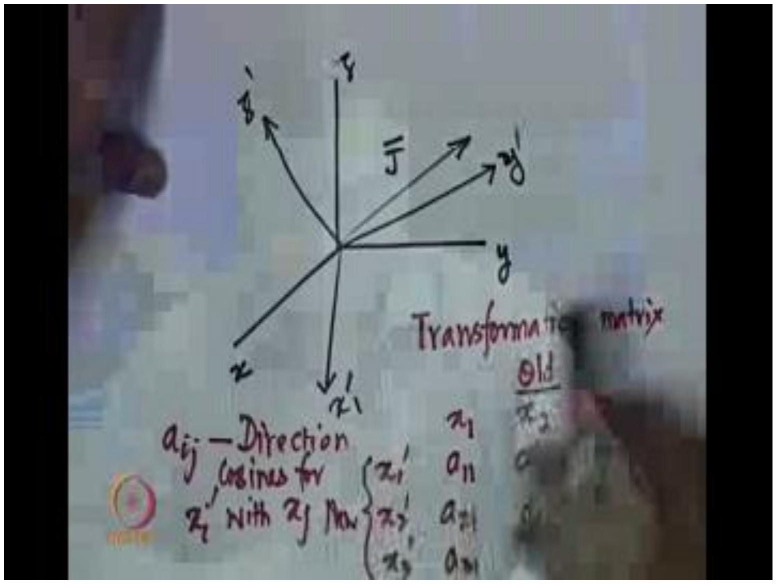

So let us do this exercise let us say that I have a vector now. I have chosen a frame of reference describe it let us say that this is X YZ somebody else decides to choose another frame of reference but it is not any arbitrary frame of reference in most of this exercise that we are going to do we would assume that the origin for the frame of reference reminds the same and you have another three mutually perpendicular directions.

Right now the relationship so how r this X prime Y prime and z prime related to XYZ that is the given by what is known as the transformation matrix. What is the transformation matrix so you have all the old frame of reference here x1 x2 x3and you have the new frame of reference here x1 prime x 25 x 3 prime the a11 is for example the direction cosine between1 and 1 prime a 12 is the direction cosine brief between 2 and 1 Prime and A13 is the direction cosine routine 1Prime and three similarly A21 A22 A23 A31 A32 A33 right these a IJ s or direction cosines for X I prime with XJ where prime is the new one and unprimed or old coordinates.

So when you do a coordinate transformation from X1 X2 X3 X1prime X2 prime X3 prime to R X Y Z to X prime Y Prime that fine then we want to know what is the effect on the components that we are going to find out for this vector which is the flux vector let us say so J is my flux vector which is being described by me by using XYZ by somebody else using X prime Y prime Z Prime except that the origin of these coordinates are the same that so that I am assuming under that condition if I want to know how the components X1 X2 X3 are related to X1prime Y1 prime Z 1 prime.

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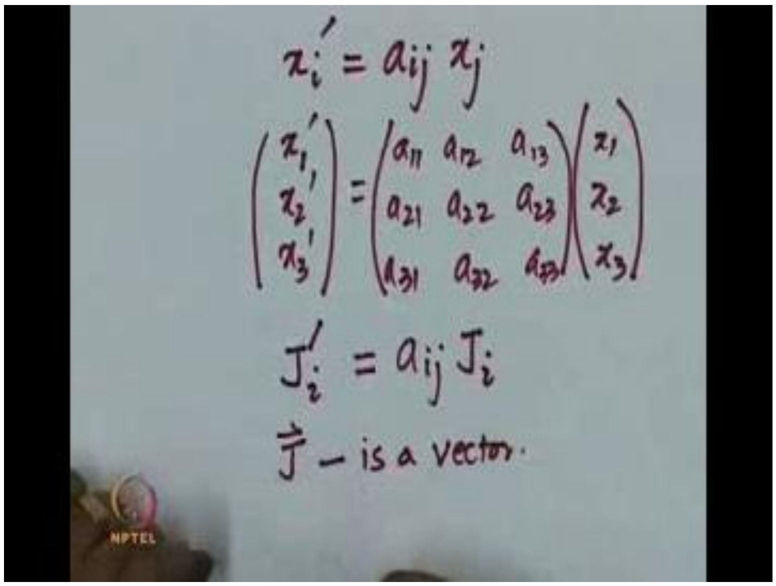

That is basically given by the following expression so we say that X_i prime = a i j x j remember we are using Einstein summation convention which means that this will be summed over in terms of matrix notation what we are saying is that X_1 prime X_2 prime X_3 prime is equal to(A_{11} A_{12} A_{13} A_{21} A_{22} A_{23} A_{31} A_{32} A_{33})(X 1 X 2 X3). So in other words what we are saying is that if you think of this quantity X 1 X 2 X3 right it could be j1 j2 j-3.

For example that is the components of the J vector along X 1 X fixable if that quantity if you know and if you know the direction cosines between the new coordinate directions and the old coordinate direction then if you make a matrix and if you multiply that if you get the components if this happens to be the components of the same vector in the new frame of reference that you are dealing with.

Then you can define this quantity so this is then you can define this X as a vector this is the third definition what is the definition of a vector we say that suppose if I have J and the components of j in a new frame of reference if they are related to the old frame of reference through the matrix which is the transformation matrix consisting of the direction cosines ji are the components of the jet vector using old frame of reference J_i prime are the components of the J vector using the new frame of reference if suppose they are related through this then we say that j is a vector.

This is another way of defining vectors with respect to coordinate transformations so what is the idea here we are thinking of vectors as quantities which have their own existence and you can define them as quantities with direction and magnitude but that is not generalize. But if you refer to the vector so to some pacific frame of reference then you can represent vectors in column matrix fashion or by giving some components which can be represented using indices so you can define vectors as quantities with one index.

For example this is generalize because then you will define a second rank tensor as quantity with two indices a 0 thread pen ser is a quantity with no indices and so on and so forth so it helps us to generalize but it leads us to the problem that we have a quantity which has an existence of its own but when we use this index based definition of the quantity.

Then we are necessarily defining it with respect to a frame of reference and the frame of reference is arbitrary because the existence of the vector has got nothing to do with what frame of reference we decide to use to describe it so we want to see what is the effect of a change of frame of reference on the vector okay. So then we define what is known as the transformation matrix which is given by this a a 11 basically is the direction cosine between 1 prime direction the new 1 direction and the world 1 direction okay.

We have 12 is the direction cosine between 1 prime direction and 2 direction so 1 2 3 or the world directions and the 1 prime 2 prime 3 prime with respect to what is the direction cosines they make or defined as 1a 12 a 13 a 2 1 a 2 3 etcetera and this transformation then tells if you take the components of a vector if you do this transformation matrix left multiplication on that you get the new components so any three numbers for example or suppose i take the j vector the components of the j vector along x1x2 x3 Ii can find out.

Because I have the frame of reference somebody else uses some other frame of reference which has the same origin as mine and the components that they have is given by x1prime x 2 prime x 3 prime and if I can find out the relationship between my frame of reference and their frame of reference in terms of the transformation matrix and if I find that my components x the transformation matrix gives their components then I know that this quantity is a vector.

So this is another definition of a vector a vector is a quantity which transforms under a frame of reference change in a particular fashion namely if I change the frame of reference from some X to some X prime then the components change according to J I and J I prime related through AIJ where a IJ is the transformation matrix which is consisting of the direction cosines between I prime direction and j direction so that is how we form the transformation matrix and this is the relationship.

So this is another way of defining vectors which is again generalizable okay if we are going to show how this generalizes to higher order tensors okay so of course scalars are quantities which are invariant so a scalar is invariant under coordinate transformation right I have the density of a material it does not matter whether I refer to some frame of reference or some other frame of reference okay or if I have temperature at some point it does not matter what frame of reference use the temperature is a quantity which is independent of any frame of reference description.

Because it is a quantity which does not require any direction information because it is just number so it does not matter what frame of reference. But the moment you are describing a vector if you use one frame of reference you will get some components if you use some other frame of reference you get some other components now if these two componentsare related through a specific relationship then that is the true vector if that the relationship does not exist no you can take three numbers and put them in a column vector and say that this is a vector it will become a better only under accordingly transformation if it transforms according to these relationships.

For example I cannot take the temperature at the three points right 123 and then put them as a column vector and say that okay this is a vector now it cannot be because if I do some other frame of reference with respect to which then if I try to look at what happens to these numbers if they remain the same then that is really not a vector but a true vector quantity will change its components according to the frame of reference you use in such a way that this relationship is always obeyed so that is how we define a vector.

Which is generalizable to higher order tensors which is what we will do in the next lecture of course it is also important to distinguish this transformation matrix from the second rank tensor that we define you know inform this looks quite close to that d11 d12 d13 that we wrote ok what is the difference between that and this so all these things we will take it up to be next. Thank you.

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