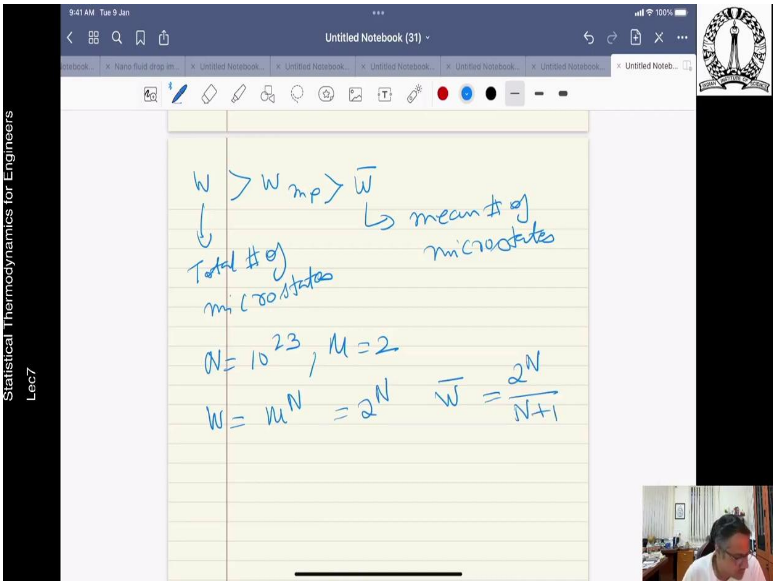
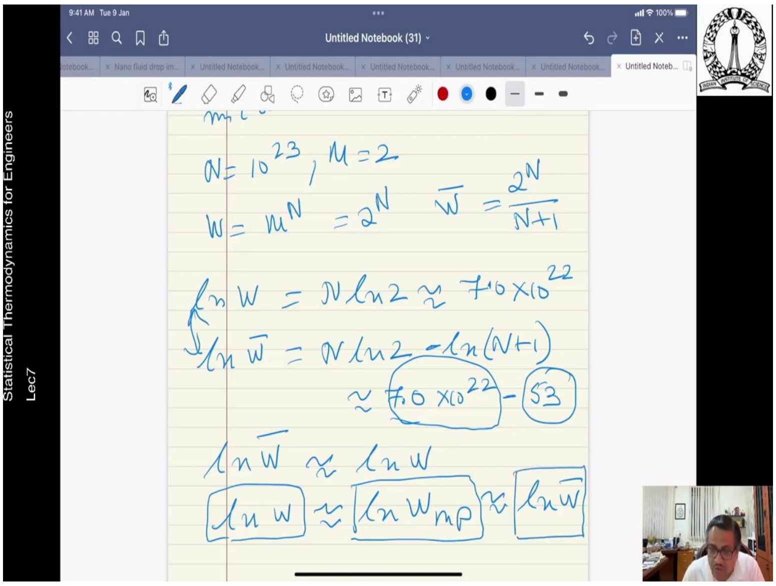
Statistical Thermodynamics for Engineers
Professor Saptarshi Basu
Indian Institute of Science, Bangalore
Lecture 07
Bose Einstein and Fermi Dirac Statistics

(Refer Slide Time: 00:01)



So, welcome to lecture 7 of the Statistical Thermodynamics for Engineers. So, just a brief recap. If you recall what we did in the last lecture is that W is greater than the most probable that greater than W bar, W was the total number of microstates, these are the total number of microstates of a system and this is the mean of microstate. So, this was already there now, if your N is 10 to the power 23 and M is equal to 2. So, W that is the total number of microstates is given as M to the N then 2 the power N, then W bar, the mean number of microstates is basically 2 to the power of N divided by N plus 1.

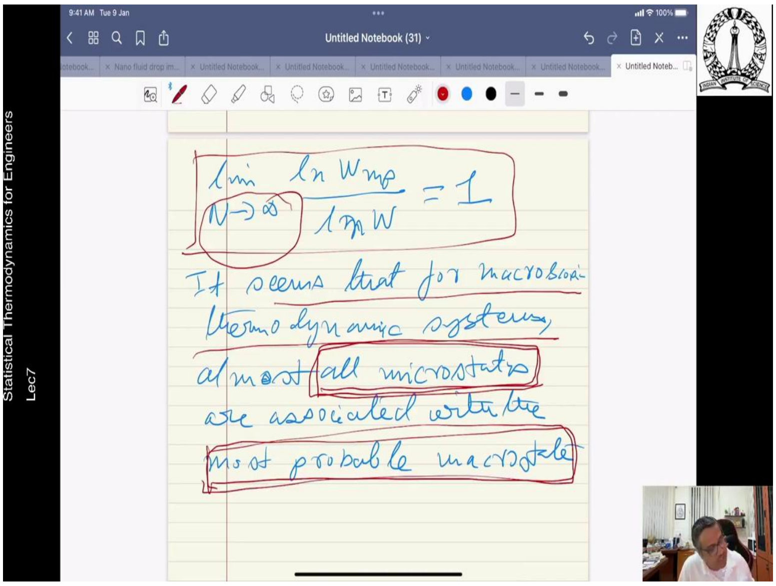
(Refer Slide Time: 01:13)



Now, if we now look at it carefully, ln W that is if we take the natural log of W, which will be in N ln 2, which roughly translates to about 7.0 into 10 to the power 22, ln W bar which is basically the logarithm mean microstates and up to minus plus 1, which also translates to 7.0 into 10 to the power 22 minus 53. So, what we can see here is ln W bar is almost the same as in W, because you can see that this particular number is so large compared to this, that these two are basically one in the same. So, in other words, this also means that ln W is therefore almost equal to ln most probable, which is almost equal to ln W bar.

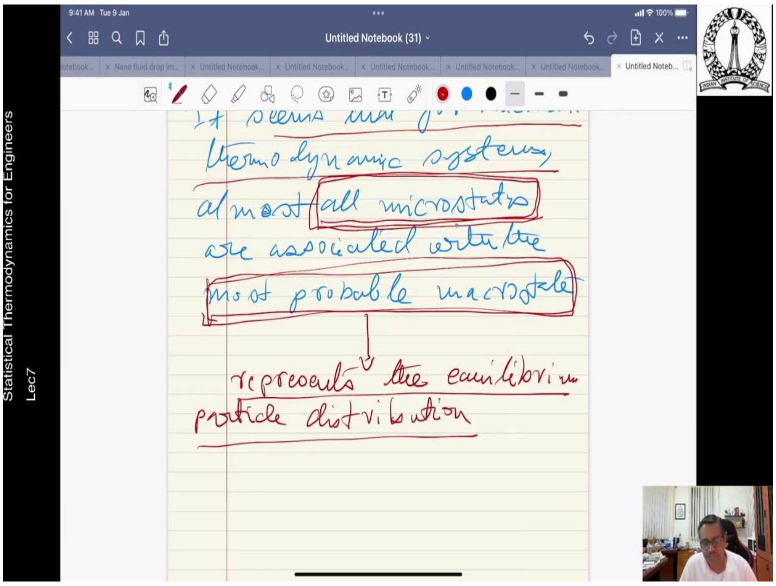
So, all it means is that the logarithmic of the total number of microstates is almost the same as the most probable number, the number of microstates associated with the most probable microstate and that is almost equal to the mean number of microstates. So, that is eliminating.

(Refer Slide Time: 02:52)



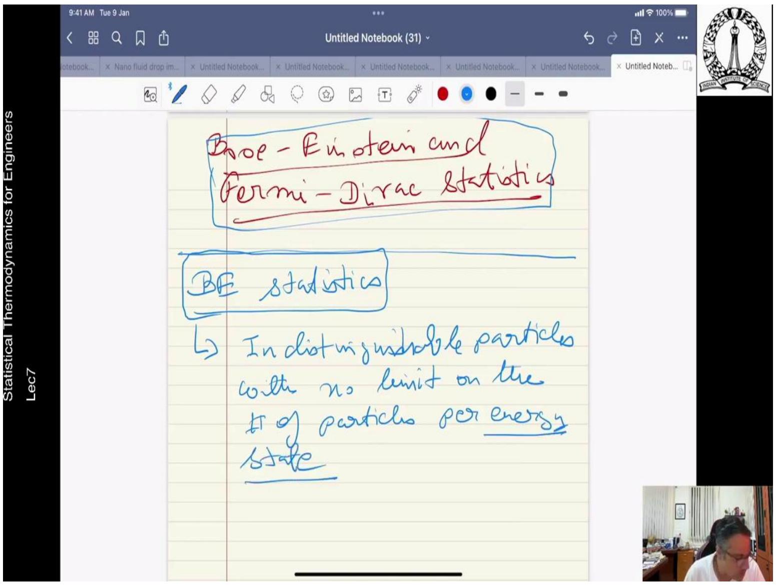
So, what we can say is that in the limit as N approaches infinity, ln of Wmp divided by ln W is almost equal to one or in other words, it seems that for macroscopic thermodynamics that was to be thermodynamic systems comma almost all microstates are associated with the most probable microstate. So, this is the most important inference that because this is true, and because N is pretty large, it seems that for macroscopic thermodynamic systems, almost all microstates. This is the most important part almost all microstates are associated with the most probable microstate, all microstates are associated with the most probable microstate. So, the most probable distribution must represent the equilibrium particle distribution.

(Refer Slide Time: 04:34)



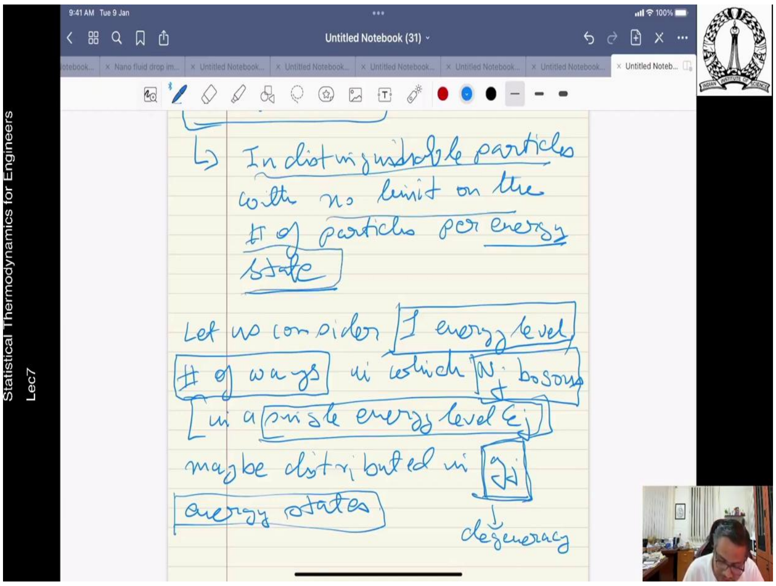
So, this therefore, represents the equilibrium particle distribution. So, this therefore represents the equilibrium particle distribution. So, that is eliminated to begin with.

(Refer Slide Time: 05:03)



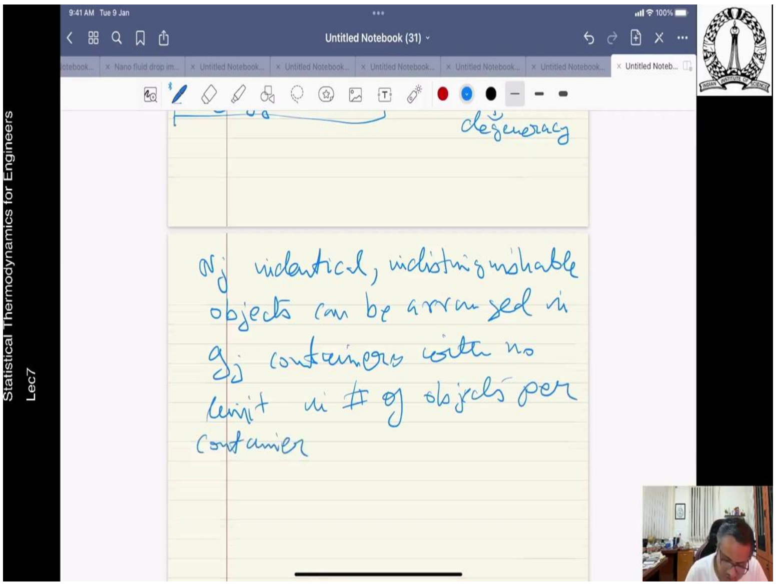
So, let us now look at the first level of statistics because we have now we have covered the different aspects of macrostate microstate we know a lot distinguishable indistinguishable particles also. So, we are going to look at Bose Einstein, Einstein and Fermi Dirac statistics. So, first let us look at the Bose Einstein statistics or the BE statistics. So, the BE statistics that is the Bose Einstein statistics after S.N. Bose and Albert Einstein. So, this the premise is that indistinguishable particles with no limit on the number of particles per energy state.

(Refer Slide Time: 06:34)



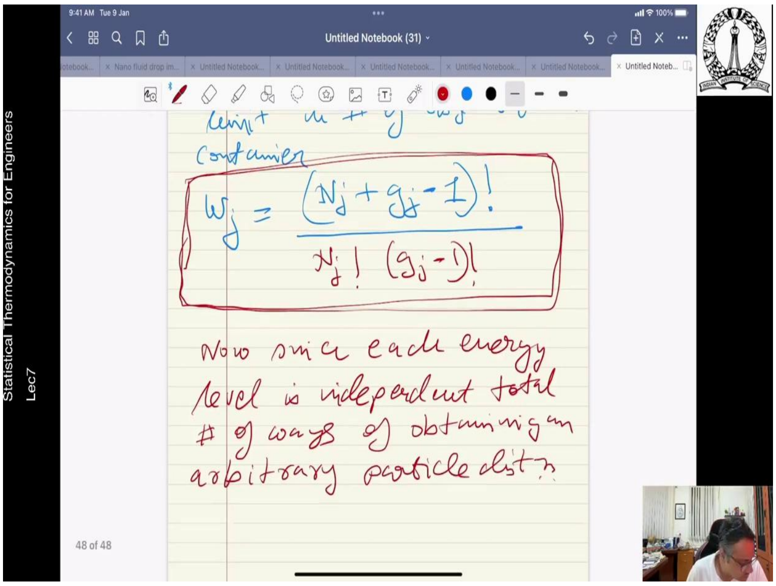
So, let us consider do it slowly one energy level and let us consider the number of ways in which Nj bosons which is the number of particles a single energy level Ej maybe distributed gj. So, we consider 1 energy level and we are going to see the number of ways in which Nj bosons because both sides test statistics that bosons so, these are the number of results in a single energy level Ej may be distributed in gj energy states where gj is the degeneracy. So, you have a physically gj compartments you have Nj indistinguishable particles there is no limit on the number of particles or energy state. So, 1 energy state can have as many number of particles as you want.

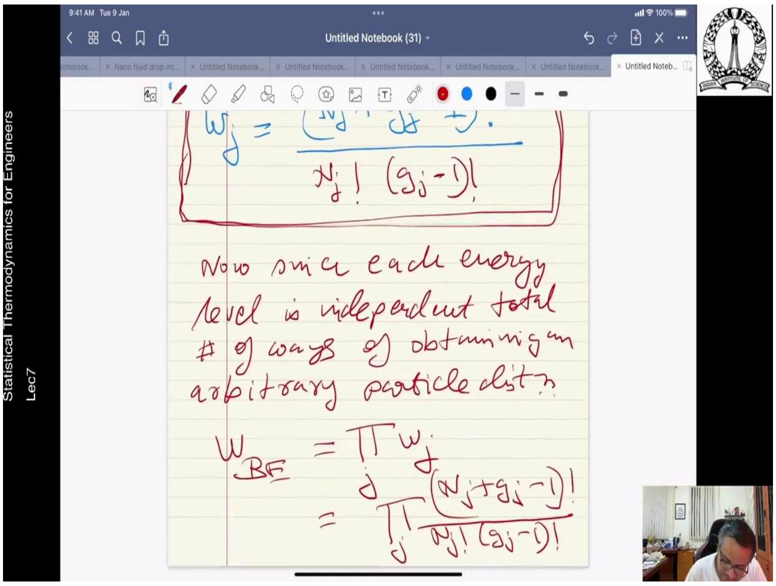
(Refer Slide Time: 08:16)



So, we have to consider that how this distribution is possible. So, this is the problem is therefore, equivalent to Nj identical indistinguishable objects we already have done this objects can be arranged in gj containers it is like what we knew as M containers with with no limit in the number of objects per container.

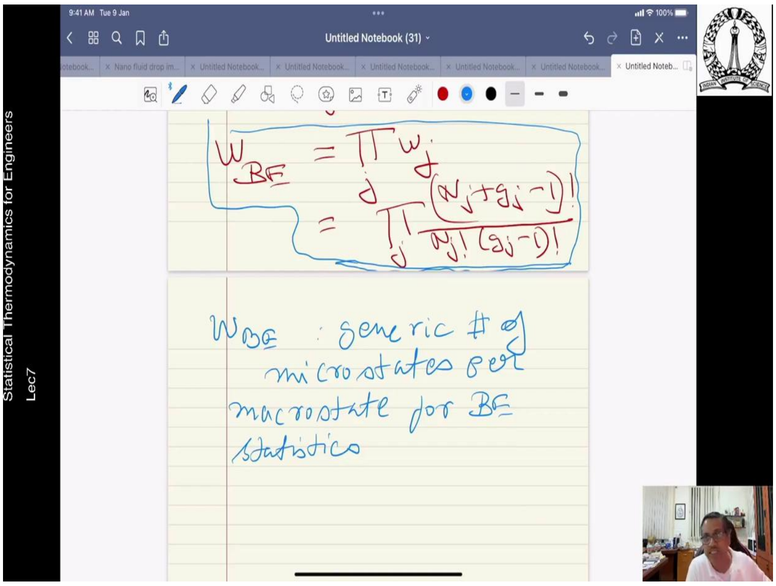
(Refer Slide Time: 09:08)





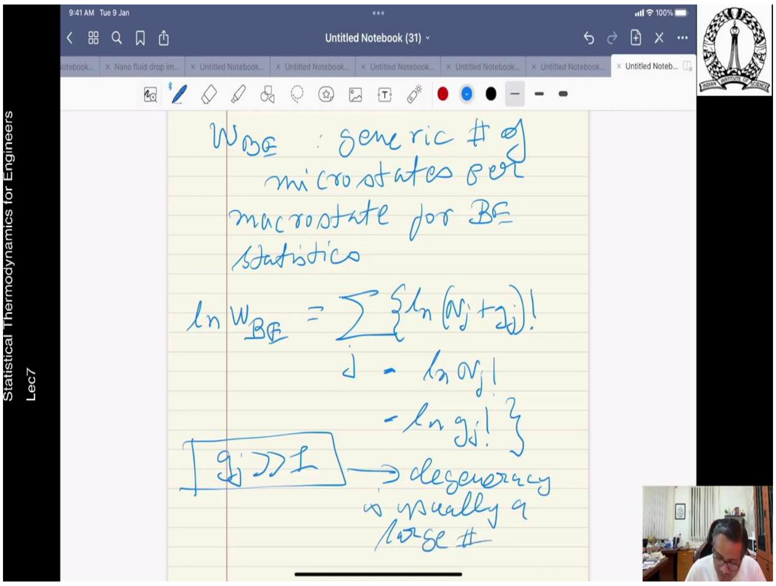
So, therefore, we write it as Wj is equal to Nj plus gj minus 1 factorial divided by Nj factorial gj minus 1. So, these are the number of arrangements that we have. So, since each energy level is independent, so, now, since, each energy level is independent because it does not depend on other one, each energy level is independent. This is important, independent. The total number of ways obtaining an arbitrary particle distribution, write it as WBE as Bose Einstein is basically will be the multiplicative of all the j energy levels that Wj which essentially translates to j, Nj plus gj minus 1 factorial divided by Nj factorial gj minus 1 factorial.

(Refer Slide Time: 10:56)



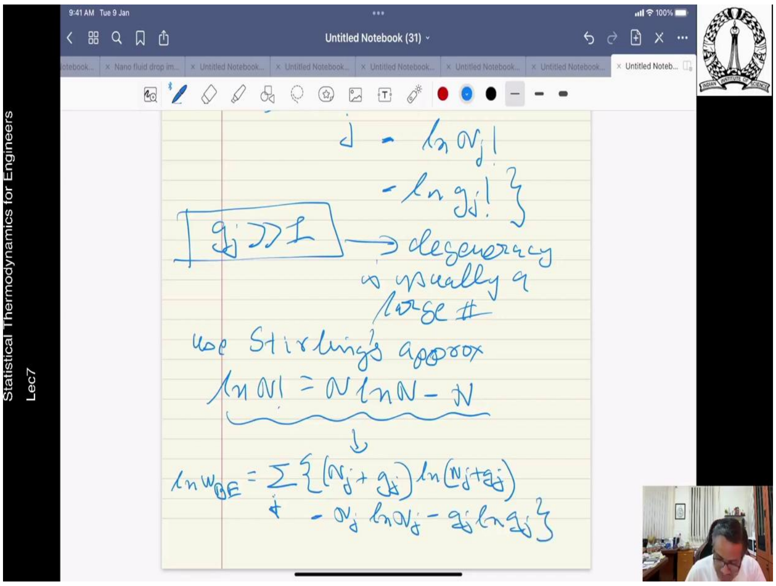
So, this is this is the, this is essentially what the distribution should look like. So, WBE is basically the generic number of microstates per macrostate for BE statistics, the generic number of microstates per macrostate for Bose Einstein statistics, they got the point that these are indistinguishable particles, they are identical and how they can be distributed in gj containers, there is nj number of particles in energy levels. So, what do you do after this?

(Refer Slide Time: 11:58)



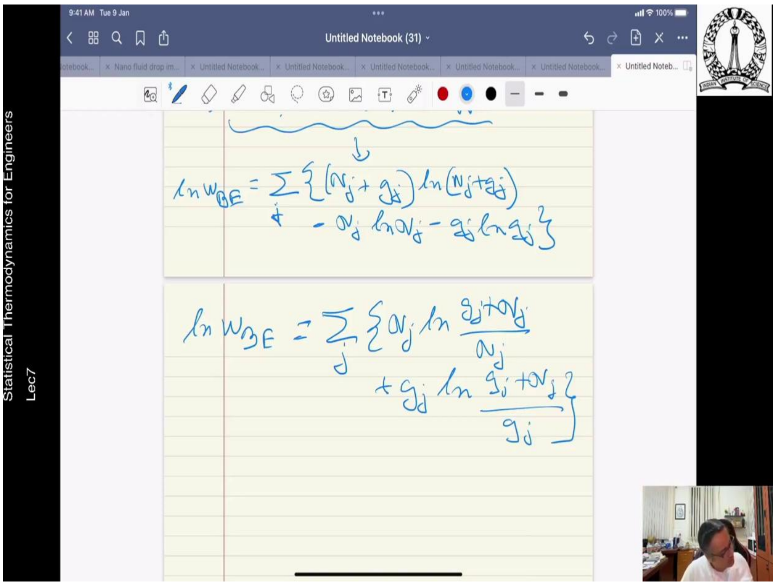
We follow the protocol that means, we take the logarithm of WBE, which therefore, converts this multiplicative sign into an additive sign. So, this becomes ln Nj plus gj factorial. So, these are all the brackets by the way so, and then Nj factorial minus ln gj. So, gj which was greater than 1 because we said that degeneracies usually a large number, degeneracy is usually large number.

(Refer Slide Time: 12:55)



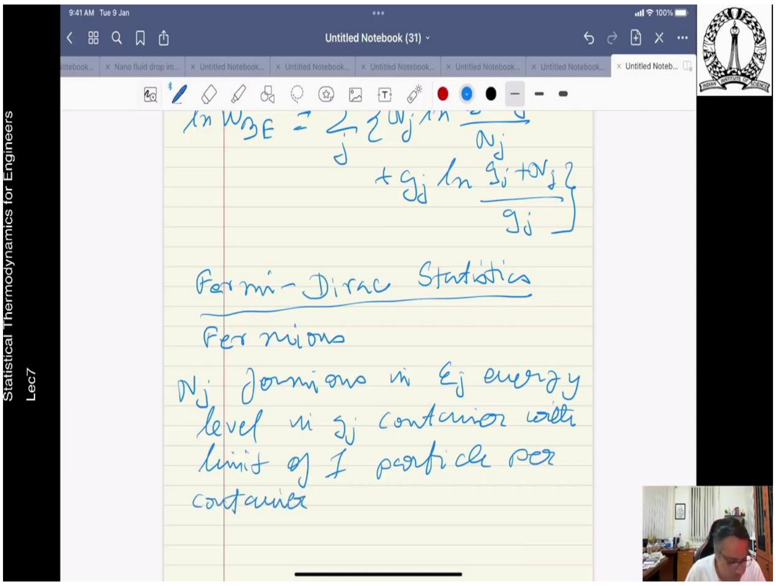
So, use the Stirling's approximation which is basically ln N factorial is equal to N ln N minus N, so this is Stirling's approximation when you apply it to this particular context ln WBE summation j, Nj plus gj, ln Nj plus gj minus Nj, ln Nj minus gj ln gj.

(Refer Slide Time: 13:59)



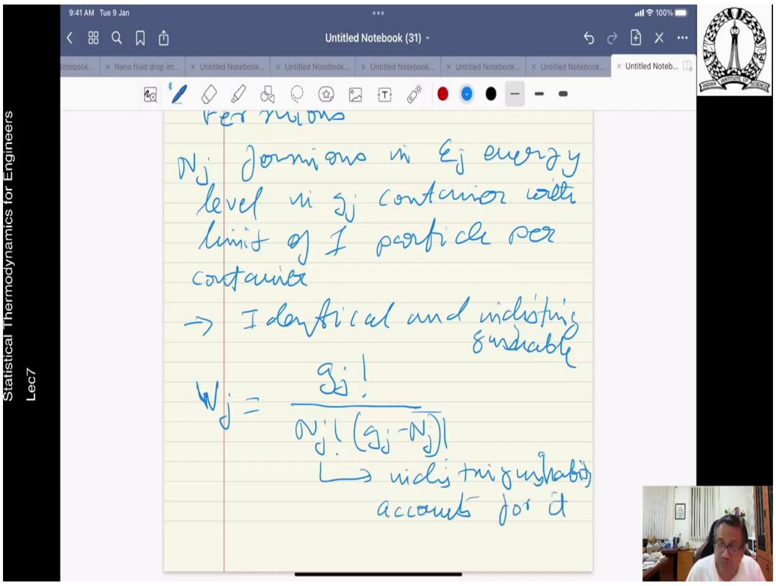
Now, simplifying it a little bit further so, this gives you j Nj gj plus ln gj plus Nj by Nj plus gj ln gj plus Nj by gj. So, this will be your Bose Einstein statistics. So, this up to this particular point worked out because this is what the statistics would look like but we are not done yet.

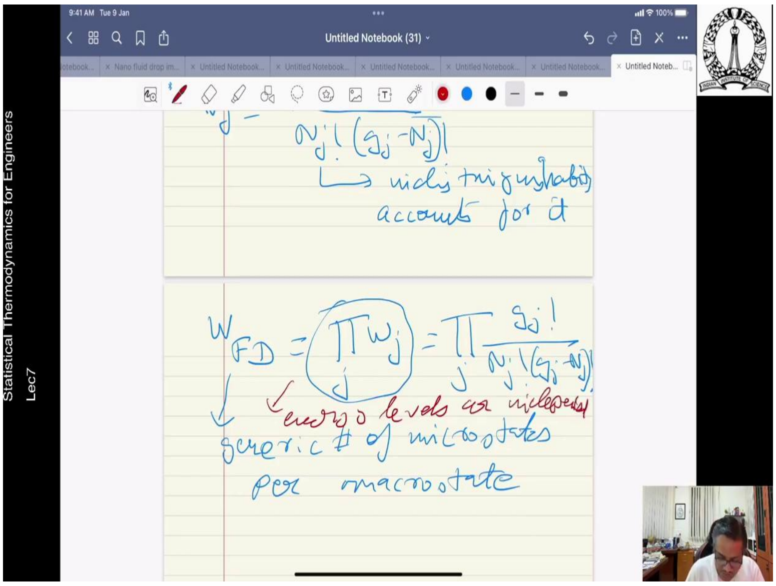
(Refer Slide Time: 14:49)



So, we will do the Fermi Dirac, now take it up to the same level and then we will see most probable distributions as the Fermi Dirac statistics. Next is the Fermi Dirac statistics. So, these are valid for fermions, small fermions, so class of Fermions. So, again the same thing Nj fermions in Ej energy level in gj containers for this time we have a limit with the limit of one particle bar per container.

(Refer Slide Time: 15:39)

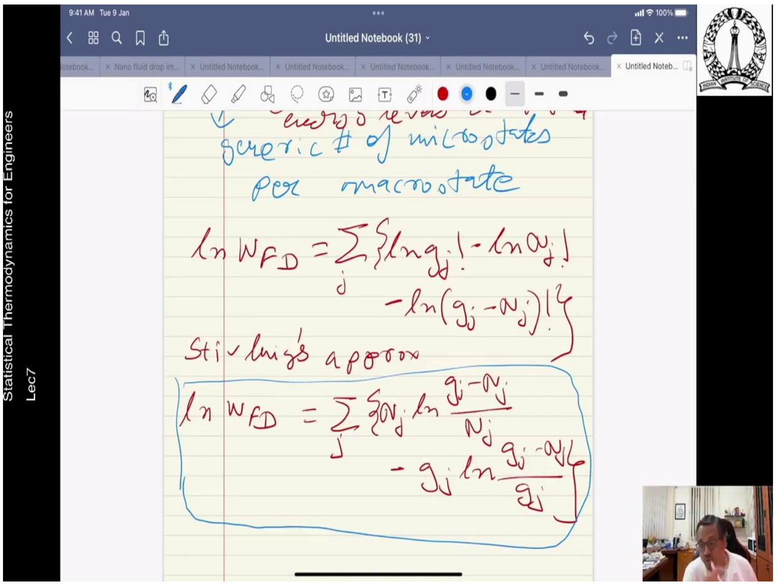




And they are, of course identical and indistinguishable so, Wj is equal to gj factorial by Nj factorial into gj minus Nj factorial obviously already know, this accounts for it, for the indistinguishability accounts for it, this already know as well and so, the Fermi Dirac statistics, therefore (())(16:20) down to the multiplicative of Wj's which essentially translates to this gj factorial divided by Nj factorial gj minus Nj.

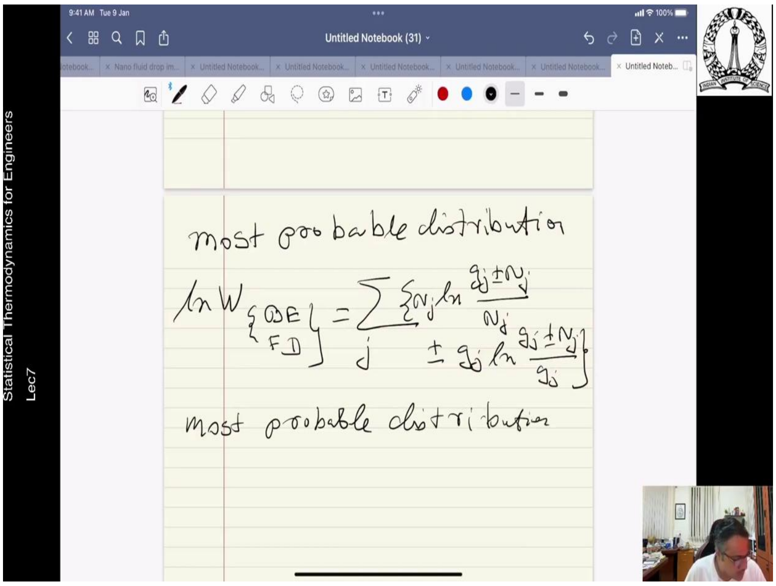
So, this is once again the generic number of microstates per macrostates also the same assumption valid because the energy levels this essentially translates to the energy levels are independent.

(Refer Slide Time: 17:15)



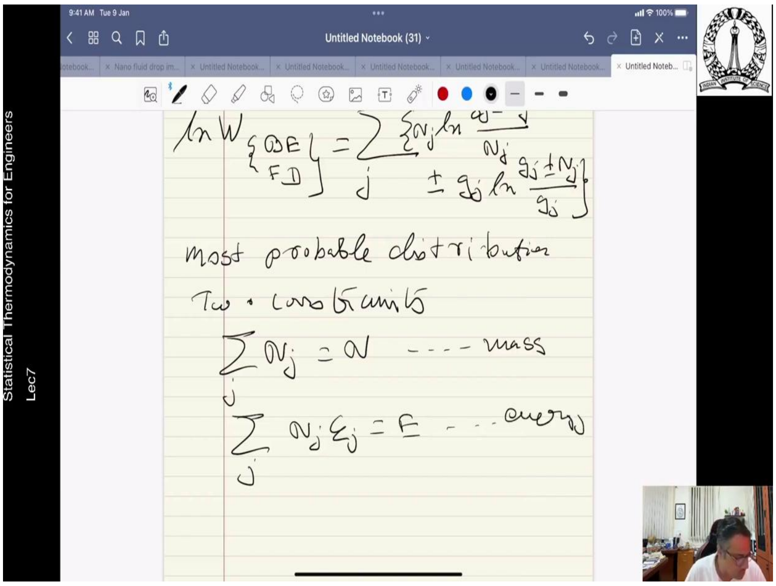
So, now if you cast ln WFD, once again we take the log j, ln gj factorial minus ln Nj factorial minus ln gj minus Nj factorial, so this is the bracket. Again, use Stirling's approximation which essentially translates ln WFD is equal to summation j Nj ln gj minus Nj divided by minus Nj minus gj ln gj minus Nj by gj. So, this will be Fermi Dirac statistics. So, you saw how we use the statistical tools that we developed already generic situations and how it is applicable when you actually do the all side and the Fermi Dirac statistics.

(Refer Slide Time: 18:43)



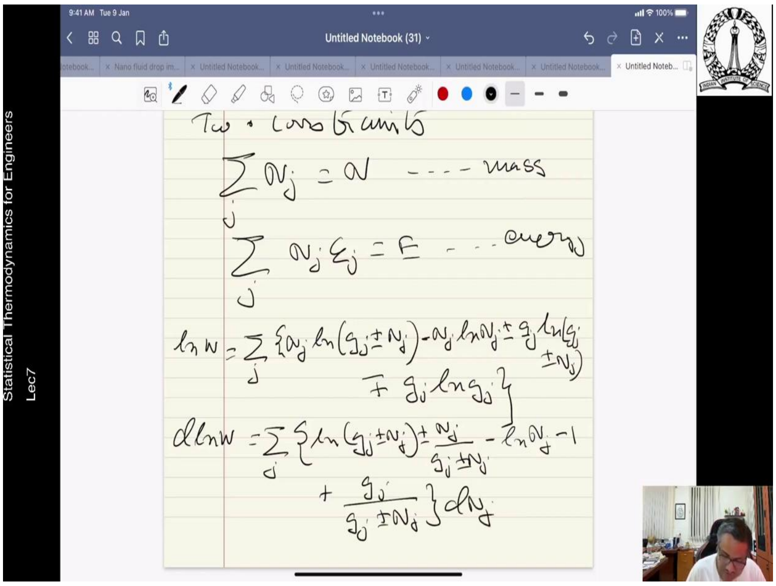
So, now, we choose the most probable distribution that is what you want to find out. So, this ln W, whether it is Bose Einstein or Fermi Dirac, it is like the distributions are like this is Nj ln depends on which distribution you are looking at this plus minus gj ln gj plus minus Nj by gj, so, this is the distribution that you are looking at the most probable distribution in order to find that out, our task is to find out the most probable distribution. So, these are the Fermi Dirac statistics for any arbitrary distribution.

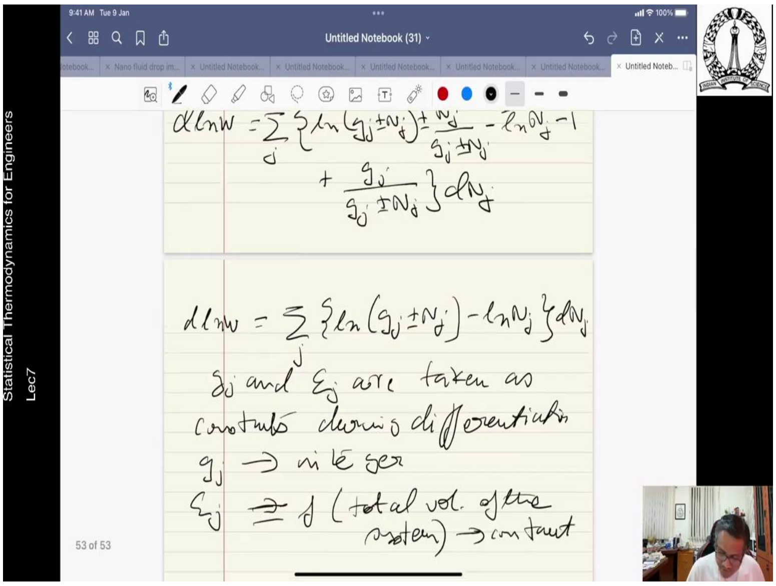
(Refer Slide Time: 19:52)

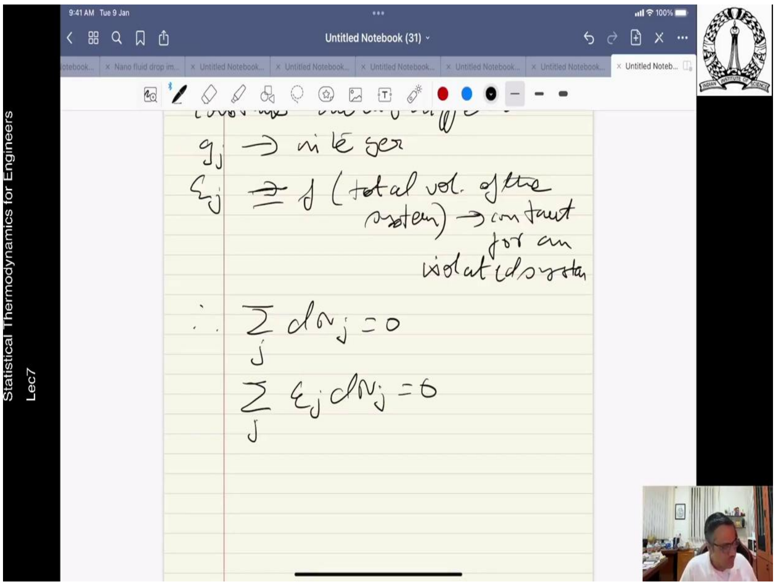


So, for that we need we introduced two constraints. The first one is that summation of Nj is equal to N, which is basically mass observation then you have sum over Nj Ej is equal to E is basically the energy conservation. So, these are both constraints.

(Refer Slide Time: 20:19)



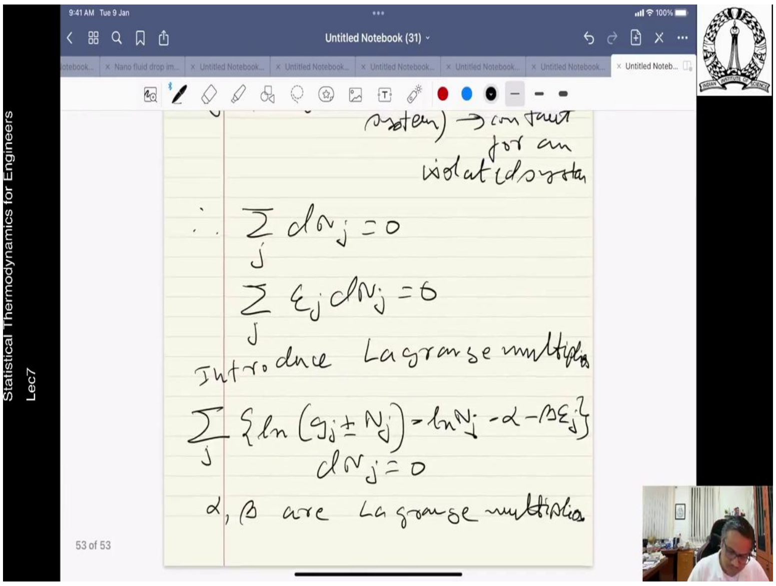




So, we saw that now if we once again ln W is summation Nj ln gj minus plus minus Nj minus Nj ln Nj plus minus gj ln gj minus plus gj ln gj, so this is the expression. What you do is that you differentiate us because you have to find out the most probable distribution. So, once you do that you get ln smaller gj plus minus Nj dNj so, that is what we have done so, if you now do it further ln gj plus minus, you work out the steps, so these are the things that you get. Now, gj and Ej are constants.

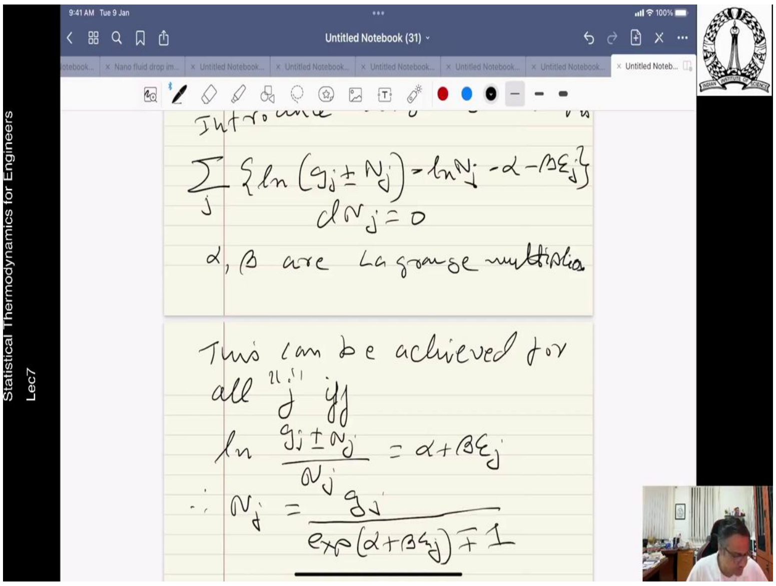
So, we have taken gj and Ej are taken as constants during the differentiation. So, gj is an integer this degeneracy and Ej is of course it is basically a function of the total volume of the system and it is constant for an isolated system. Therefore, summation dNj is equal to 0, summation j Ej dNj equal to 0.

(Refer Slide Time: 23:56)



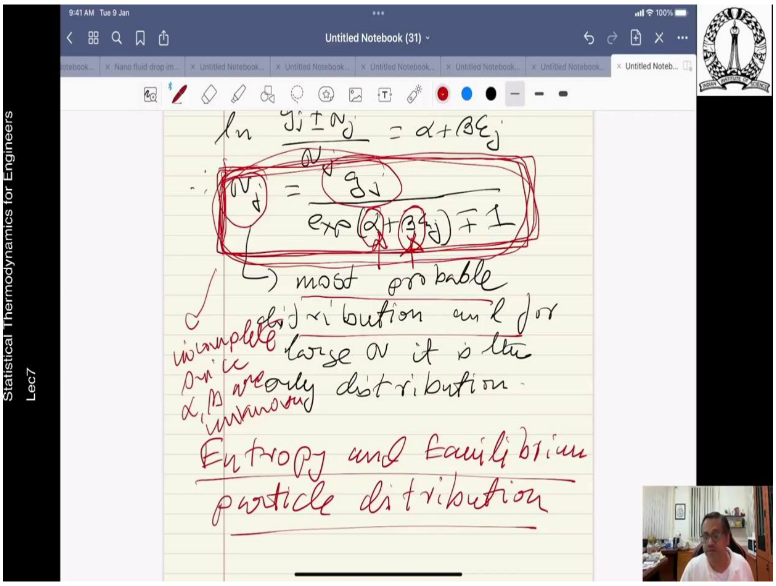
So, now we have the situation and you know so, introducing the concept of Lagrange multipliers now, introduce Lagrange multipliers and what you have is summation j ln gj plus minus Nj minus ln Nj minus alpha minus beta Ej dNj equal to 0. Alpha and beta are Langrage multipliers.

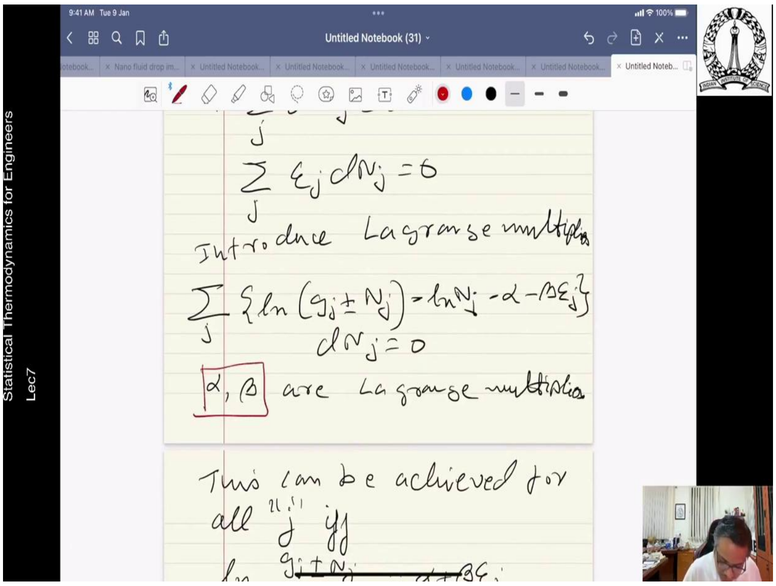
(Refer Slide Time: 24:42)

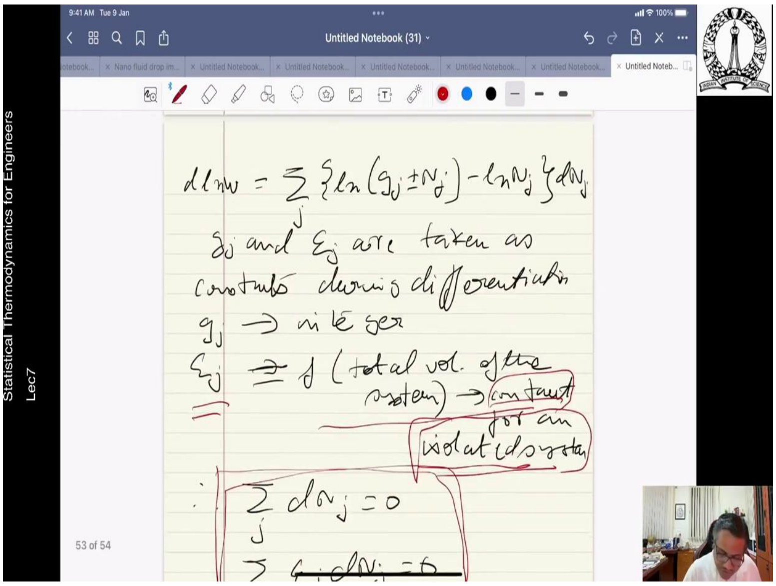


This particular thing can only be achieved for all j and only if ln gj plus minus Nj divided by Nj is equal to alpha plus beta Ej. So, therefore, Nj is equal to gj divided by exponential alpha plus beta gj minus plus 1.

(Refer Slide Time: 25:36)







This will be the most probable distribution and as we saw is only distribution and for large it is for large N, it is the only distribution. So, as you can see now, so, this is depending on which one you are looking at this is the Bose Einstein is the most probable distribution this is how Nj particles are distributed in gj energy level, so, this is this is another interesting thing and just to recap our data Lagrange multipliers, this is valid as you can see why, Eg is basically the function of a system that is constant system and that comes from Formula macroscopic definition and we have done a lot of stuff using Stirling's approximation stuff like that to arrive at this particular session.

So, the next class we are going to look at the next lecture we are going to look at entropy and the equilibrium particle distribution. So, coming up is entropy and equilibrium distribution. So, this is what is coming up next, in the next lecture, which will be lecture 8.

And there we are going to look at how the most probable distribution can be found out, but here in this case you can see we possibly do much more alpha or beta is an unknown. So, therefore, this distribution that whatever I have given you is incomplete. This is incomplete. Since have been, so hence they are hence you cannot really have a fair idea of what is your Nj to be.

So, Nj if it is the most probable distribution, we still need to see what is this alpha and beta all about. Because only then you can find out the most probable distribution and for that we need entropy and we need a few other things, which we are going to cover in the next lecture. So, thank you.